CAPTURE AND RECREATION OF HIGHER ORDER 3D SOUND FIELDS VIA RECIPROCITY

Zhiyun Li, Ramani Duraiswami, Nail A. Gumerov

Perceptual Interfaces and Reality Laboratory, UMIACS
University of Maryland, College Park, MD 20742
zli@cs.umd.edu;{ramani,gumerov}@umiacs.umd.edu

ABSTRACT

We propose a unified and simple approach of capturing and recreating 3D sound fields by exploring the reciprocity principle that is satisfied between the two processes. Our approach makes the system easy to build, and practical. Using this approach, we can capture the 3D sound field by a spherical microphone array and recreate it using a spherical loudspeaker array, and ensure that the recreated sound field matches the recorded field up to a high order of spherical harmonics. A design example and simulation results are presented. For some regular or semi-regular microphone layouts, we design an efficient parallel implementation of the multi-directional spherical beamformer by using the rotational symmetries of the beampattern and of the spherical microphone array. This can be implemented in either software or hardware. A simple design example is presented to demonstrate the idea. It can be easily adapted for other regular or semi-regular layouts of microphones.

1. INTRODUCTION

Technologies for capturing and recreating 3D sound fields have numerous applications in film, music, computer games, virtual reality and telepresence. Recently, researchers also have begun to use these technologies to design 3D auditory user interfaces for the visually impaired [1][2][3][4]. The goal of these systems is the accurate reproduction of the captured sound so that when a listener listens to the recreated sound field she perceives herself to be in the original sound field.

In this paper we propose a new approach for achieving this that ensures that the reconstructed field closely approximates the recorded field to a high order. We first introduce the microphone and loudspeaker array we suggest separately. We then present an overview of the microphone-loudspeaker integrated systems for capturing and recreating the 3D sound field.

This work was partially supported by NSF Award 0205271.

1.1. Previous Work

There are various designs for microphone arrays to capture and analyze sound fields. Geometrically, most such arrays fall into one of three designs: linear, circular and spherical. Perhaps the most straightforward design is the linear microphone array. In [5], general linear sensor arrays for beamforming are elaborated. A more symmetric and compact configuration is the circular microphone array such as the one for speech acquisition in [6]. To capture the 3D sound field, we prefer a 3D symmetric configuration: the spherical microphone array. In [14], the authors captured the sound field using a spherical microphone array in free space. Microphones can also be positioned on the surface of a rigid sphere to make use of the scattering effect. In [13], the beamformer using such configuration has the same shape of beampattern in all directions. In section 2 and 3, we will explore the nice property of this configuration for capturing and recreating sound fields.

Loudspeaker arrays have similar configurations to recreate the sound field. In [9][10], a linear loudspeaker array is designed recreate the sound field on a 2D plane. In [19], a theoretical analysis for using a spherical loudspeaker array to recreate 3D sound field was provided. In section 3, we will apply this to analyze and design our microphone-loudspeaker integrated system.

People have proposed several schemes to build the microphone-loudspeaker array system. In [10], based on the Kirchhoff-Helmholtz integral on a plane, the sound field is captured by a directive microphone array, and recreated by a loudspeaker array. That is called the Wave Field Synthesis (WFS) method. While this system works well in an auditorium environment where the listening area is separated from the primary source area by a plane, it is hard to render an immersive perception in a 3D sound field. In [11], a general framework was proposed, which uses a microphone array beamformer with a localization and tracking system to identify the sound sources, then uses the loudspeaker array to recreate them with the correct spatial cues. To work properly, however, it requires a robust and accurate localiza-

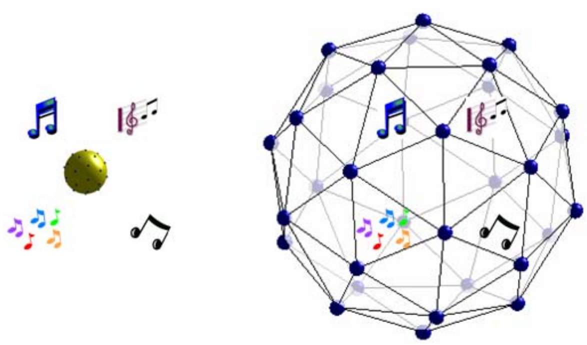

Fig. 1. Left: the microphone array is capturing a 3D sound field remotely. Right: the loudspeaker array is recreating the captured sound field.

tion and tracking system and a highly directive beamformer which are usually expensive, if available.

