Optimizing Support Vector Machines

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LECTURE 10

Slides adapted from David Page

Content Questions

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Administrivia

- SVM homework and Boosting homework's posted
- Dates moved a week later for both

SMO Algorithm

Positive

(-2, 2)
(0, 4)
(2, 1)

Negative

(-2, -3)
(0, -1)
(2, -3)

SMO Algorithm

Positive	Negative
(-2, 2)	(-2, -3)
(0, 4)	(0, -1)
(2, 1)	(2, -3)

- Initially, all alphas are zero

$$\vec{\alpha} = < 0, 0, 0, 0, 0, 0 > \tag{1}$$

SMO Algorithm

Positive	Negative
(-2, 2)	(-2, -3)
(0, 4)	(0, -1)
(2, 1)	(2, -3)

- Initially, all alphas are zero

$$\vec{\alpha} = < 0, 0, 0, 0, 0, 0 > \quad (1)$$

- Intercept b is also zero
- Regularization $C = \pi$

SMO Optimization for $i = 0, j = 4$: Predictions and Step

- Prediction: $f(x_0)$
- Prediction: $f(x_4)$
- Error: E_0
- Error: E_4
- Step η

(2)

SMO Optimization for $i = 0, j = 4$: Predictions and Step

- Prediction: $f(x_0) = 0$
- Prediction: $f(x_4)$
- Error: E_0
- Error: E_4
- Step η

$$ \tag{2} $$

SMO Optimization for $i = 0, j = 4$: Predictions and Step

- Prediction: $f(x_0) = 0$
- Prediction: $f(x_4) = 0$
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SMO Optimization for $i = 0, j = 4$: Predictions and Step

- Prediction: $f(x_0) = 0$
- Prediction: $f(x_4) = 0$
- Error: $E_0 = -1$
- Error: $E_4 = +1$
- Step η

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SMO Optimization for $i = 0, j = 4$: Predictions and Step

- Prediction: $f(x_0) = 0$
- Prediction: $f(x_4) = 0$
- Error: $E_0 = -1$
- Error: $E_4 = +1$
- Step η

$$\eta = 2\langle x_0, x_4 \rangle - \langle x_0, x_0 \rangle - \langle x_4, x_4 \rangle \tag{2}$$

SMO Optimization for $i = 0, j = 4$: Predictions and Step

- Prediction: $f(x_0) = 0$
- Prediction: $f(x_4) = 0$
- Error: $E_0 = -1$
- Error: $E_4 = +1$
- Step η

$$\eta = 2\langle x_0, x_4 \rangle - \langle x_0, x_0 \rangle - \langle x_4, x_4 \rangle = 2 \cdot -2 - 8 - 1 = -13 \quad (2)$$

SMO Optimization for $i = 0, j = 4$: Bounds

- Lower and upper bounds for α_j

$$L = \max(0, \alpha_j - \alpha_i) \tag{3}$$

$$H = \min(C, C + \alpha_j - \alpha_i) \tag{4}$$

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SMO Optimization for $i = 0, j = 4$: Bounds

- Lower and upper bounds for α_j

$$L = \max(0, \alpha_j - \alpha_i) = 0 \tag{3}$$

$$H = \min(C, C + \alpha_j - \alpha_i) = \pi \tag{4}$$

SMO Optimization for $i = 0, j = 4$: α update

New value for α_j

$$\alpha_j^* = \alpha_j - \frac{y_j(E_i - E_j)}{\eta} \tag{5}$$

$$\tag{6}$$

SMO Optimization for $i = 0, j = 4$: α update

New value for α_j

$$\alpha_j^* = \alpha_j - \frac{y_j(E_i - E_j)}{\eta} = \frac{-2}{\eta} = \frac{2}{13} \tag{5}$$

$$\tag{6}$$

SMO Optimization for $i = 0, j = 4$: α update

New value for α_j

$$\alpha_j^* = \alpha_j - \frac{y_j(E_i - E_j)}{\eta} = \frac{-2}{\eta} = \frac{2}{13} \tag{5}$$

New value for α_i

$$\tag{6}$$

SMO Optimization for $i = 0, j = 4$: α update

New value for α_j

$$\alpha_j^* = \alpha_j - \frac{y_j(E_i - E_j)}{\eta} = \frac{-2}{\eta} = \frac{2}{13} \tag{5}$$

New value for α_i

$$\alpha_i^* = \alpha_i + y_i y_j \left(\alpha_j^{(old)} - \alpha_j\right) \tag{6}$$

SMO Optimization for $i = 0, j = 4$: α update

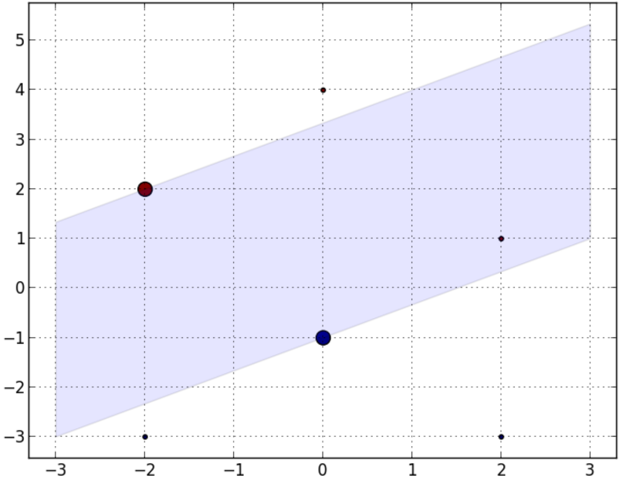
New value for α_j

$$\alpha_j^* = \alpha_j - \frac{y_j(E_i - E_j)}{\eta} = \frac{-2}{\eta} = \frac{2}{13} \tag{5}$$

