Department of Computer Science
UNIVERSITY OF COLORADO **BOULDER**

Reinforcement Learning

Machine Learning: Jordan Boyd-Graber
University of Colorado Boulder
LECTURE 25

Slides adapted from Tom Mitchell and Peter Abeel

Reinforcement Learning

- Control learning
- Control policies that choose optimal actions
- Q learning
- Feature-based representations
- Policy Search

Plan

Control Learning

Q-Learning

Policy Search

Control Learning

Consider learning to choose actions, e.g.,

- Roomba learning to dock on battery charger
- Learning to choose actions to optimize factory output
- Learning to play Backgammon

Note several problem characteristics:

- Delayed reward
- Opportunity for active exploration
- Possibility that state only partially observable
- Possible need to learn multiple tasks with same sensors/effectors

One Example: TD-Gammon

[Tesauro, 1995]

Learn to play Backgammon

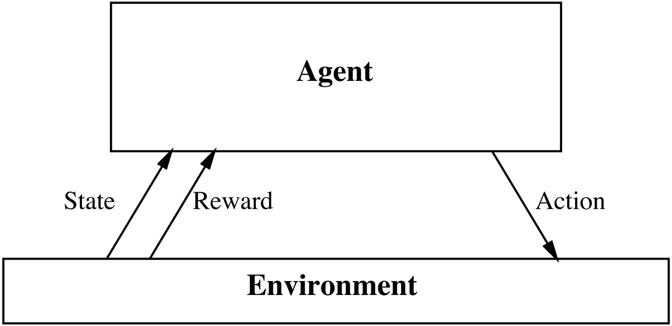
Immediate reward

- $+100$ if win
- -100 if lose
- 0 for all other states

Trained by playing 1.5 million games against itself

Now approximately equal to best human player

Reinforcement Learning Problem



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} \dots$$

Markov Decision Processes

Assume

- finite set of states S
- set of actions A
- at each discrete time agent observes state $s_t \in S$ and chooses action $a_t \in A$
- then receives immediate reward r_t
- and state changes to s_{t+1}
- Markov assumption: $s_{t+1} = \delta(s_t, a_t)$ and $r_t = r(s_t, a_t)$
  - i.e., r_t and s_{t+1} depend only on *current* state and action
  - functions δ and r may be nondeterministic
  - functions δ and r not necessarily known to agent

Agent's Learning Task

Execute actions in environment, observe results, and

- learn action policy $\pi : S \to A$ that maximizes

$$\mathbb{E}\left[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots\right]$$

  from any starting state in S
- here $0 \leq \gamma < 1$ is the discount factor for future rewards

Note something new:

- Target function is $\pi : S \to A$
- but we have no training examples of form $\langle s, a \rangle$
- training examples are of form $\langle \langle s, a \rangle, r \rangle$

Plan

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Policy Search

Value Function

To begin, consider deterministic worlds ...
For each possible policy π the agent might adopt, we can define an evaluation function over states

$$
\begin{aligned}
V^{\pi}(s) &\equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots \\
&\equiv \sum_{i=0}^{\infty} \gamma^i r_{t+i}
\end{aligned}
$$

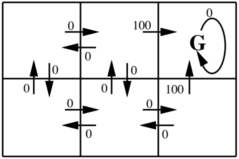
where $r_t, r_{t+1}, \ldots$ are from following policy π starting at state s

Q-learning

Restated, the task is to learn the optimal policy π^*

$$\pi^* \equiv \arg\max_{\pi} V^{\pi}(s), (\forall s)$$

- $r(s, a)$ (immediate reward) values



- $Q(s, a)$ values
- One optimal policy

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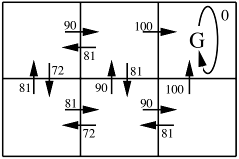

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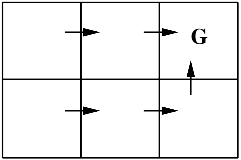


What to Learn

We might try to have agent learn the evaluation function V^{π^*} (which we write as V^*)
It could then do a lookahead search to choose best action from any state s because

$$\pi^*(s) = \arg\max_a [r(s,a) + \gamma V^*(\delta(s,a))]$$

A problem:

- This works well if agent knows $\delta : S \times A \rightarrow S$, and $r : S \times A \rightarrow \Re$
- But when it doesn't, it can't choose actions this way

Q Function

Define new function very similar to V^*

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$

If agent learns Q, it can choose optimal action even without knowing δ!