1.2. Present Work

In this paper, we design an integrated system that records and reproduces the sound scene using a microphone array mounted on a rigid sphere and a spherical loudspeaker array in free space. A scenario is shown in Fig. 1. The main advantages of our system are: (a) reciprocity between capturing and recreating processes makes the system easy to build; (b) the performance is robust and optimal with the given number of microphones and loudspeakers; (c) the capturing part is compact and portable which is convenient to record the immersive 3D sound field; (d) the system is highly scalable.

In addition, for some regular or semi-regular microphone layouts, there exists efficient parallel implementations of the multi-directional spherical beamformer. We will illustrate this design in Section 6.

Our system can be seen as an extension of the Ambisonics system, which only captures and recreates the 3D sound field to the first order of spherical harmonics [7][8].

2. CAPTURING THE 3D SOUND FIELD

We first introduce acoustical scattering off a sphere, and spherical beamforming. We then present a concept of viewing the beamforming as a form of projection which leads to the design of the proposed system.

2.1. Scattering off a sphere

For a unit magnitude plane wave $\mathbf{k}$, incident from direction (θ_k, φ_k), the incident field at a point $(r_s, \theta_s, \varphi_s)$ can be written as

$$\begin{aligned} p_i &= e^{i\mathbf{k}\cdot\mathbf{r}_s} \qquad (1) \\ &= 4\pi \sum_{n=0}^{\infty} i^n j_n(kr_s) \sum_{m=-n}^{n} Y_n^m(\theta_k,\varphi_k) Y_n^{m*}(\theta_s,\varphi_s), \end{aligned}$$

where j_n is nth-order spherical Bessel function, Y_n^m is the spherical harmonics of order n and degree m. At the same point, the field scattered by the rigid sphere of radius a is [12]:

$$p_s = -4\pi \sum_{n=0}^{\infty} i^n \frac{j_n'(ka)}{h_n'(ka)} h_n(kr_s) \qquad (2)$$
$$\times \sum_{m=-n}^{n} Y_n^m(\theta_k,\varphi_k) Y_n^{m*}(\theta_s,\varphi_s).$$

The total field on the surface ($r_s = a$) of the rigid sphere is:

$$\begin{aligned} p_t &= (p_s + p_i)|_{r_s=a} \qquad (3) \\ &= 4\pi \sum_{n=0}^{\infty} i^n b_n(ka) \sum_{m=-n}^{n} Y_n^m(\theta_k,\varphi_k) Y_n^{m*}(\theta_s,\varphi_s), \end{aligned}$$

where

$$b_n(ka) = j_n(ka) - \frac{j_n'(ka)}{h_n'(ka)} h_n(ka), \qquad (4)$$

and h_n is the spherical Hankel function of the first kind.

2.2. Soundfield Decomposition and Beamforming

The basic principle of spherical microphone array beamforming is to make use of the orthonormality of the spherical harmonics to decompose the soundfield arriving at a spherical array. Then the orthogonal components of the soundfield are linearly combined to approximate a desired beampattern.

If we assume that the pressure recorded at each point (θ_s, φ_s) on the surface of the sphere Ω_s, is weighted by

$$W_{n'}^{m'}(\theta_s,\varphi_s,ka) = \frac{Y_{n'}^{m'}(\theta_s,\varphi_s)}{4\pi i^{n'} b_{n'}(ka)}. \qquad (5)$$

Making use of orthonormality of the spherical harmonics:

$$\int_{\Omega_s} Y_n^{m*}(\theta_s,\varphi_s) Y_{n'}^{m'}(\theta_s,\varphi_s) d\Omega_s = \delta_{nn'}\delta_{mm'}, \qquad (6)$$

the total output from a pressure-sensitive spherical surface is:

$$P = \int_{\Omega_s} p_t W_{n'}^{m'}(\theta_s,\varphi_s,ka) d\Omega_s = Y_{n'}^{m'}(\theta_k,\varphi_k). \quad (7)$$

This shows that the gain of the plane wave coming from direction (θ_k, φ_k), for a continuous pressure-sensitive

