Regression

Machine Learning: Jordan Boyd-Graber
University of Maryland
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Content Questions

Linear Regression Predictions

dimension	weight
b	1
w_1	2.0
w_2	-1.0
σ	1.0

1. $\mathbf{x}_1 = \{0.0, 0.0\}$; y_1=
2. $\mathbf{x}_2 = \{1.0, 1.0\}$; y_2=
3. $\mathbf{x}_3 = \{.5, 2\}$; y_3=

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Probabilities

dimension	weight
w_0	1
w_1	2.0
w_2	-1.0
σ	1.0

$$p(y \mid x) = y \sim N\left(b + \sum_{j=1}^{p} w_j x_j, \sigma^2\right)$$

$$p(y \mid x) = \frac{\exp\left\{-\frac{(y-\hat{y})^2}{2}\right\}}{\sqrt{2\pi}}$$

1. $p(y_1 = 1 \mid \mathbf{x}_1 = \{0.0, 0.0\}) =$
2. $p(y_2 = 3 \mid \mathbf{x}_2 = \{1.0, 1.0\}) =$
3. $p(y_3 = -1 \mid \mathbf{x}_3 = \{.5, 2\}) =$

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1. $p(y_1 = 1\,|\,\mathbf{x}_1 = \{0.0, 0.0\}) = 0.399$
2. $p(y_2 = 3\,|\,\mathbf{x}_2 = \{1.0, 1.0\}) =$
3. $p(y_3 = -1\,|\,\mathbf{x}_3 = \{.5, 2\}) =$

Probabilities

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w_0	1
w_1	2.0
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1. $p(y_1 = 1 \mid \mathbf{x}_1 = \{0.0, 0.0\}) = 0.399$
2. $p(y_2 = 3 \mid \mathbf{x}_2 = \{1.0, 1.0\}) = 0.242$
3. $p(y_3 = -1 \mid \mathbf{x}_3 = \{.5, 2\}) =$

Probabilities

dimension	weight
w_0	1
w_1	2.0
w_2	-1.0
σ	1.0

$$p(y \mid x) = y \sim N\left(b + \sum_{j=1}^{p} w_j x_j, \sigma^2\right)$$

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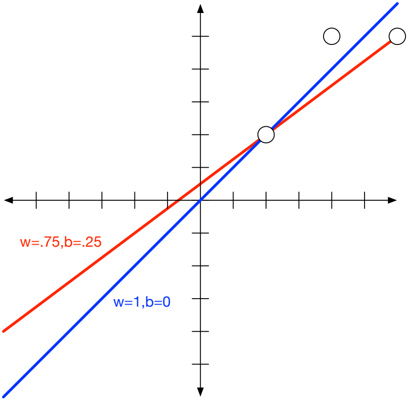
Consider these points and data



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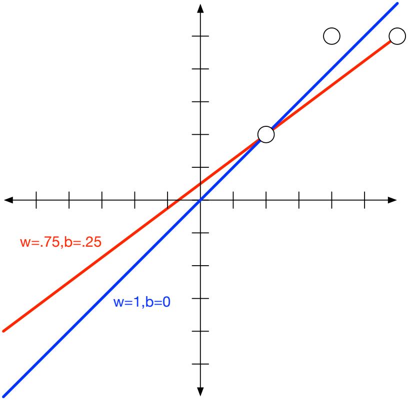
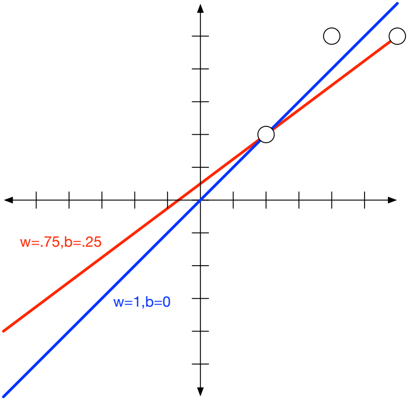


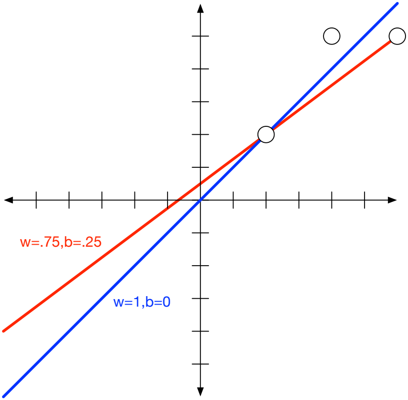
Which is the better OLS solution?