$$\pi^*(s) = \arg\max_a [r(s,a) + \gamma V^*(\delta(s,a))]$$

$$\pi^*(s) = \arg\max_a Q(s,a)$$

Q is the evaluation function the agent will learn

Training Rule to Learn Q

Note Q and V^* closely related:

$$V^*(s) = \max_{a'} Q(s, a')$$

Which allows us to write Q recursively as

$$\begin{aligned} Q(s_t, a_t) &= r(s_t, a_t) + \gamma V^*(\delta(s_t, a_t))) \\ &= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a') \end{aligned}$$

Nice! Let $\hat{Q}$ denote learner's current approximation to Q. Consider training rule

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

where s' is the state resulting from applying action a in state s

Q Learning for Deterministic Worlds

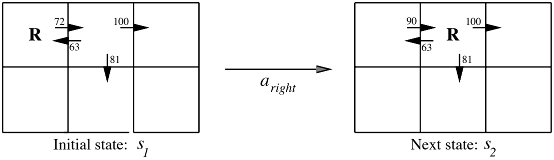
For each s, a initialize table entry $\hat{Q}(s, a) \leftarrow 0$
Observe current state s
Do forever:

- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $\hat{Q}(s, a)$ as follows:

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

- $s \leftarrow s'$

Updating $\hat{Q}$



$$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a')$$
$$\leftarrow 0 + 0.9 \ \max\{63, 81, 100\} = 90$$

if rewards non-negative, then

$$(\forall s, a, n) \ \ \hat{Q}_{n+1}(s, a) \geq \hat{Q}_n(s, a)$$

and

$$(\forall s, a, n) \ \ 0 \leq \hat{Q}_n(s, a) \leq Q(s, a)$$

$\hat{Q}$ converges to Q.

Nondeterministic Case

What if reward and next state are non-deterministic?
We redefine V, Q by taking expected values

$$
\begin{aligned}
V^{\pi}(s) &\equiv E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots] \\
&\equiv E[\sum_{i=0}^{\infty} \gamma^i r_{t+i}]
\end{aligned}
$$

$$Q(s,a) \equiv E[r(s,a) + \gamma V^*(\delta(s,a))]$$

Nondeterministic Case

Q learning generalizes to nondeterministic worlds
Alter training rule to

$$\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n[r + \max_{a'} \hat{Q}_{n-1}(s',a')]$$

where

$$\alpha_n = \frac{1}{1 + visits_n(s,a)}$$

Can still prove convergence of $\hat{Q}$ to Q [Watkins and Dayan, 1992]

Temporal Difference Learning

Q learning: reduce discrepancy between successive Q estimates
One step time difference:

$$Q^{(1)}(s_t, a_t) \equiv r_t + \gamma \max_a \hat{Q}(s_{t+1}, a)$$

Why not two steps?

$$Q^{(2)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + \gamma^2 \max_a \hat{Q}(s_{t+2}, a)$$

Or n?

$$Q^{(n)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + \cdots + \gamma^{(n-1)} r_{t+n-1} + \gamma^n \max_a \hat{Q}(s_{t+n}, a)$$

Blend all of these:

$$Q^{\lambda}(s_t, a_t) \equiv (1-\lambda) \left[Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \cdots \right]$$

Temporal Difference Learning

$$Q^{\lambda}(s_t, a_t) \equiv (1-\lambda)\left[Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \cdots\right]$$

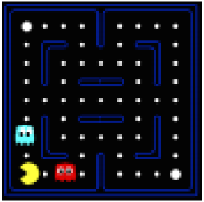
Equivalent expression:

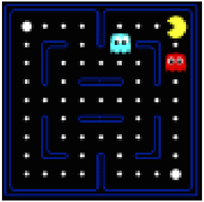
$$\begin{aligned} Q^{\lambda}(s_t, a_t) \quad &= r_t + \gamma[\quad (1-\lambda) \max_a \hat{Q}(s_t, a_t) \\ &\qquad\qquad + \lambda \; Q^{\lambda}(s_{t+1}, a_{t+1})] \end{aligned}$$

TD(λ) algorithm uses above training rule

- Sometimes converges faster than Q learning
- converges for learning V^* for any $0 \leq \lambda \leq 1$ (Dayan, 1992)
- Tesauro's TD-Gammon uses this algorithm

What if the number of states is huge and/or structured?





- Let's say we discover that state is bad
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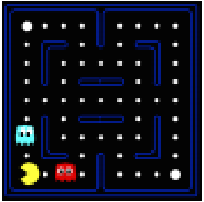

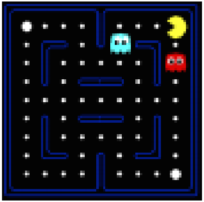

- Let's say we discover that state is bad
- In Q learning, we know nothing about similar states
- Solution: Feature-based Representation
  - Distance to closest ghost
  - Distance to closest dot
  - Number of ghosts
  - Is Pacman in a tunnel?

