Department of Computer Science
UNIVERSITY OF COLORADO **BOULDER**

Boosting

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LECTURE 12

Content Questions

Content Questions

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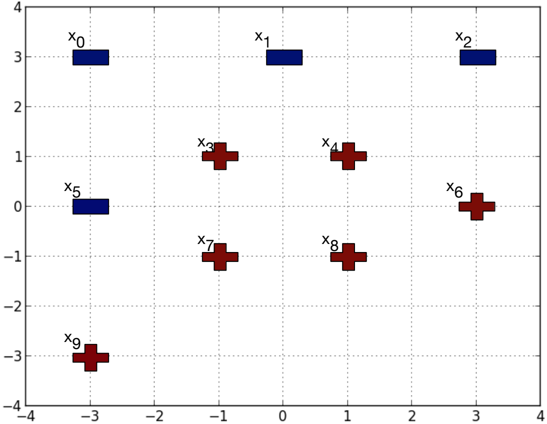
Administrivia Questions

Boosting Example



Hypothesis 1

- Find the best weak learner weighted by D_1

Hypothesis 1

- Find the best weak learner weighted by D_1
- Return 1.0 if x_1 is less than 2.0, -1.0 otherwise

Iteration 1

- Error: $\epsilon_1 = \sum_{i=1}^{m} D_1(i)\mathbb{1}\left[y_i \neq h_1(x_i)\right]$

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$$\epsilon_1 = 0.10_5 = 0.10 \tag{1}$$

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- Update distribution: $D_2(i) \propto D_1(i) \exp(-\alpha_1 y_i h_1(x_i))$

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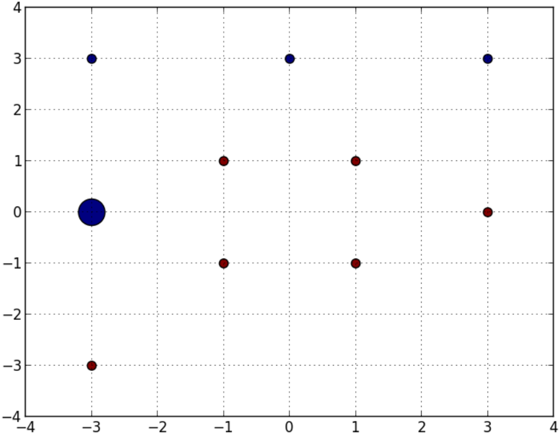
0	1	2	3	4	5	6	7	8	9
0.06	0.06	0.06	0.06	0.06	0.50	0.06	0.06	0.06	0.06

Distribution 2



Hypothesis 2

- Find the best learner weighted by D_2
- Return 1.0 if x_0 is greater than -2.0, -1.0 otherwise

Iteration 2

- Error: $\epsilon_2 = \sum_{i=1}^{m} D_2(i)\mathbb{1}\left[y_i \neq h_2(x_i)\right]$

Iteration 2

- Error: $\epsilon_2 = \sum_{i=1}^{m} D_2(i) \mathbb{1}\left[y_i \neq h_2(x_i)\right]$

$$\epsilon_2 = 0.06_1 + 0.06_2 + 0.06_9 = 0.17 \tag{2}$$

Iteration 2

- Error: $\epsilon_2 = \sum_{i=1}^{m} D_2(i)\mathbb{1}\left[y_i \neq h_2(x_i)\right]$

$$\epsilon_2 = 0.06_1 + 0.06_2 + 0.06_9 = 0.17 \tag{2}$$

- $\alpha_2 = \frac{1}{2} \ln \left(\frac{1-\epsilon_2}{\epsilon_2} \right)$

Iteration 2

- Error: $\epsilon_2 = \sum_{i=1}^{m} D_2(i) \mathbb{1}\left[y_i \neq h_2(x_i)\right]$

$$\epsilon_2 = 0.06_1 + 0.06_2 + 0.06_9 = 0.17 \tag{2}$$

- $\alpha_2 = \frac{1}{2} \ln \left(\frac{1-\epsilon_2}{\epsilon_2} \right) = 0.80$
- Update distribution: $D_3(i) \propto D_2(i) \exp(-\alpha_2 y_i h_2(x_i))$