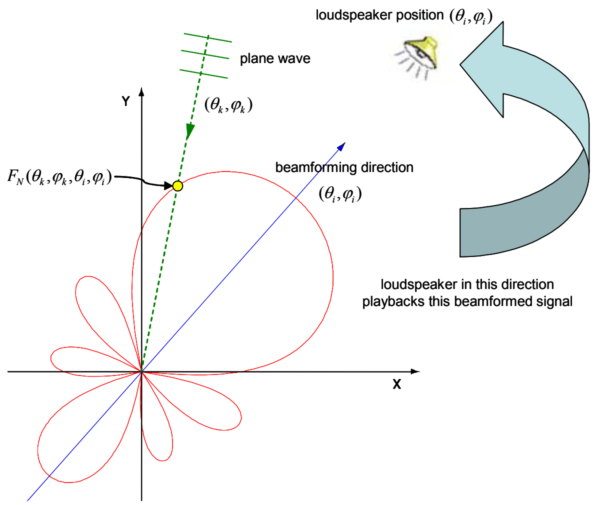

Fig. 2. System workflow: for each chosen direction (θ_i, φ_i), we first beamform the 3D sound field into that direction using our N-order beamformer, then simply playback the resulted signal from the loudspeaker in that direction.

spherical microphone is $Y_{n'}^{m'}(\theta_k, \varphi_k)$. Since an arbitrary real function $F(\theta, \varphi)$ can be expanded in terms of spherical harmonics, we can implement an arbitrary beampattern. For example, an ideal beampattern directed at direction (θ_0, φ_0) is:

$$F_\infty(\theta, \varphi, \theta_0, \varphi_0) = \begin{cases} 1, & (\theta, \varphi) = (\theta_0, \varphi_0) \\ 0, & otherwise \end{cases} . \quad (8)$$

This can be expanded into:

$$2\pi \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^{m*}(\theta_0, \varphi_0) Y_n^m(\theta, \varphi). \quad (9)$$

So from (5) the weight for each point (θ_s, φ_s) for this function is:

$$f(\theta_s, \varphi_s) = \sum_{n=0}^{\infty} \frac{1}{2i^n b_n(ka)} \sum_{m=-n}^{n} Y_n^{m*}(\theta_0, \varphi_0) Y_n^m(\theta_s, \varphi_s). \quad (10)$$

The advantage of this system is that it can be steered into any 3D directions *digitally* with the same beampattern. This is for an ideal continuous microphone array on spherical surface.

2.3. Beamforming as Projection

In a real-world system, we only have a finite number of microphones distributed on the spherical surface discretely. With S microphones the following discrete version of (6) can be satisfied:

$$\frac{4\pi}{S} \sum_{s=1}^{S} Y_n^{m*}(\theta_s, \varphi_s) Y_{n'}^{m'}(\theta_s, \varphi_s) = \delta_{nn'} \delta_{mm'}, \quad (11)$$

where the equation holds to order N. The achieved beampattern of order N is a truncated version of (9):

$$F_N(\theta, \varphi, \theta_0, \varphi_0) = 2\pi \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^{m*}(\theta_0, \varphi_0) Y_n^m(\theta, \varphi). \quad (12)$$

Each beamformed signal not only picks up the sound from the desired direction (θ_0, φ_0), but also from other directions. We assume the 3D sound field is composed of plane waves. The beamforming can be viewed as a *projection* process. This is illustrated in Fig.2, where it is shown in 2D for clarity. In this example, the beamformer ($N = 3$) is pointing to (θ_i, φ_i). We can think of the plane wave from (θ_k, φ_k) as weighted by

$$F_N(\theta_k, \varphi_k, \theta_i, \varphi_i) = 2\pi \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^{m*}(\theta_i, \varphi_i) Y_n^m(\theta_k, \varphi_k) \quad (13)$$

before it is projected into the (θ_i, φ_i) direction. We view this projected plane wave as a *subsource*.

Suppose we beamform the recorded signals in D directions. This creates D subsources for the 3D sound field.

3. RECREATING THE RECORDED 3D SOUND FIELD

As shown in Fig. 2, if the D beamforming directions are well chosen, we can simply playback the D subsources from their respective directions. We wish to recreate the original 3D sound field to order N.

In this section, we first derive a theoretical condition for achieving the recreation. We then find an approximate and optimal solution that provides us a simple and unified way to recreate the 3D sound field from the subsources.