Consider these points and data



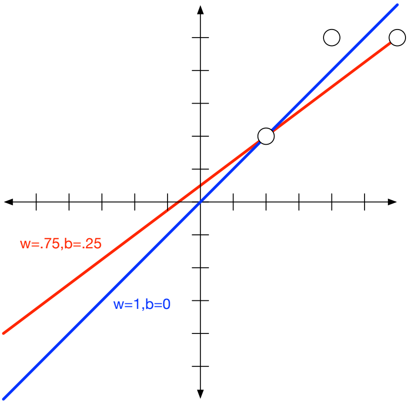
Blue! It has lower RSS.

Consider these points and data



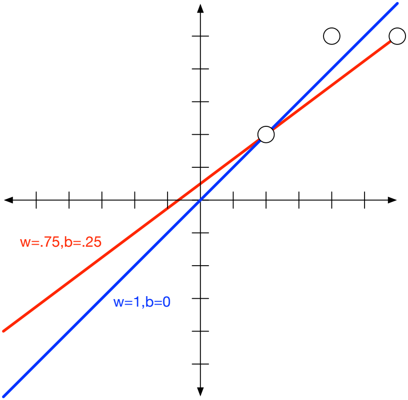
What is the RSS of the better solution?

Consider these points and data



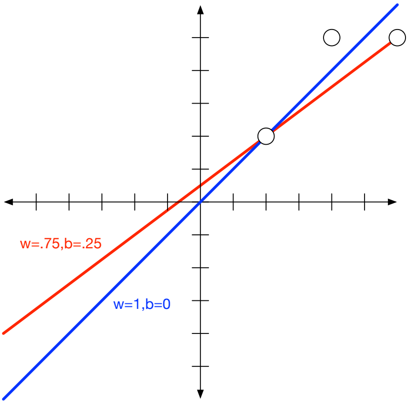
$$\frac{1}{2}\sum_i r_i^2 = \frac{1}{2}\left((1-1)^2 + (2.5-2)^2 + (2.5-3)^2\right) = \frac{1}{4}$$

Consider these points and data



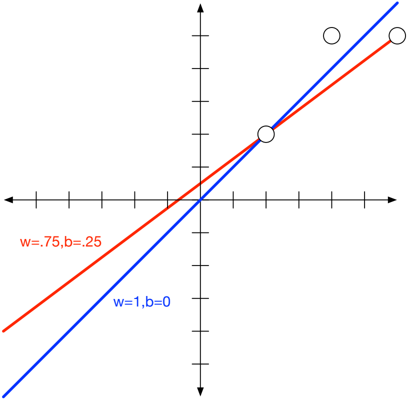
What is the RSS of the red line?

Consider these points and data



$$\frac{1}{2}\sum_i r_i^2 = \frac{1}{2}\left((1-1)^2 + (2.5-1.75)^2 + (2.5-2.5)^2\right) = \frac{3}{8}$$

Consider these points and data



For what λ does the blue line have a better regularized solution with L_2 and L_1?

When Regularization Wins

L_2

L_1

When Regularization Wins

L_2

$$\mathrm{RSS}(x, y, w) + \lambda \sum_d w_d^2 > \mathrm{RSS}(x, y, w) + \lambda \sum_d w_d^2$$

L_1

When Regularization Wins

L_2

$$\text{RSS}(x,y,w) + \lambda \sum_d w_d^2 > \text{RSS}(x,y,w) + \lambda \sum_d w_d^2$$

$$\frac{1}{4} + \lambda 1 > \frac{3}{8} + \lambda \frac{9}{16}$$

L_1

When Regularization Wins

L_2

$$\frac{1}{4} + \lambda 1 > \frac{3}{8} + \lambda \frac{9}{16}$$
$$\frac{7}{16}\lambda > \frac{1}{8}$$

