Department of Computer Science
UNIVERSITY OF COLORADO **BOULDER**

Supervised Learning

Introduction to Data Science Algorithms

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Linear Regression Predictions

dimension	weight
b	1
w_1	2.0
w_2	−1.0
σ	1.0

1. $\mathbf{x}_1 = \{0.0, 0.0\}$; y_1=
2. $\mathbf{x}_2 = \{1.0, 1.0\}$; y_2=
3. $\mathbf{x}_3 = \{.5, 2\}$; y_3=

Linear Regression Predictions

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3. $\mathbf{x}_3 = \{.5, 2\}$; y_3=

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2. $\mathbf{x}_2 = \{1.0, 1.0\}$; y_2=2.0
3. $\mathbf{x}_3 = \{.5, 2\}$; y_3=0.0

Probabilities

dimension	weight
w_0	1
w_1	2.0
w_2	-1.0
σ	1.0

$$p(y\,|\,x) = y \sim N\left(b + \sum_{j=1}^{p} w_j x_j, \sigma^2\right)$$

$$p(y\,|\,x) = \frac{\exp\left\{-\frac{(y-\hat{y})^2}{2}\right\}}{\sqrt{2\pi}}$$

1. $p(y_1 = 1\,|\,\mathbf{x}_1 = \{0.0, 0.0\}) =$
2. $p(y_2 = 3\,|\,\mathbf{x}_2 = \{1.0, 1.0\}) =$
3. $p(y_3 = -1\,|\,\mathbf{x}_3 = \{.5, 2\}) =$

Probabilities

dimension	weight
w_0	1
w_1	2.0
w_2	−1.0
σ	1.0

$$p(y \,|\, x) = y \sim N\left(b + \sum_{j=1}^{p} w_j x_j, \sigma^2\right)$$

$$p(y \,|\, x) = \frac{\exp\left\{-\frac{(y-\hat{y})^2}{2}\right\}}{\sqrt{2\pi}}$$

1. $p(y_1 = 1 \,|\, \mathbf{x}_1 = \{0.0, 0.0\}) = 0.399$
2. $p(y_2 = 3 \,|\, \mathbf{x}_2 = \{1.0, 1.0\}) =$
3. $p(y_3 = -1 \,|\, \mathbf{x}_3 = \{.5, 2\}) =$

Probabilities

dimension	weight
w_0	1
w_1	2.0
w_2	−1.0
σ	1.0

$$p(y \mid x) = y \sim N\left(b + \sum_{j=1}^{p} w_j x_j, \sigma^2\right)$$

$$p(y \mid x) = \frac{\exp\left\{-\frac{(y-\hat{y})^2}{2}\right\}}{\sqrt{2\pi}}$$

1. $p(y_1 = 1 \mid \mathbf{x}_1 = \{0.0, 0.0\}) = 0.399$
2. $p(y_2 = 3 \mid \mathbf{x}_2 = \{1.0, 1.0\}) = 0.242$
3. $p(y_3 = -1 \mid \mathbf{x}_3 = \{.5, 2\}) =$

Probabilities

dimension	weight
w_0	1
w_1	2.0
w_2	−1.0
σ	1.0

$$p(y \mid x) = y \sim N\left(b + \sum_{j=1}^{p} w_j x_j, \sigma^2\right)$$

$$p(y \mid x) = \frac{\exp\left\{-\frac{(y-\hat{y})^2}{2}\right\}}{\sqrt{2\pi}}$$

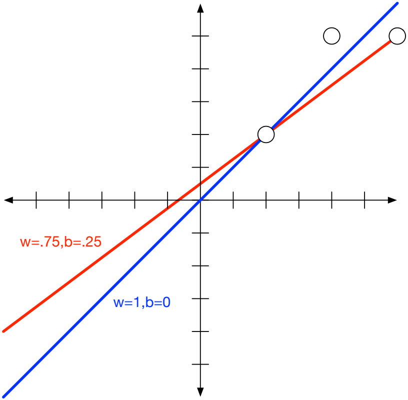
1. $p(y_1 = 1 \mid \mathbf{x}_1 = \{0.0, 0.0\}) = 0.399$
2. $p(y_2 = 3 \mid \mathbf{x}_2 = \{1.0, 1.0\}) = 0.242$
3. $p(y_3 = -1 \mid \mathbf{x}_3 = \{.5, 2\}) = 0.242$

