Linear Regression

Introduction to Data Science Algorithms

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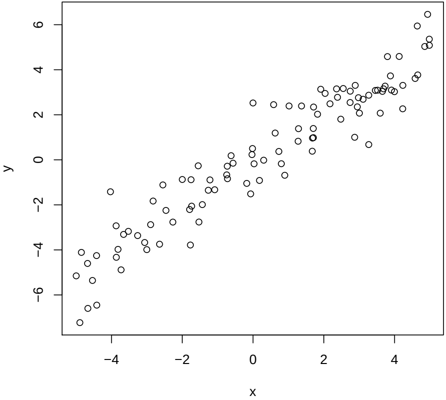
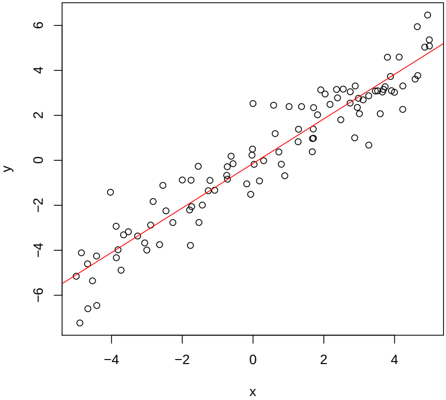
SLIDES ADAPTED FROM LAUREN HANNAH

Linear Regression



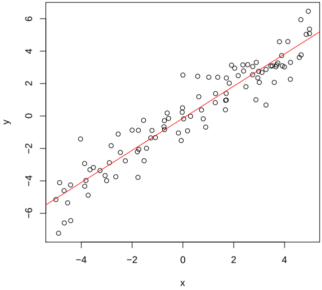
Data are the set of inputs and outputs, $\mathscr{D} = \left\{(x_i, y_i)\right\}_{i=1}^{n}$

Linear Regression



In *linear regression*, the goal is to predict y from x using a linear function

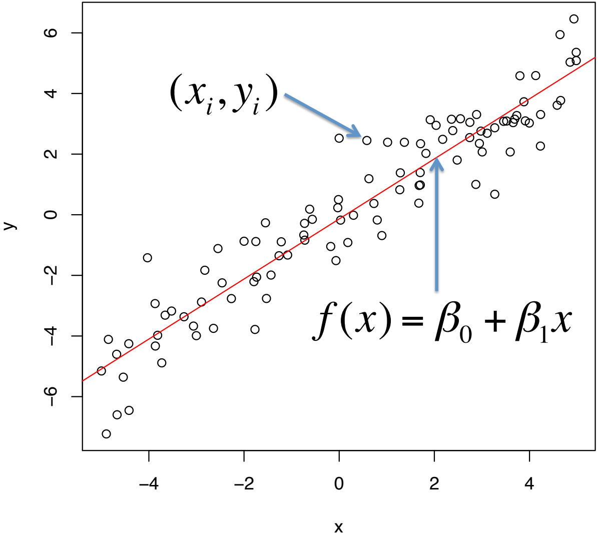
Linear Regression



Examples of linear regression:

- given a child's age and gender, what is his/her height?
- given unemployment, inflation, number of wars, and economic growth, what will the president's approval rating be?
- given a browsing history, how long will a user stay on a page?

Linear Regression



Multiple Covariates

Often, we have a vector of inputs where each represents a different *feature* of the data

$$\mathbf{x} = (x_1, \ldots, x_p)$$

The function fitted to the response is a linear combination of the covariates

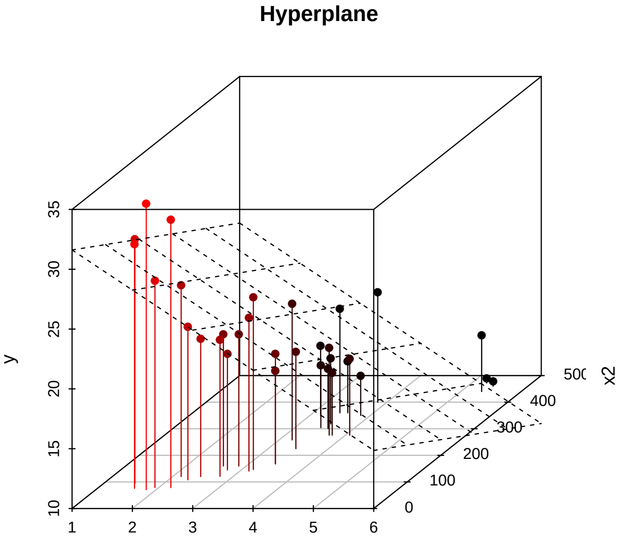
$$f(\mathbf{x}) = \beta_0 + \sum_{j=1}^{p} \beta_j x_j$$