3.1. Theoretical Condition for Recreation

We assume all loudspeakers are positioned in free space and arranged as a spherical array with a radius large enough to produce plane waves at the observation points (this will be relaxed in the next section). Each loudspeaker plays back the identical signal weighted by a complex coefficient $a_l(k)$. The resulted sound field inside this sphere is:

$$p_c = 4\pi \sum_{n=0}^{\infty} i^n j_n(kr_s) \sum_{m=-n}^{n} \sum_{l=1}^{L} a_l(k) Y_n^m(\theta_l, \varphi_l) Y_n^{m*}(\theta_s, \varphi_s),$$

(14)

where $(\theta_l, \varphi_l), l = 1, ..., L$ are the angular positions of L loudspeakers and $(\theta_s, \varphi_s, r_s)$ is the observation point inside the sphere.

To recreate the 3D sound field exactly, we let (1) equal (14):

$$p_i = p_c \tag{15}$$

Let Ω_s be the spherical surface of radius r_s. Here we only require r_s to be less than the radius of the loudspeaker array, not necessarily equal to the radius of the microphone array. We have:

$$\int_{\Omega_s} p_i Y_{n'}^{m'}(\theta_s, \varphi_s) d\Omega_s = \int_{\Omega_s} p_c Y_{n'}^{m'}(\theta_s, \varphi_s) d\Omega_s. \tag{16}$$

Using (6), we get:

$$Y_{n'}^{m'}(\theta_k, \varphi_k) = \sum_{l=1}^{L} a_l(k) Y_{n'}^{m'}(\theta_l, \varphi_l), \tag{17}$$

$$(n' = 0, ..., \infty, \quad m' = -n', ..., n').$$

Please note (17) is independent of the observation point only if it is inside the sphere of the loudspeaker array. To recreate the 3D sound field to order N, we need (adapted from [19]):

$$\mathbf{Pa} = c\mathbf{b} \tag{18}$$

where c is a constant, $\mathbf{a}$ is the vector of unknown weights to be assigned to each loudspeaker;

$$\mathbf{P} = \begin{bmatrix} Y_0^0(\theta_1, \varphi_1) & Y_0^0(\theta_2, \varphi_2) & \cdots & Y_0^0(\theta_L, \varphi_L) \\ Y_1^{-1}(\theta_1, \varphi_1) & Y_1^{-1}(\theta_2, \varphi_2) & \cdots & Y_1^{-1}(\theta_L, \varphi_L) \\ Y_1^0(\theta_1, \varphi_1) & Y_1^0(\theta_2, \varphi_2) & \cdots & Y_1^0(\theta_L, \varphi_L) \\ Y_1^1(\theta_1, \varphi_1) & Y_1^1(\theta_2, \varphi_2) & \cdots & Y_1^1(\theta_L, \varphi_L) \\ \vdots & \vdots & \ddots & \vdots \\ Y_N^{-N}(\theta_1, \varphi_1) & Y_N^{-N}(\theta_2, \varphi_2) & \cdots & Y_N^{-N}(\theta_L, \varphi_L) \\ \vdots & \vdots & \ddots & \vdots \\ Y_N^N(\theta_1, \varphi_1) & Y_N^N(\theta_2, \varphi_2) & \cdots & Y_N^N(\theta_L, \varphi_L) \end{bmatrix}; \tag{19}$$

and

$$\mathbf{b} = \begin{bmatrix} Y_0^0(\theta_k, \varphi_k) \\ Y_1^{-1}(\theta_k, \varphi_k) \\ Y_1^0(\theta_k, \varphi_k) \\ Y_1^1(\theta_k, \varphi_k) \\ \vdots \\ Y_N^{-N}(\theta_k, \varphi_k) \\ \vdots \\ Y_N^N(\theta_k, \varphi_k) \end{bmatrix}. \tag{20}$$

Here (θ_k, φ_k) is the direction from which the original plane wave is incident.

3.2. Reproduction as the Reciprocal of Beamforming

If we choose the beamforming directions (θ_i, φ_i) and loudspeaker angular positions (θ_l, φ_l) to all be the same L angular positions, in such a way that the discrete orthonormality of spherical harmonics can be satisfied to order N, we have:

$$\frac{4\pi}{L} \sum_{i=1}^{L} F_N(\theta_k, \varphi_k, \theta_i, \varphi_i) Y_{n'}^{m'}(\theta_i, \varphi_i) = Y_{n'}^{m'}(\theta_k, \varphi_k). \tag{21}$$

So we have the solution of $\mathbf{a}$ as:

$$\mathbf{a} = \begin{bmatrix} F_N(\theta_k, \varphi_k, \theta_1, \varphi_1) \\ F_N(\theta_k, \varphi_k, \theta_2, \varphi_2) \\ \vdots \\ F_N(\theta_k, \varphi_k, \theta_L, \varphi_L) \end{bmatrix}. \tag{22}$$

This clearly shows that each loudspeaker plays back exactly the beamformed signal with the beampattern pointing to itself as shown in Fig. 2.