Function Approximation

- $Q(s, a) \approx w_1 f_1(s, a) + \dots$
- Q-learning now had perceptron style updates (least squares regression)

Plan

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Policy Search

Policy Search

- Problem: often feature-based policies that work well aren't those that approximate V/Q best
- Solution: Find policies that maximize rewards rather than the value that predicts rewards
- Successful

Example: Imitation Learning

- Take examples of experts $\{(s_1, a_1)\dots\}$
- Learn a classifier mapping $s \rightarrow a$
- Create loss as the negative reward

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- Create loss as the negative reward
- Problems?

How do we find a good policy?

- Find optimal policies through dynamic programming $\pi_0 \equiv \pi*$
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SEARN: Searching to Learn (Daumé & Marcu, 2006)

Applications of Imitation Learning

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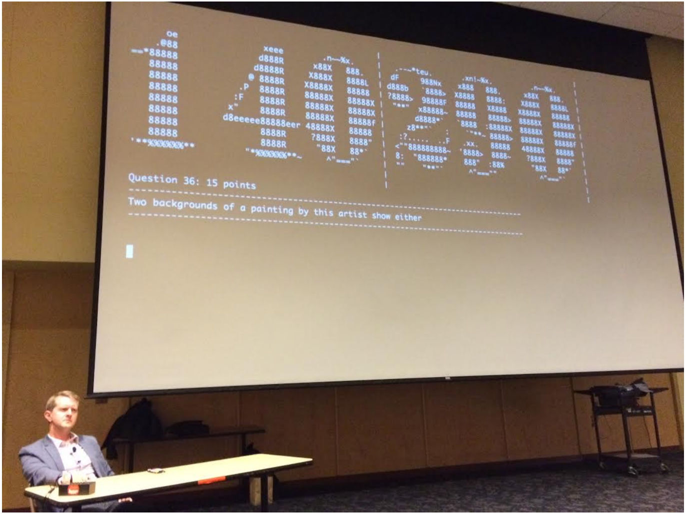
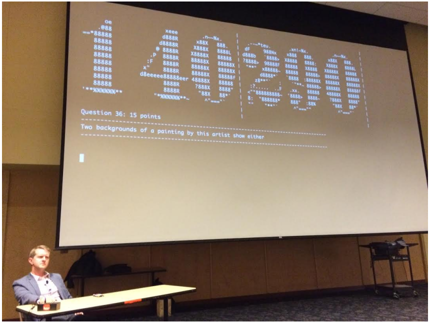
Question Answering



Question Answering

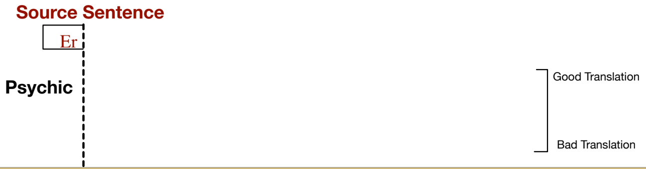


- **State**: The words seen, opponent
- **Action**: Buzz or wait
- **Reward**: Points

Why machine translation really hard is

- **State**: The words you've seen, output of machine translation system
- **Action**: Take translation, predict the verb
- **Reward**: Translation quality

Comparing Policies



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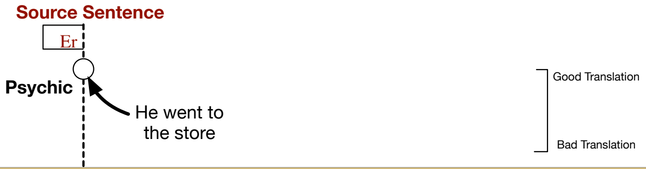

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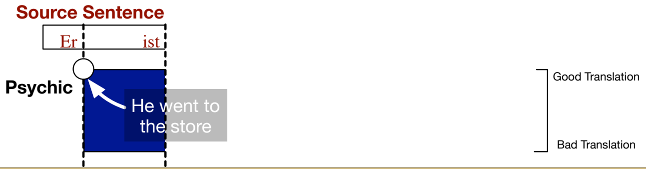

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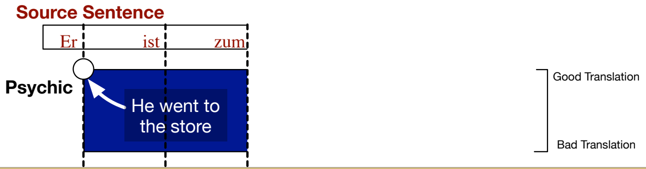

