Classification: Logistic Regression

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LECTURE 2B

Slides adapted from Hinrich Schütze and Lauren Hannah

What are we talking about?

- Statistical classification: $p(y|x)$
- Classification uses: ad placement, spam detection
- Building block of other machine learning methods

Logistic Regression: Definition

- Weight vector β_i
- Observations X_i
- “Bias” β_0 (like intercept in linear regression)

$$P(Y=0|X) = \frac{1}{1+\exp\left[\beta_0+\sum_i \beta_i X_i\right]} \tag{1}$$

$$P(Y=1|X) = \frac{\exp\left[\beta_0+\sum_i \beta_i X_i\right]}{1+\exp\left[\beta_0+\sum_i \beta_i X_i\right]} \tag{2}$$

- For shorthand, we’ll say that

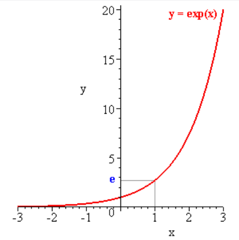
$$P(Y=0|X) = \sigma(-(\beta_0+\sum_i \beta_i X_i)) \tag{3}$$

$$P(Y=1|X) = 1-\sigma(-(\beta_0+\sum_i \beta_i X_i)) \tag{4}$$

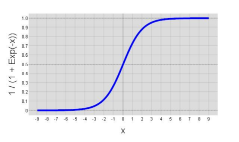
- Where $\sigma(z) = \frac{1}{1+exp[-z]}$

What's this "exp" doing?

Exponential



Logistic



- $\exp[x]$ is shorthand for e^x
- e is a special number, about 2.71828
  - e^x is the limit of compound interest formula as compounds become infinitely small
  - It's the function whose derivative is itself
- The "logistic" function is $\sigma(z) = \frac{1}{1+e^{-z}}$
- Looks like an "S"
- Always between 0 and 1.

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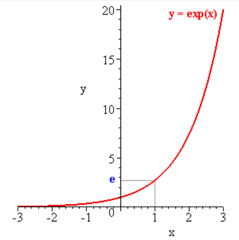

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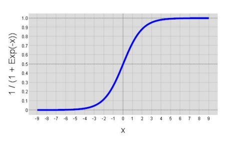

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- Looks like an "S"
- Always between 0 and 1.
  - Allows us to model probabilities
  - Different from **linear** regression

Outline

1. Logistic Regression Example

Logistic Regression Example

feature	coefficient	weight
bias	β_0	0.1
"viagra"	β_1	2.0
"mother"	β_2	-1.0
"work"	β_3	-0.5
"nigeria"	β_4	3.0

- What does $Y = 1$ mean?

Example 1: Empty Document?

$X = \{\}$

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- $P(Y = 1) = \frac{\exp[0.1]}{1 + \exp[0.1]} =$

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- $P(Y = 0) = \frac{1}{1+\exp[0.1]} = 0.48$
- $P(Y = 1) = \frac{\exp[0.1]}{1+\exp[0.1]} = 0.52$
- Bias β_0 encodes the prior probability of a class

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$X = \{\text{Mother}, \text{Nigeria}\}$

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- $P(Y=0) = \frac{1}{1+\exp\left[0.1-1.0+3.0\right]} =$
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- Include bias, and sum the other weights

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- $P(Y=1) = \frac{\exp\left[0.1-1.0+3.0\right]}{1+\exp\left[0.1-1.0+3.0\right]} = 0.88$
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- $P(Y = 0) = \frac{1}{1+\exp[0.1-1.0-0.5+2.0-1.0]} = 0.60$
- $P(Y = 1) = \frac{\exp[0.1-1.0-0.5+2.0-1.0]}{1+\exp[0.1-1.0-0.5+2.0-1.0]} = 0.30$
- Multiply feature presence by weight

How is Logistic Regression Used?

- Given a set of weights $\vec{\beta}$, we know how to compute the conditional likelihood $P(y|\beta, x)$
- Find the set of weights $\vec{\beta}$ that maximize the conditional likelihood on training data (next week)
- **Intuition**: higher weights mean that this feature implies that this feature is a good this is the class you want for this observation

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- Naïve Bayes is a special case of logistic regression that uses Bayes rule and conditional probabilities to set these weights

$$\arg\max_{c_j \in \mathbb{C}} \left[\ln \hat{P}(c_j) + \sum_{1 \le i \le n_d} \ln \hat{P}(w_i | c_j) \right]$$

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Contrasting Naïve Bayes and Logistic Regression

- Naïve Bayes easier
- Naïve Bayes better on smaller datasets
- Logistic regression better on medium-sized datasets
- On huge datasets, it doesn't really matter (data always win)
    - Optional reading by Ng and Jordan has proofs and experiments
- Logistic regression allows arbitrary features (biggest difference!)

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  - Optional reading by Ng and Jordan has proofs and experiments
- Logistic regression allows arbitrary features (biggest difference!)
- Don't need to memorize (or work through) previous slide—just understand that naïve Bayes is a special case of logistic regression

Next time . . .

- How to learn the best setting of weights
- Regularizing logistic regression to encourage sparse vectors
- Extracting features