Iteration 2

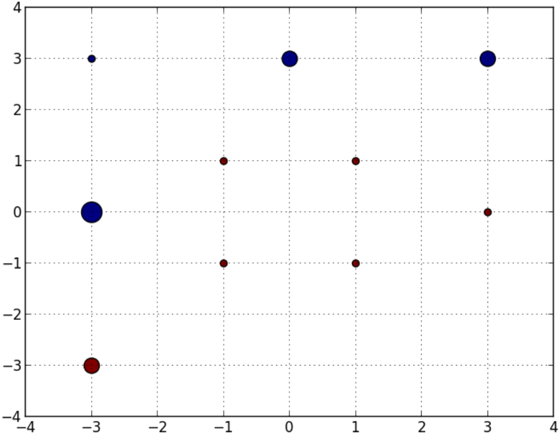
- Error: $\epsilon_2 = \sum_{i=1}^{m} D_2(i) \mathbb{1}\left[y_i \neq h_2(x_i)\right]$

$$\epsilon_2 = 0.06_1 + 0.06_2 + 0.06_9 = 0.17 \tag{2}$$

- $\alpha_2 = \frac{1}{2} \ln \left(\frac{1-\epsilon_2}{\epsilon_2} \right) = 0.80$
- Update distribution: $D_3(i) \propto D_2(i) \exp(-\alpha_2 y_i h_2(x_i))$

0	1	2	3	4	5	6	7	8	9
0.03	0.17	0.17	0.03	0.03	0.30	0.03	0.03	0.03	0.17

Distribution 3



Hypothesis 3

- Find the best learner weighted by D_3
- Return 1.0 if x_1 is less than -0.5, -1.0 otherwise

Iteration 3

- Error: $\epsilon_3 = \sum_{i=1}^{m} D_3(i) \mathbb{1}\left[y_i \neq h_3(x_i)\right]$

Iteration 3

- Error: $\epsilon_3 = \sum_{i=1}^{m} D_3(i)\mathbb{1}\left[y_i \neq h_3(x_i)\right]$

$$\epsilon_3 = 0.03_3 + 0.03_4 + 0.03_6 = 0.10 \tag{3}$$

Iteration 3

- Error: $\epsilon_3 = \sum_{i=1}^{m} D_3(i) \mathbb{1}\left[y_i \neq h_3(x_i)\right]$

$$\epsilon_3 = 0.03_3 + 0.03_4 + 0.03_6 = 0.10 \tag{3}$$

- $\alpha_3 = \frac{1}{2} \ln \left(\frac{1-\epsilon_3}{\epsilon_3} \right)$

Iteration 3

- Error: $\epsilon_3 = \sum_{i=1}^{m} D_3(i)\mathbb{1}\left[y_i \neq h_3(x_i)\right]$

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- $\alpha_3 = \frac{1}{2} \ln \left(\frac{1-\epsilon_3}{\epsilon_3} \right) = 1.10$
- Update distribution: $D_4(i) \propto D_3(i) \exp(-\alpha_3 y_i h_3(x_i))$

Iteration 3

- Error: $\epsilon_3 = \sum_{i=1}^{m} D_3(i) \mathbb{1}\left[y_i \neq h_3(x_i)\right]$

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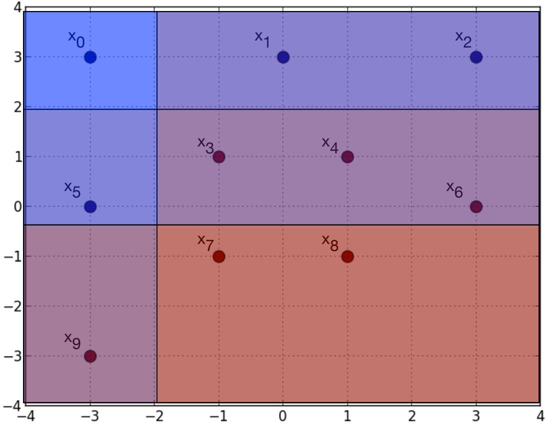
0	1	2	3	4	5	6	7	8	9
0.02	0.09	0.09	0.17	0.17	0.17	0.17	0.02	0.02	0.09

Classifier



Final Predictions

$$H(x) = \text{sign}\left(\sum_t \alpha_t h_t(x)\right) \tag{4}$$

- $H(x_0) =$

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- $H(x_0) = \text{sign}(-1.10 + -0.80 + -1.10) = \text{sign}(-3.00) = -1.0$
- $H(x_1) =$

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- $H(x_5) = \text{sign}(1.10 + -0.80 + -1.10) = \text{sign}(-0.80) = -1.0$
- $H(x_6) =$

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