4. EXTENSION TO POINT-SOURCE FORM

Up to now, we have assumed the loudspeakers are far enough to generate plane waves. In this section, we will extend the algorithms to the general point-source model.

If the loudspeakers can be modeled as point sources, the soundfield recreated by the spherical loudspeaker array with radius r becomes [18]:

$$p_c = 4\pi \sum_{n=0}^{\infty} i^n j_n(kr_s) R_n(kr) \times \sum_{m=-n}^{n} \sum_{l=1}^{L} a_l(k) Y_n^m(\theta_l, \varphi_l) Y_n^{m*}(\theta_s, \varphi_s), \tag{23}$$

where

$$R_n(kr) = -ikre^{ikr} i^{-n} h_n(kr). \tag{24}$$

The condition for recreation, (17), then becomes:

$$Y_{n'}^{m'}(\theta_k, \varphi_k) = R_{n'}(kr) \sum_{l=1}^{L} a_l(k) Y_{n'}^{m'}(\theta_l, \varphi_l), \tag{25}$$

$$(n' = 0, ..., \infty, \quad m' = -n', ..., n').$$

To have an optimal solution to order N, we have to modify our spherical beamformer. Specifically, we normalize the spherical harmonics expansion of the beampattern (13) to:

$$\tilde{F}_N(\theta_k, \varphi_k, \theta_i, \varphi_i) = 2\pi \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^{m*}(\theta_i, \varphi_i) \frac{Y_n^m(\theta_k, \varphi_k)}{R_n(kr)}.$$

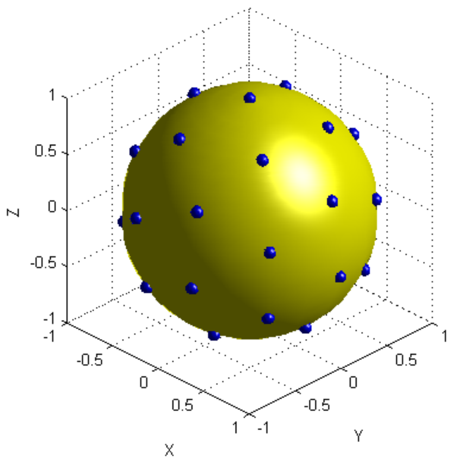

Fig. 3. The layout of 32 microphones on a rigid sphere with radius of $5\,\mathrm{cm}$. (The sphere is lighted to show the 3D effect, NOT the sound scattering.)

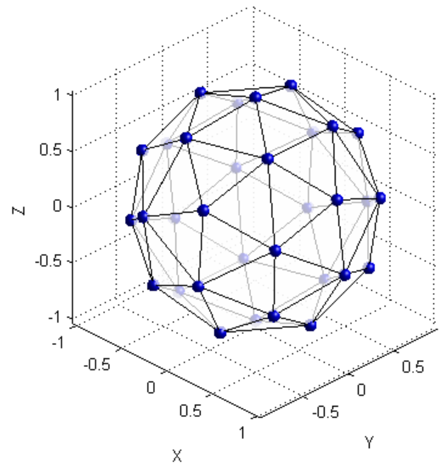

Fig. 4. The layout of 32 loudspeakers in free space arranged as a sphere of large radius.

(26)

So there is still a solution of (25):

$$\mathbf{a} = \begin{bmatrix} \tilde{F}_N(\theta_k, \varphi_k, \theta_1, \varphi_1) \\ \tilde{F}_N(\theta_k, \varphi_k, \theta_2, \varphi_2) \\ \vdots \\ \tilde{F}_N(\theta_k, \varphi_k, \theta_L, \varphi_L) \end{bmatrix}. \tag{27}$$

5. DESIGN EXAMPLE AND SIMULATIONS

To start our design, we need a layout of discrete points on a spherical surface which satisfy (11) to some order. The angular positions specified by the centers of the faces of a truncated icosahedron can satisfy (11) to order 4 according to [13]. Other layouts can be used also such as in [15][16][17]. We just use these 32 angular positions to design our system. Our microphone array is as shown in Fig. 3 where the 32 omnidirectional microphones are positioned on a rigid sphere of radius $5\,\mathrm{cm}$. Our loudspeaker array is as shown in Fig. 4 with the same 32 angular positions. We capture the 3D sound field using the microphone array, and then use the (normalized) spherical beamformer to steer the signals to these 32 directions. Finally, we recreate the 3D sound field using the loudspeaker array.