New value for α_i

$$\alpha_i^* = \alpha_i + y_i y_j \left(\alpha_j^{(old)} - \alpha_j \right) = \alpha_j = \frac{2}{13} \tag{6}$$

Margin



Find weight vector and bias

- Weight vector

$$\vec{w} = \sum_{i=1}^{m} \alpha_i y_i \vec{x}_i \tag{7}$$

- Bias

$$b = b^{(old)} - E_i - y_i(\alpha_i^* - \alpha_i^{(old)}) x_i \cdot x_i - y_j(\alpha_j^* - \alpha_j^{(old)}) x_i \cdot x_j \tag{8}$$

$$\tag{9}$$

Find weight vector and bias

- Weight vector

$$\vec{w} = \sum_{i=1}^{m} \alpha_i y_i \vec{x}_i = \frac{2}{13}\begin{bmatrix} -2 \\ 2 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad (7)$$

- Bias

$$b = b^{(old)} - E_i - y_i(\alpha_i^* - \alpha_i^{(old)})x_i \cdot x_i - y_j(\alpha_j^* - \alpha_j^{(old)})x_i \cdot x_j \quad (8)$$

$$(9)$$

Find weight vector and bias

- Weight vector

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Find weight vector and bias

- Weight vector

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- Bias

$$b = b^{(old)} - E_i - y_i(\alpha_i^* - \alpha_i^{(old)}) x_i \cdot x_i - y_j(\alpha_j^* - \alpha_j^{(old)}) x_i \cdot x_j \tag{8}$$

$$= 1 - \frac{2}{13} \cdot 8 + \frac{2}{13} \cdot -2 = -0.54 \tag{9}$$

SMO Optimization for $i = 2, j = 4$

Let's skip the boring stuff

- $E_2 = -1.69$
- $E_4 = 0.00$
- $\eta = -8$

- $\alpha_4 = \alpha_j^{(old)} - \frac{y_j(E_i - E_j)}{\eta}$
- $\alpha_2 = \alpha_i^{(old)} + y_i y_j \left(\alpha_j^{(old)} - \alpha_j \right)$

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- $\alpha_4 = \alpha_j^{(old)} - \frac{y_j(E_i - E_j)}{\eta} = 0.15 + \frac{-1.69}{-8} = 0.37$
- $\alpha_2 = \alpha_i^{(old)} + y_i y_j \left(\alpha_j^{(old)} - \alpha_j \right)$

SMO Optimization for $i = 2, j = 4$

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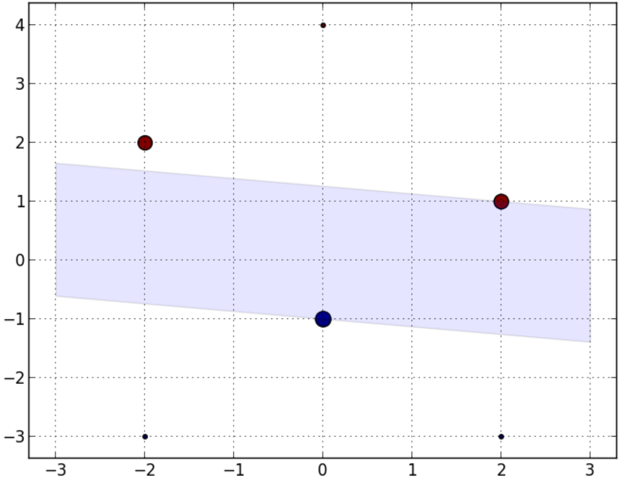
- $E_4 = 0.00$
- $\eta = -8$

- $\alpha_4 = \alpha_j^{(old)} - \frac{y_j(E_i - E_j)}{\eta} = 0.15 + \frac{-1.69}{-8} = 0.37$
- $\alpha_2 = \alpha_i^{(old)} + y_i y_j \left(\alpha_j^{(old)} - \alpha_j\right) = 0 - (0.15 - 0.37) = 0.21$

Margin



Weight vector and bias

- Bias $b = -0.12$
- Weight vector

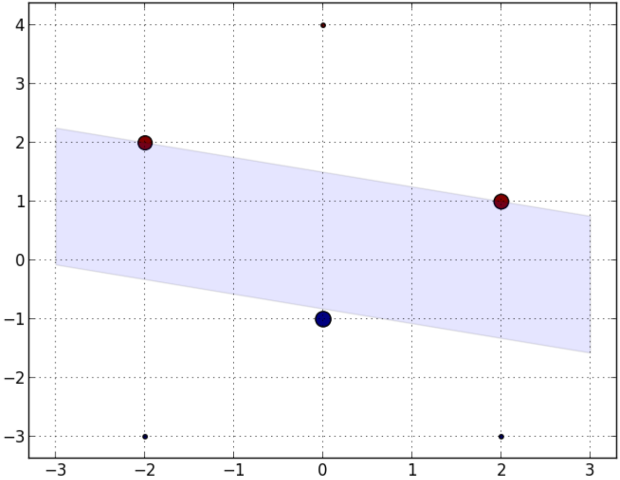
$$\vec{w} = \sum_{i=1}^{m} \alpha_i y_i \vec{x}_i \tag{10}$$

Weight vector and bias

- Bias $b = -0.12$
- Weight vector

$$\vec{w} = \sum_{i=1}^{m} \alpha_i y_i \vec{x}_i = \begin{bmatrix} 0.12 \\ 0.88 \end{bmatrix} \tag{10}$$

Another Iteration $(i = 0, j = 2)$



SMO Algorithm

- Convenient approach for solving: vanilla, slack, kernel approaches
- Convex problem
- Scalable to large datasets (implemented in scikit learn)
- What we didn't do:
  - Check KKT conditions
  - Randomly choose indices