Outline

1. Linear Regression Objective

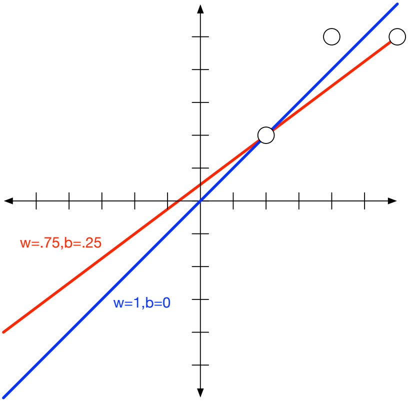
Consider these points and data

Data: $(1,1);(2,2.5);(3,2.5)$



Consider these points and data

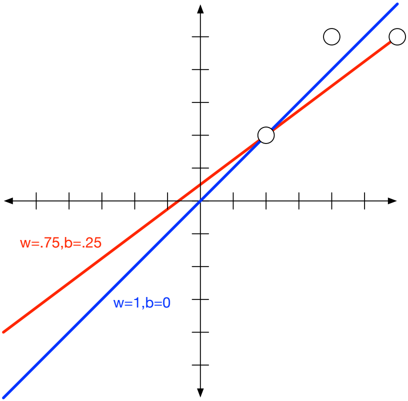
Data: $(1,1); (2,2.5); (3,2.5)$



Which is the better OLS solution?

Consider these points and data

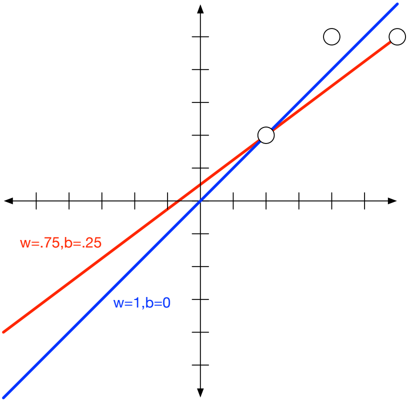
Data: $(1,1); (2,2.5); (3,2.5)$



Blue! It has lower RSS.

Consider these points and data

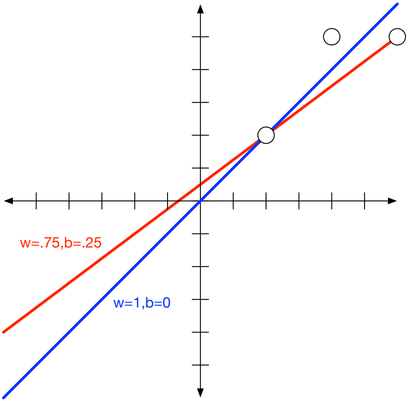
Data: $(1,1); (2,2.5); (3,2.5)$



What is the RSS of the better solution?

Consider these points and data

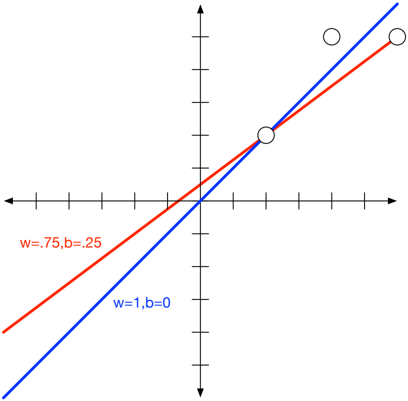
Data: $(1,1);(2,2.5);(3,2.5)$



$$\frac{1}{2}\sum_i r_i^2 = \frac{1}{2}\left((1-1)^2 + (2.5-2)^2 + (2.5-3)^2\right) = \frac{1}{4}$$

Consider these points and data

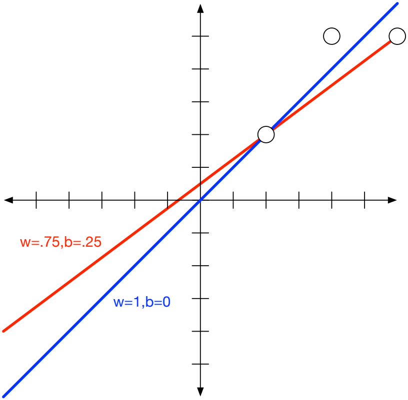
Data: $(1, 1); (2, 2.5); (3, 2.5)$



What is the RSS of the red line?

