



Slides adapted from Emily Fox

Introduction to Machine Learning

Machine Learning: Jordan Boyd-Graber

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LOGISTIC REGRESSION FROM TEXT

Logistic Regression: Regularized Objective

$$\mathcal{L}' \equiv \ln p(Y|X, \beta) = \sum_j \ln p(y^{(j)} | x^{(j)}, \beta) \quad (1)$$

$$= \sum_j y^{(j)} \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) - \ln \left[1 + \exp \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) \right] \quad (2)$$

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$$\mathcal{L} = \mathcal{L}' - \mu \sum_i \beta_i^2 \quad (3)$$

New Stochastic Gradient

For document i :

$$\frac{\partial \mathcal{L}_i}{\partial \beta_j} = (y - \pi_i) - 2\mu\beta_j \quad (4)$$

Our gradient from before minus a term that brings feature weights to zero (opposite sign of β_j)

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$$\beta_j = \beta'_j + \lambda \left((y - \pi_i) x_j - 2\mu \beta'_j \right) \quad (5)$$

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$$\beta_j = \beta_j' + \lambda (y - \pi_i) x_j - 2\lambda \mu \beta_j' \quad (6)$$

$$\beta_j = \beta_j' (1 - 2\lambda \mu) + \lambda (y - \pi_i) x_j \quad (7)$$

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- Thus, break the update into two steps:

- $\beta'_j = \beta''_j \cdot (1 - 2\lambda \mu)$
- $\beta_j = \beta'_j + \lambda (y - \pi_i) x_j$

Revised Algorithm

1. Initialize a vector β to be all zeros
2. Initialize a vector A to be all zeros
3. For $t = 1, \dots, T$
 - For each example \vec{x}_i, y_i and feature j :
 - Simulate regularization updates: $\beta[j] = \beta[j] \cdot (1 - 2\lambda\mu)^{k-A[j]-1}$
 - Compute $\pi_i \equiv \Pr(y_i = 1 | \vec{x}_i)$
 - Set $\beta[j] = (\beta[j] + \lambda(y_i - \pi_i)x_i)(1 - 2\lambda\mu)$
 - Keep track of last update for feature $A[j] = k$
4. For each parameter, catch up on missing updates
$$\beta[j] = \beta[j] \cdot (1 - 2\lambda\mu)^{T-A[j]}$$
5. Output the parameters β_1, \dots, β_d .