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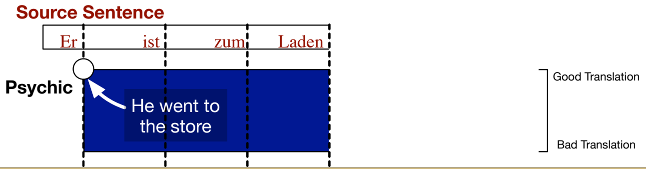

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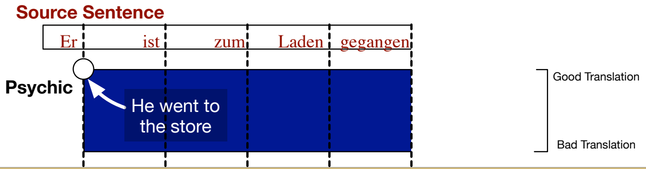

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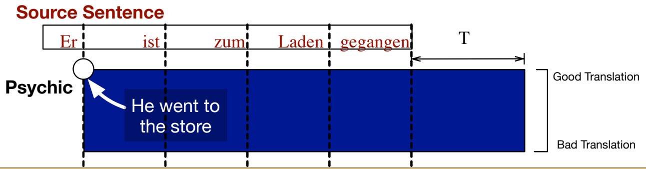

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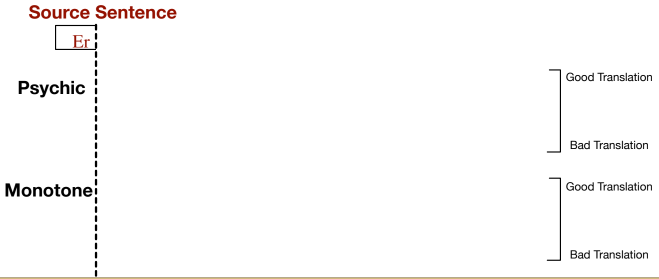

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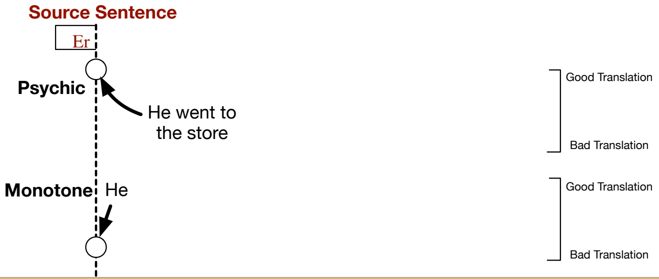

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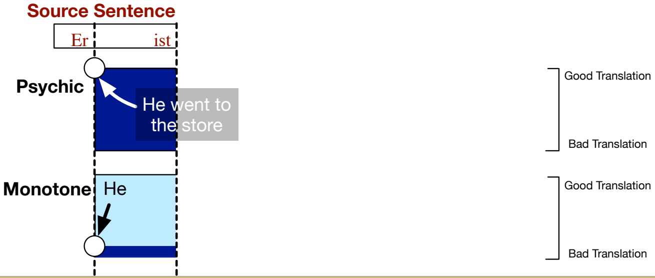

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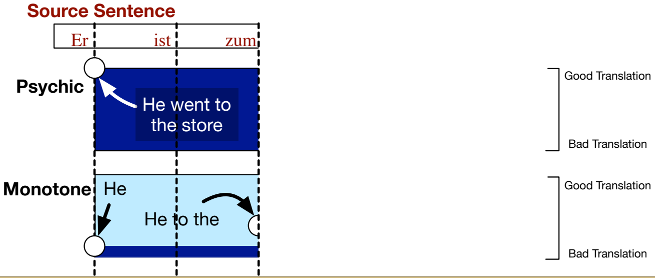

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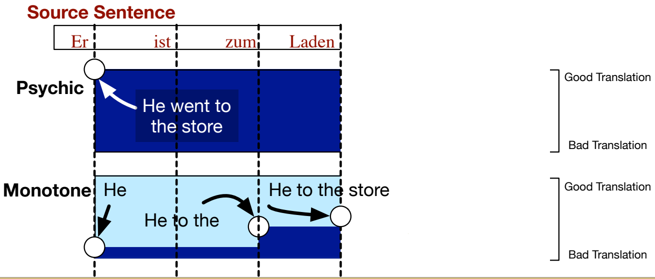