Consider these points and data

Data: $(1,1);(2,2.5);(3,2.5)$



$$\frac{1}{2}\sum_i r_i^2 = \frac{1}{2}\left((1-1)^2 + \left(\frac{10}{4}-\frac{7}{4}\right)^2 + (2.5-2.5)^2\right) = \frac{9}{32}$$

Reminder: Logistic Regression

$$P(Y=0|X) = \frac{1}{1+\exp\left[\beta_0 + \sum_i \beta_i X_i\right]} \quad (1)$$

$$P(Y=1|X) = \frac{\exp\left[\beta_0 + \sum_i \beta_i X_i\right]}{1+\exp\left[\beta_0 + \sum_i \beta_i X_i\right]} \quad (2)$$

- Discriminative prediction: $p(y|x)$
- Classification uses: ad placement, spam detection
- What we didn't talk about is how to learn β from data

Logistic Regression: Objective Function

$$\mathscr{L} \equiv \ln p(Y \,|\, X, \beta) = \sum_j \ln p(y^{(j)} \,|\, x^{(j)}, \beta) \tag{3}$$

$$= \sum_j y^{(j)} \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) - \ln \left[1 + \exp \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) \right] \tag{4}$$

Algorithm

1. Initialize a vector B to be all zeros
2. For $t = 1, \dots, T$
   - For each example $\vec{x}_i, y_i$ and feature j:
     - Compute $\pi_i \equiv \Pr(y_i = 1 \,|\, \vec{x}_i)$
     - Set $\beta[j] = \beta[j]' + \eta(y_i - \pi_i)x_i$
3. Output the parameters $\beta_1, \dots, \beta_d$.

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$

$$\vec{\beta} = \langle \beta_{bias} = 0, \beta_A = 0, \beta_B = 0, \beta_C = 0, \beta_D = 0 \rangle$$

$y_1 =$ 1

A A A A B B B C

$y_2 =$ 0

B C C C D D D D

(Assume step size $\eta = 1.0$.)

You first see the positive example. First, compute π_1

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$

$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

You first see the positive example. First, compute π_1

$$\pi_1 = \Pr(y_1 = 1 \,|\, \vec{x}_1) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} =$$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

You first see the positive example. First, compute π_1

$$\pi_1 = \Pr(y_1 = 1 \mid \vec{x}_1) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp 0}{\exp 0 + 1} = 0.5$$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

$\pi_1 = 0.5$ What's the update for β_{bias}?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_{bias}?

$$\beta_{bias} = \beta'_{bias} + \eta \cdot (y_1 - \pi_1) \cdot x_{1,bias} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 1.0$$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_{bias}?

$\beta_{bias} = \beta'_{bias} + \eta \cdot (y_1 - \pi_1) \cdot x_{1,bias} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 1.0$ =0.5

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_A?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_A?

$$\beta_A = \beta'_A + \eta \cdot (y_1 - \pi_1) \cdot x_{1,A} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 4.0$$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_A?

$\beta_A = \beta'_A + \eta \cdot (y_1 - \pi_1) \cdot x_{1,A} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 4.0 \quad =2.0$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$

$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_B?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_B?

$\beta_B = \beta'_B + \eta \cdot (y_1 - \pi_1) \cdot x_{1,B} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 3.0$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_B?

$\beta_B = \beta'_B + \eta \cdot (y_1 - \pi_1) \cdot x_{1,B} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 3.0$ =1.5

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_C?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_C?

$\beta_C = \beta'_C + \eta \cdot (y_1 - \pi_1) \cdot x_{1,C} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 1.0$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_C?