L_1

When Regularization Wins

L_2

$$\frac{7}{16}\lambda > \frac{1}{8}$$
$$\lambda > \frac{2}{7}$$

L_1

When Regularization Wins

L_2

$$\lambda > \frac{2}{7}$$

L_1

$$\text{RSS}(x, y, w) + \lambda \sum_d |w_d| > \text{RSS}(x, y, w) + \lambda \sum_d |w_d|$$

When Regularization Wins

L_2

$$\lambda > \frac{2}{7}$$

L_1

$$\text{RSS}(x, y, w) + \lambda \sum_d |w_d| > \text{RSS}(x, y, w) + \lambda \sum_d |w_d|$$

$$\frac{1}{4} + \lambda 1 > \frac{3}{8} + \lambda \frac{3}{4}$$

When Regularization Wins

L_2

$$\lambda > \frac{2}{7}$$

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$$\frac{1}{4} + \lambda 1 > \frac{3}{8} + \lambda \frac{3}{4}$$

When Regularization Wins

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$$\lambda > \frac{2}{7}$$

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$$\frac{1}{4}\lambda > \frac{1}{8}$$

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$$\frac{1}{4}\lambda > \frac{1}{8}$$
$$\lambda > \frac{1}{2}$$

When Regularization Wins

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$$\lambda > \frac{2}{7}$$

L_1

$$\lambda > \frac{1}{2}$$

Bigger λ: preference for lower weights w

MPG Dataset

- Predict mpg from features of a car
  1. Number of cylinders
  2. Displacement
  3. Horsepower
  4. Weight
  5. Acceleration
  6. Year
  7. Country (ignore this)

Simple Regression

If $w = 0$, what's the intercept?

Simple Regression

If $w = 0$, what's the intercept?
23.4

Simple Linear Regression

What are the coefficients for OLS?

Simple Linear Regression

What are the coefficients for OLS?

Coefficients

cyl	-0.329859
dis	0.007678
hp	-0.000391
wgt	-0.006795
acl	0.085273
yr	0.753367

Simple Linear Regression

What are the coefficients for OLS?

Coefficients

cyl	-0.329859
dis	0.007678
hp	-0.000391
wgt	-0.006795
acl	0.085273
yr	0.753367

Intercept: -14.5

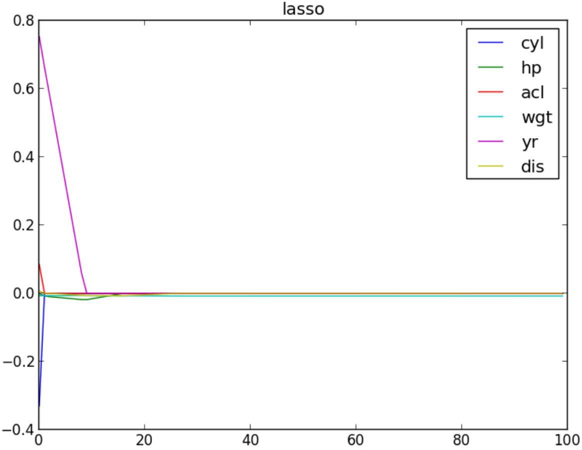
Simple Linear Regression

```
from sklearn import linear_model
linear_model.LinearRegression()
fit = model.fit(x, y)
```

Lasso

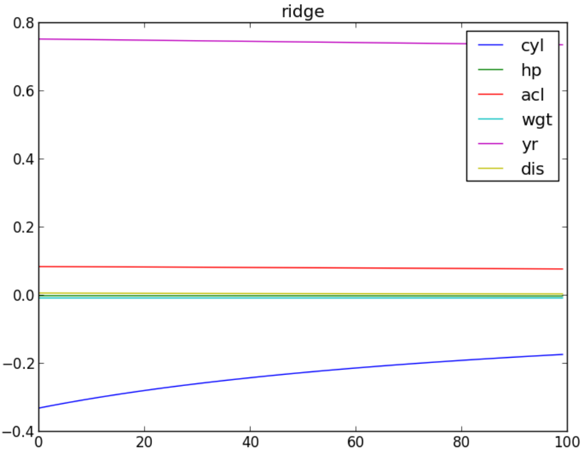
- As you increase the weight of alpha, what feature dominates?
- What happens to the other features?

Weight is Everything



mpg = 46 - 0.01 Weight

How is ridge different?



Regression isn't special

- Feature engineering
- Regularization
- Overfitting
- Development / Test Data