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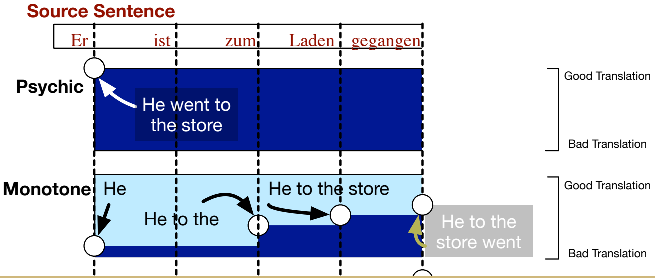

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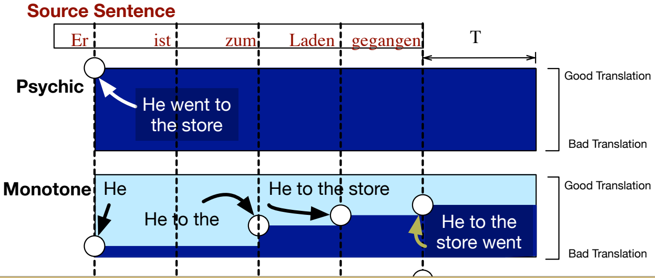

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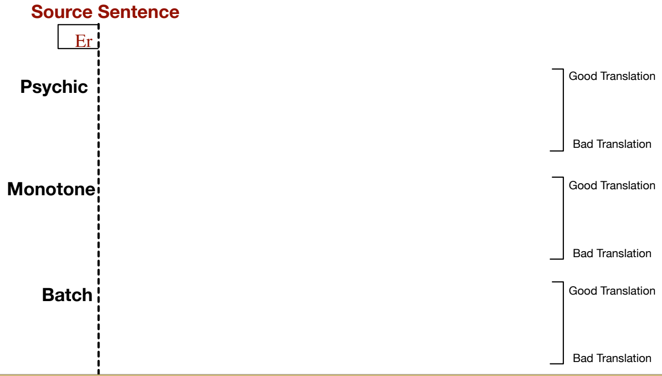

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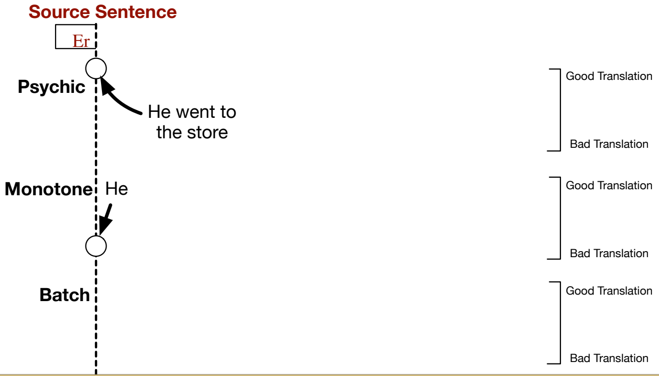

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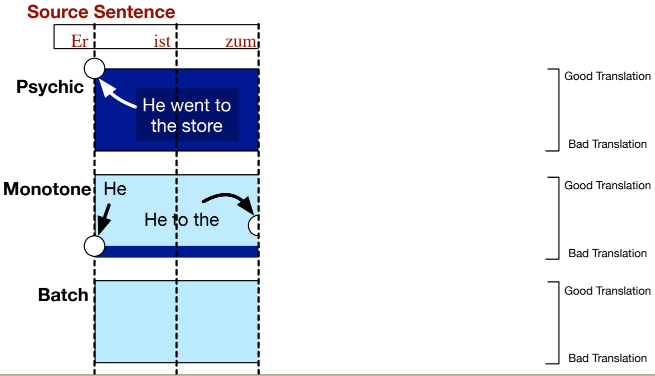

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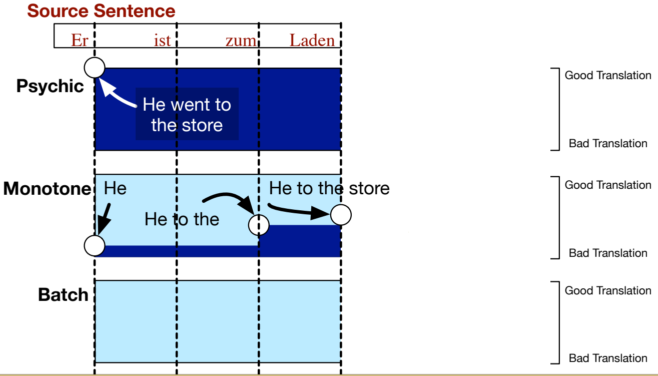