$\beta_C = \beta'_C + \eta \cdot (y_1 - \pi_1) \cdot x_{1,C} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 1.0$ =0.5

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_D?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_D?

$\beta_D = \beta'_D + \eta \cdot (y_1 - \pi_1) \cdot x_{1,D} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 0.0$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$

A A A A B B B C

$y_2 = 0$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_D?

$\beta_D = \beta'_D + \eta \cdot (y_1 - \pi_1) \cdot x_{1,D} = 0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 0.0 \quad = 0.0$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

Now you see the negative example. What's π_2?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

Now you see the negative example. What's π_2?

$$\pi_2 = \Pr(y_2 = 1 \,|\, \vec{x}_2) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp\{.5 + 1.5 + 1.5 + 0\}}{\exp\{.5 + 1.5 + 1.5 + 0\} + 1} =$$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

Now you see the negative example. What's π_2?

$$\pi_2 = \Pr(y_2 = 1 \mid \vec{x}_2) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp\{.5+1.5+1.5+0\}}{\exp\{.5+1.5+1.5+0\}+1} = 0.97$$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

Now you see the negative example. What's π_2?

$\pi_2 = 0.97$

What's the update for β_{bias}?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$

$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_{bias}?

$\beta_{bias} = \beta'_{bias} + \eta \cdot (y_2 - \pi_2) \cdot x_{2,bias} = 0.5 + 1.0 \cdot (0.0 - 0.97) \cdot 1.0$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_{bias}?

$\beta_{bias} = \beta'_{bias} + \eta \cdot (y_2 - \pi_2) \cdot x_{2,bias} = 0.5 + 1.0 \cdot (0.0 - 0.97) \cdot 1.0$ =-0.47

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_A?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_A?

$\beta_A = \beta'_A + \eta \cdot (y_2 - \pi_2) \cdot x_{2,A} = 2.0 + 1.0 \cdot (0.0 - 0.97) \cdot 0.0$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_A?

$\beta_A = \beta'_A + \eta \cdot (y_2 - \pi_2) \cdot x_{2,A} = 2.0 + 1.0 \cdot (0.0 - 0.97) \cdot 0.0$ $= 2.0$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_B?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$	$y_2 = \mathbf{0}$
A A A A B B B C	B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_B?

$\beta_B = \beta'_B + \eta \cdot (y_2 - \pi_2) \cdot x_{2,B} = 1.5 + 1.0 \cdot (0.0 - 0.97) \cdot 1.0$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$

$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_B?

$\beta_B = \beta'_B + \eta \cdot (y_2 - \pi_2) \cdot x_{2,B} = 1.5 + 1.0 \cdot (0.0 - 0.97) \cdot 1.0$ =0.53

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$

$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_C?

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What's the update for β_C?

$\beta_C = \beta'_C + \eta \cdot (y_2 - \pi_2) \cdot x_{2,C} = 0.5 + 1.0 \cdot (0.0 - 0.97) \cdot 3.0$

Example Documents

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(Assume step size $\eta = 1.0$.)

What's the update for β_C?

$\beta_C = \beta'_C + \eta \cdot (y_2 - \pi_2) \cdot x_{2,C} = 0.5 + 1.0 \cdot (0.0 - 0.97) \cdot 3.0$ =-2.41

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_D?

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_D?

$$\beta_D = \beta_D' + \eta \cdot (y_2 - \pi_2) \cdot x_{2,D} = 0.0 + 1.0 \cdot (0.0 - 0.97) \cdot 4.0$$

Example Documents

$$\beta[j] = \beta[j] + \eta(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle .5, 2, 1.5, 0.5, 0 \rangle$$

$y_1 = \mathbf{1}$

A A A A B B B C

$y_2 = \mathbf{0}$

B C C C D D D D

(Assume step size $\eta = 1.0$.)

What's the update for β_D?

$\beta_D = \beta'_D + \eta \cdot (y_2 - \pi_2) \cdot x_{2,D} = 0.0 + 1.0 \cdot (0.0 - 0.97) \cdot 4.0$ =-3.88

Recap

- Linear Regression
- Logistic Regression
- HW5: Implement SGD for logistic regression