We simulate our system by a plane wave of $4\mathrm{kHz}$ incident from the positive direction of X-axis in Fig. 3. Fig. 5 shows the scattered sound field on the equator of the spherical microphone array. Fig. 6 is the recreated plane wave to order 4 on the $z = 0$ plane.

Please note that a spherical microphone array can also be rotated arbitrarily without affecting the results. Because of the reciprocity between the beamformer and the loudspeaker arrays, the beamforming directions and the loudspeaker arrays must have the same angular positions. However, there may exist efficient parallel implementation of the multi-directional beamformer if the microphone array locations can be made consistent with the beamforming directions in some way. We will illustrate this in Section 6.

When we only decompose the 3D sound field to the first order of spherical harmonics, our system mimics the Ambisonics system by pointing the bidirectional beampattern (the first order spherical harmonics) to the left-right, front-back, and up-down along with the omnidirectional beampattern (the zeroth order spherical harmonics) [7][8].

Like most microphone arrays that sample the 3D space discretely, the spherical microphone array has a spatial aliasing problem [13]. Low frequency requires larger sphere to decompose higher orders of spherical harmonics while high frequency requires denser microphones to avoid spatial aliasing. To broaden the frequency band, we may need a larger sphere with more microphones. Another solutions may be to use nested spherical microphone arrays, or multiple arrays.

6. EFFICIENT MULTI-DIRECTIONAL BEAMFORMER

If the layout of microphones is regular or semi-regular, we can make use of rotational symmetries of the microphone array and of the beampattern to design an efficient parallel multi-directional spherical beamformer. For specificity, we consider a regular icosahedron to illustrate the idea. It

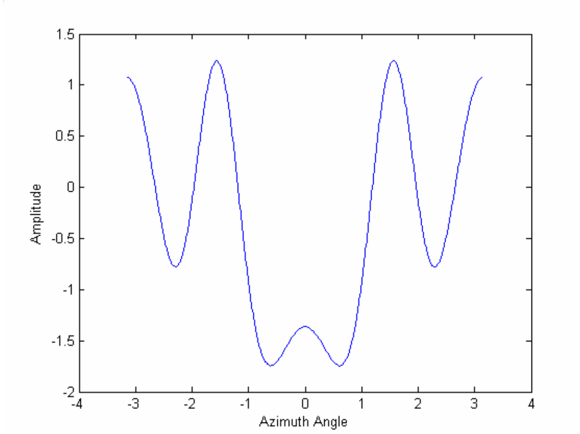

Fig. 5. The plane wave of $4\,\mathrm{kHz}$ incident from positive direction of X-axis scattered by the microphone array. Plot shows the pressure on the equator.

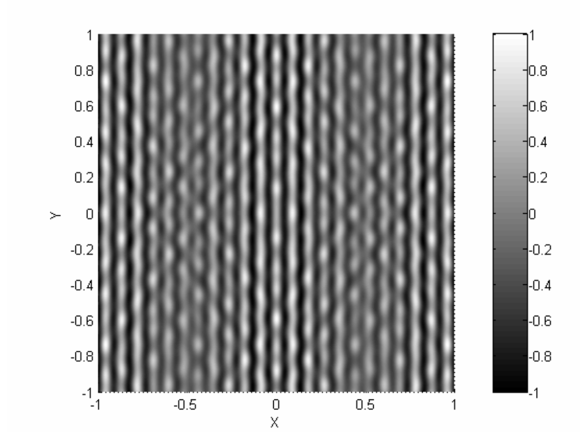

Fig. 6. The recreated plane wave to order 4. Plot shows the $2 \times 2\,\mathrm{m}^2$ area on the $z = 0$ plane.

can be easily adapted to other regular or semi-regular microphone layouts.

A regular icosahedron, as shown in Fig. 7, is rotationally symmetric around any line connecting the origin and an arbitrary node. If the symmetric axis is as shown in the figure, there are two groups of nodes $\{2, 3, 4, 5, 6\}$ and $\{7, 8, 9, 10, 11\}$ on two different cones along with the two endpoints $\{1\}$ and $\{12\}$ of the line.