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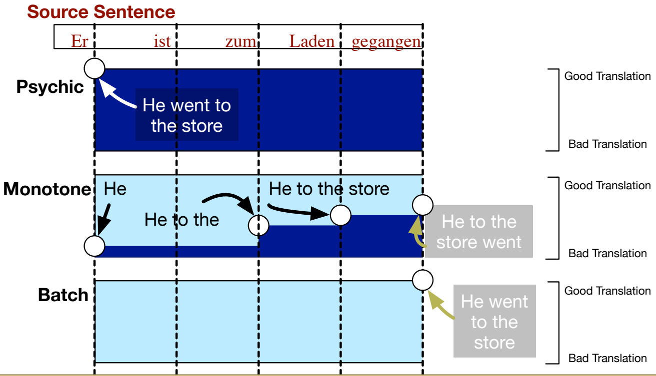

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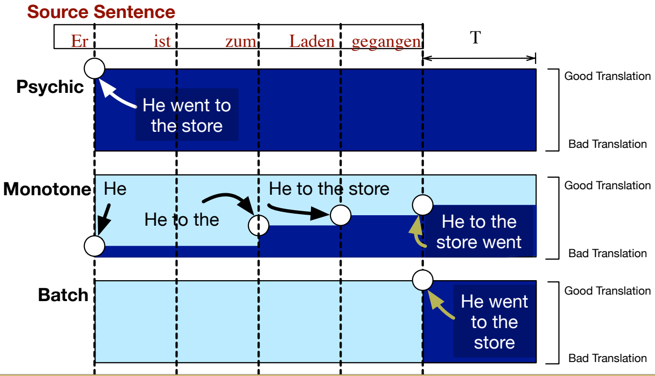

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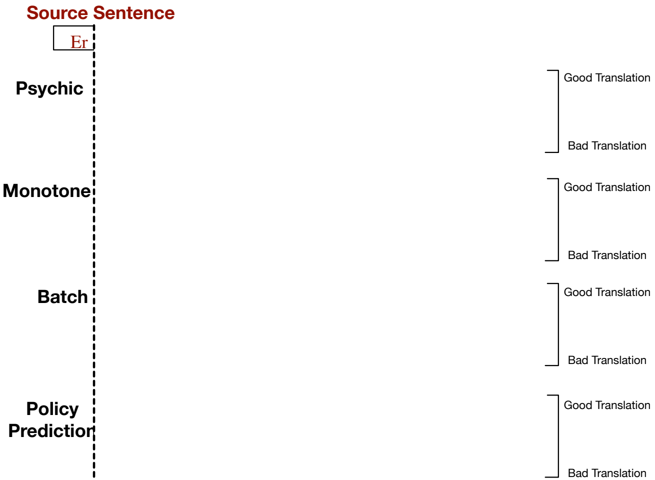

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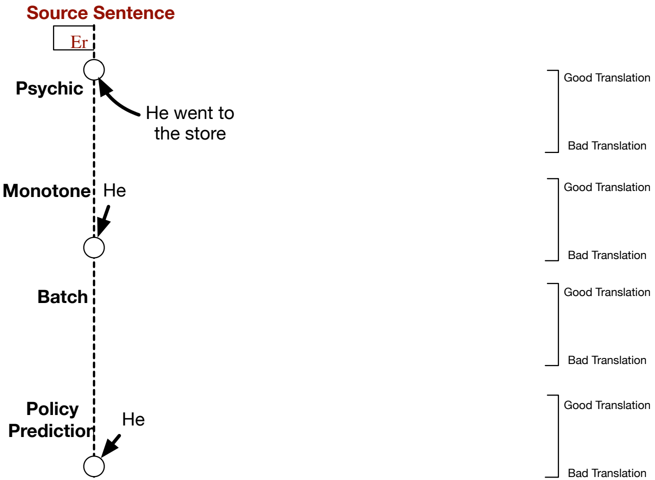

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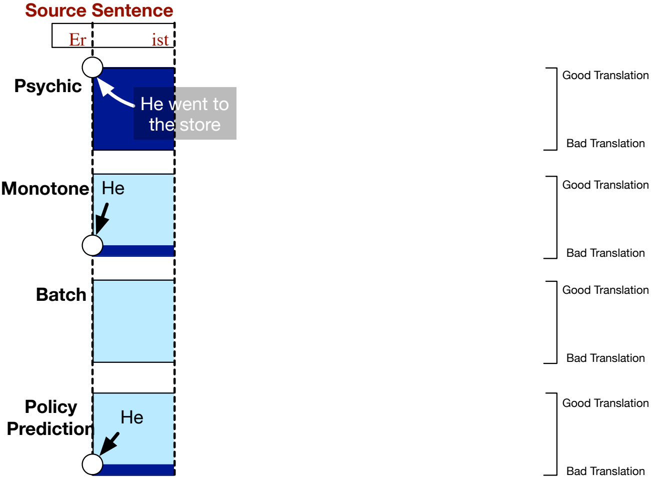