Multiple Covariates

- Often, it is convenient to represent **x** as $(1, x_1, \dots, x_p)$
- In this case **x** is a vector, and so is β (we'll represent them in bold face)
- This is the dot product between these two vectors
- This then becomes a sum (this should be familiar!)

$$\beta \mathbf{x} = \beta_0 + \sum_{j=1}^{p} \beta_j x_j$$

Hyperplanes: Linear Functions in Multiple Dimensions



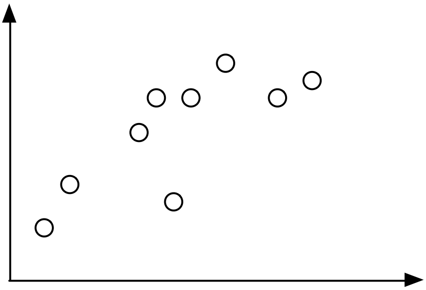
Covariates

- Do not need to be raw value of $x_1, x_2, \ldots$
- Can be any feature or function of the data:
  - Transformations like $x_2 = \log(x_1)$ or $x_2 = \cos(x_1)$
  - Basis expansions like $x_2 = x_1^2$, $x_3 = x_1^3$, $x_4 = x_1^4$, etc
  - Indicators of events like $x_2 = 1_{\{-1 \leq x_1 \leq 1\}}$
  - Interactions between variables like $x_3 = x_1 x_2$
- Because of its simplicity and flexibility, it is one of the most widely implemented regression techniques

Prediction

- After finding $\hat{\beta}$, we would like to predict an output value for a new set of covariates
- We just find the point on the line that corresponds to the new input:

$$\hat{y} = \beta_0 + \beta_1 x \tag{1}$$



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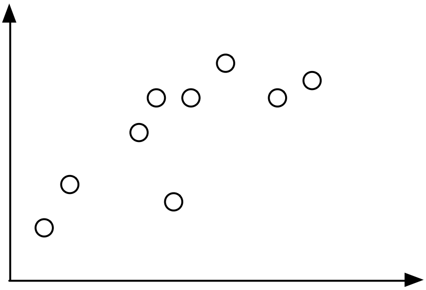
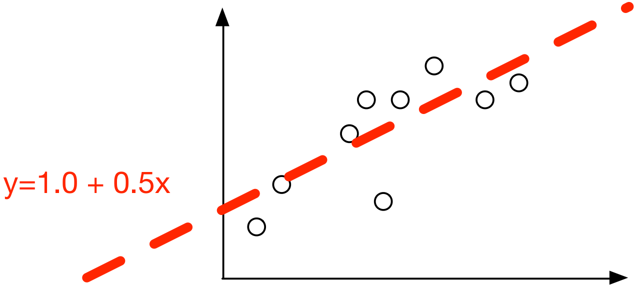


Prediction

- After finding $\hat{\beta}$, we would like to predict an output value for a new set of covariates
- We just find the point on the line that corresponds to the new input:

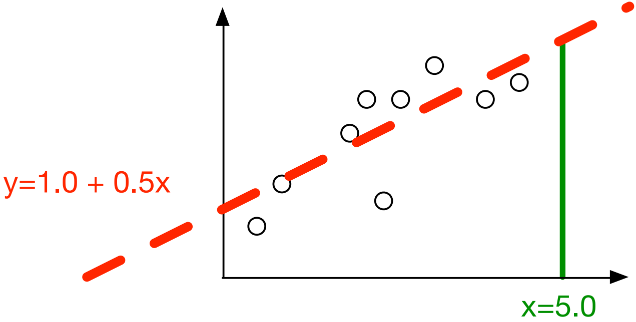
$$\hat{y} = 1.0 + 0.5x \tag{1}$$



Prediction

- After finding $\hat{\beta}$, we would like to predict an output value for a new set of covariates
- We just find the point on the line that corresponds to the new input:

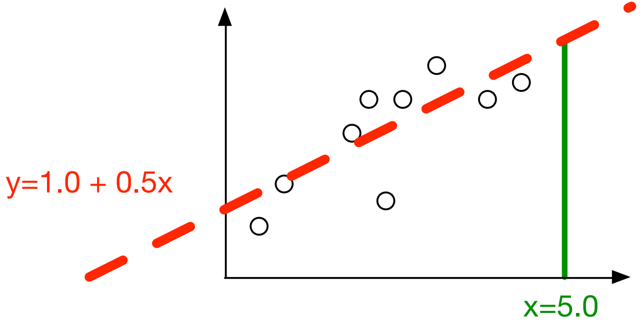
$$\hat{y} = 1.0 + 0.5 * 5 \tag{1}$$



Prediction

- After finding $\hat{\beta}$, we would like to predict an output value for a new set of covariates
- We just find the point on the line that corresponds to the new input:

$$\hat{y} = 3.5 \tag{1}$$



Outline

1. Example

Example: Old Faithful

Example: Old Faithful

We will predict the time that we will have to wait to see the next eruption given the duration of the current eruption