The rotational symmetry of the beampattern is formulated as the spherical harmonic addition theorem. Let γ be the angle between the two spherical coordinates (θ_0, φ_0) and (θ, φ). We have:

$$\sin\theta \sin\theta_0 \cos(\varphi - \varphi_0) + \cos\theta \cos\theta_0 = \cos\gamma, \quad (28)$$

which defines a cone with respect to (θ, φ). According to the spherical harmonic addition theorem, we have:

$$\sum_{m=-n}^{n} Y_n^{m*}(\theta_0, \varphi_0) Y_n^m(\theta, \varphi) = \frac{2n+1}{4\pi} P_n(\cos\gamma), \quad (29)$$

where $P_n(x)$ is the Legendre polynomial of degree n. So the value of the beampattern on the cone is defined by (28) as a function of γ:

$$2\pi \sum_{n=0}^{N} \sum_{m=-n}^{n} Y_n^{m*}(\theta_0, \varphi_0) Y_n^m(\theta, \varphi) = \sum_{n=0}^{N} \frac{2n+1}{2} P_n(\cos\gamma). \quad (30)$$

From (10), the weight for each microphone at (θ_s, φ_s) is:

$$f(\theta_s, \varphi_s) = f(\gamma_j) = \sum_{n=0}^{N} \frac{2n+1}{8\pi i^n b_n(ka)} P_n(\cos\gamma_j), \quad (31)$$

$$(s = 1, ...12; \;\; j = 1, 2, 3, 4).$$

For each beamforming direction specified by the spherical coordinates of the icosahedron's nodes, there are only four γ values for the icosahedron layout: $\{0, 1.1071, \pi - 1.1071, \pi\}$. For example, if the beampattern is pointing to node 1, node 1 has $\gamma_1 = 0$ and nodes 2, 3, 4, 5 and 6 have $\gamma_2 = 1.1071$, etc.; if the beampattern is pointing to node 2, node 2 has $\gamma_1 = 0$ and nodes 1, 3, 6, 7 and 8 have $\gamma_2 = 1.1071$, etc.. Thus, to build a twelve-directional beamformer, for each microphone, we only need to make four multiplications corresponding to four γ values instead of twelve. The structure of the multi-directional spherical beamformer is shown in Fig. 8.

7. CONCLUSION

We explored the reciprocity between capturing and recreating sound and used it to propose a simple and unified way to capture and recreate a 3D sound field to higher orders of spherical harmonics. A design example and simulation results are presented to demonstrate the effectiveness of our system. For regular or semi-regular microphone layouts, we designed an efficient parallel implementation of the multi-directional spherical beamformer which is the key signal processing unit in the whole system.

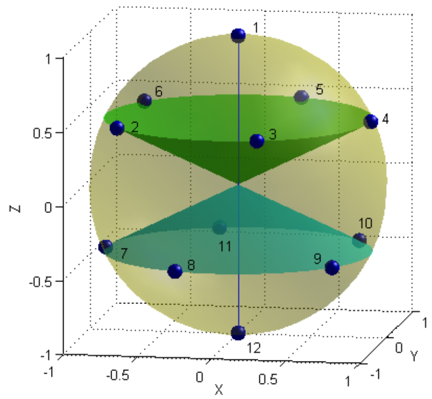

Fig. 7. The rotational symmetry of icosahedron.

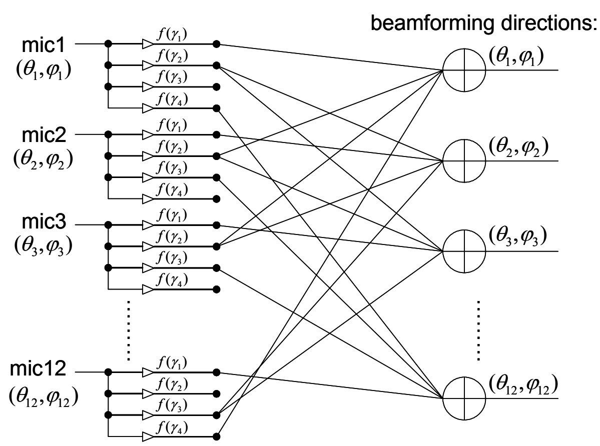

Fig. 8. The efficient structure of the multi-directional spherical beamformer.

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