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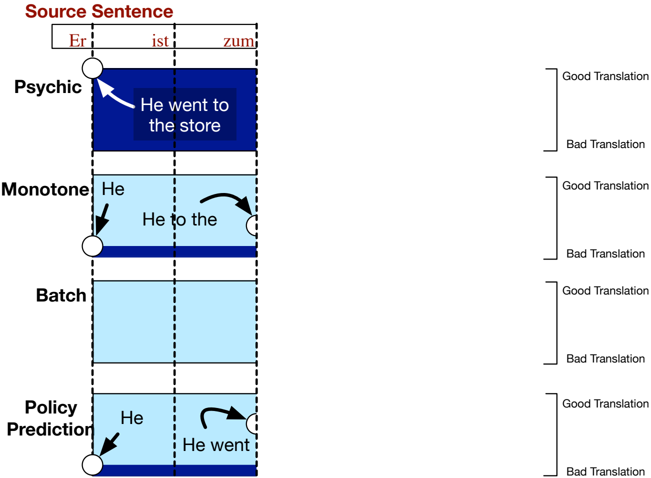

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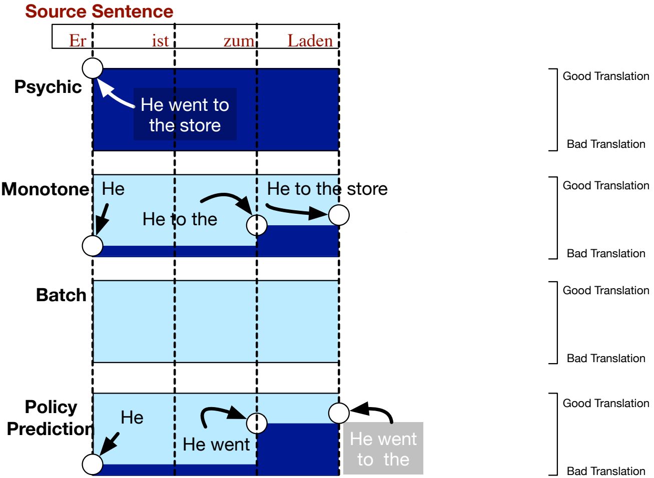

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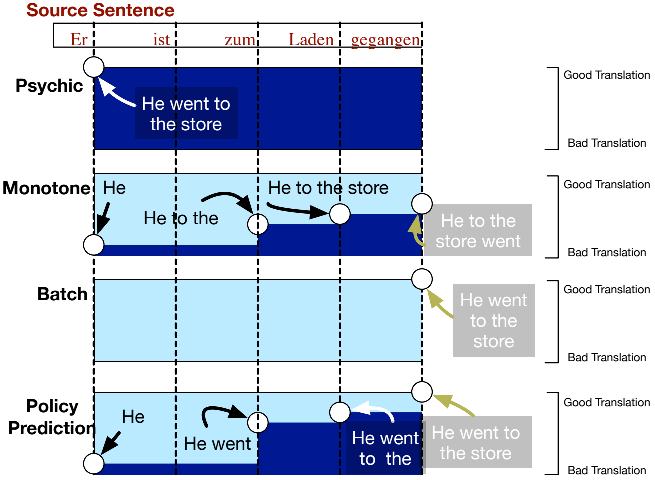

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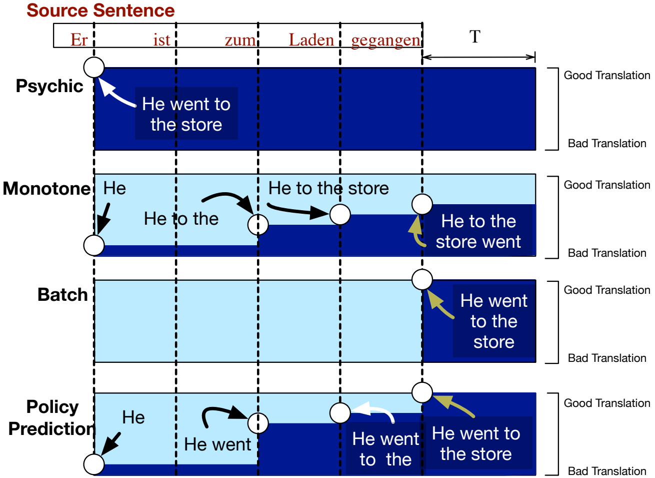
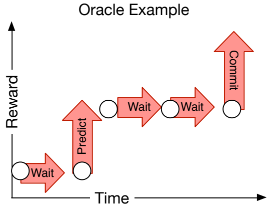


Applying SEARN



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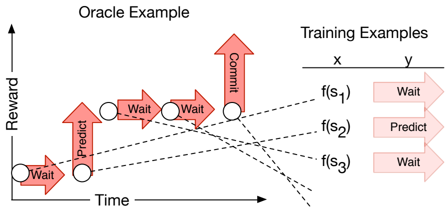

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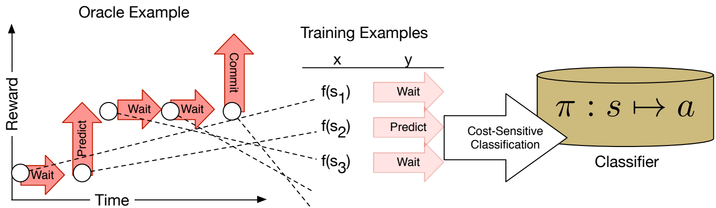
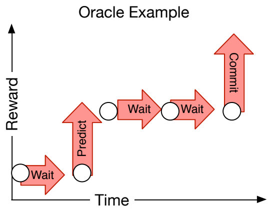
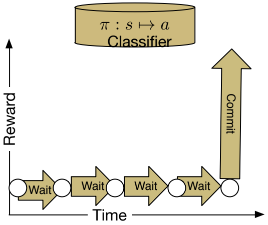


Applying SEARN





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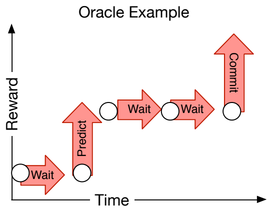

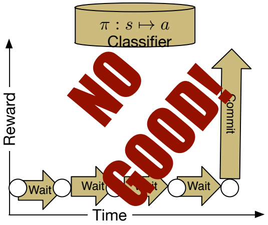

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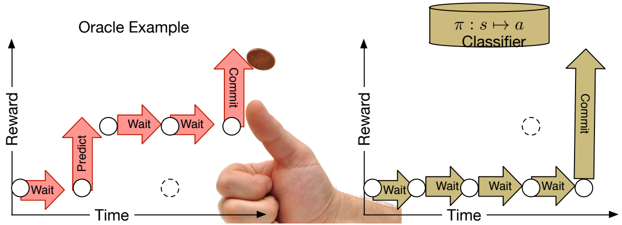

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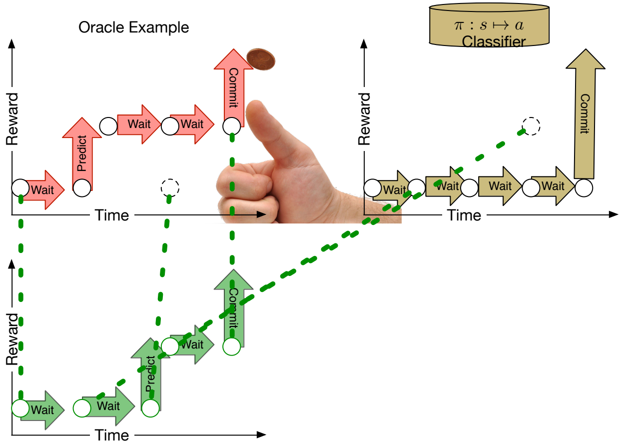

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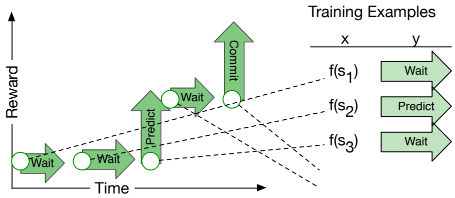

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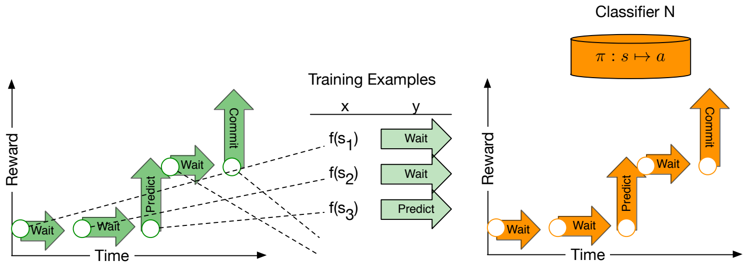

Recap

- Reinforcement learning: *interacting with environments*
- Important to scale to large state spaces
- Connection with classification
- Lets computers observe and repeat