Naïve Bayes Discussion

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Roadmap

- Content Questions
- Administrivia Questions
- NB Exercise

Content Questions

Content Questions

Content Questions

Content Questions

Administrivia Announcements

- Use Piazza
- HW2 Posted

Administrivia Questions

Administrivia Questions

Administrivia Questions

Administrivia Questions

Documents

D1: Spam

abuja man

D2: Ham

man dog

D3: Spam

cialis deal

D4: Ham

logistic mother logistic abuja

D5: Spam

abuja deal

D6: Ham

bagel deal

D7: Spam

cialis dog

Documents

D1: Spam

abuja man

D2: Ham

man dog

D3: Spam

cialis deal

D4: Ham

logistic mother logistic abuja

D5: Spam

abuja deal

D6: Ham

bagel deal

D7: Spam

cialis dog

What's $|C|$ and $|V|$?

Documents

D1: Spam

abuja man

D2: Ham

man dog

D3: Spam

cialis deal

D4: Ham

logistic mother logistic abuja

D5: Spam

abuja deal

D6: Ham

bagel deal

D7: Spam

cialis dog

$|C| = 2$ (spam vs. ham)

Documents

D1: Spam

abuja man

D2: Ham

man dog

D3: Spam

cialis deal

D4: Ham

logistic mother logistic abuja

D5: Spam

abuja deal

D6: Ham

bagel deal

D7: Spam

cialis dog

$|V| = 8$: 'deal', 'dog', 'bagel', 'logistic', 'mother', 'cialis', 'abuja', 'man'

Background Probabilities

D1: Spam

abuja man

D2: Ham

man dog

D3: Spam

cialis deal

D4: Ham

logistic mother logistic abuja

D5: Spam

abuja deal

D6: Ham

bagel deal

D7: Spam

cialis dog

Background Probabilities

D1: Spam

abuja man

D2: Ham

man dog

D3: Spam

cialis deal

D4: Ham

logistic mother logistic abuja

D5: Spam

abuja deal

D6: Ham

bagel deal

D7: Spam

cialis dog

What's $\hat{P}(c_j)$?

Background Probabilities

- For spam:

$$(1)$$

Background Probabilities

- For spam:

$$\hat{P}(c_j = \text{spam}) = \frac{N_c + 1}{N + |C|} \tag{1}$$

$$\tag{2}$$

Background Probabilities

- For spam:

$$\hat{P}(c_j = \text{spam}) = \frac{N_c + 1}{N + |C|} \tag{1}$$

$$= \frac{4 + 1}{7 + 2} \tag{2}$$

$$= \frac{5}{9} \tag{3}$$

Background Probabilities

- For spam:

$$\hat{P}(c_j = \text{spam}) = \frac{N_c + 1}{N + |C|} \tag{1}$$

$$= \frac{4 + 1}{7 + 2} \tag{2}$$

$$= \frac{5}{9} \tag{3}$$

- For ham:

$$\tag{4}$$

Background Probabilities

- For spam:

$$\hat{P}(c_j = \text{spam}) = \frac{N_c + 1}{N + |C|} \tag{1}$$

$$= \frac{4 + 1}{7 + 2} \tag{2}$$

$$= \frac{5}{9} \tag{3}$$

- For ham:

$$\hat{P}(c_j = \text{ham}) = \frac{N_c + 1}{N + |C|} \tag{4}$$

$$\tag{5}$$

Background Probabilities

- For spam:

$$\hat{P}(c_j = \text{spam}) = \frac{N_c + 1}{N + |C|} \tag{1}$$

$$= \frac{4 + 1}{7 + 2} \tag{2}$$

$$= \frac{5}{9} \tag{3}$$

- For ham:

$$\hat{P}(c_j = \text{ham}) = \frac{N_c + 1}{N + |C|} \tag{4}$$

$$= \frac{3 + 1}{7 + 2} \tag{5}$$

$$= \frac{4}{9} \tag{6}$$

Conditional Probabilities

D1: Spam

abuja man

D3: Spam

cialis deal

D5: Spam

abuja deal

D7: Spam

cialis dog

D2: Ham

man dog

D4: Ham

logistic mother logistic abuja

D6: Ham

bagel deal

What's the conditional probability $\hat{P}(w = \text{dog} \mid c)$?

Conditional Probabilities

- For spam:

$$(7)$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{dog} \,|\, c) = \frac{T_{cw} + 1}{\left(\sum_{w' \in V} T_{cw'}\right) + |V|} \tag{7}$$

$$\tag{8}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{dog} \,|\, c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{7}$$

$$= \frac{1 + 1}{8 + 8} \tag{8}$$

$$\tag{9}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{dog} \mid c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{7}$$

$$= \frac{1 + 1}{8 + 8} \tag{8}$$

$$= \frac{1}{8} \tag{9}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{dog} \,|\, c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{7}$$

$$= \frac{1 + 1}{8 + 8} \tag{8}$$

$$= \frac{1}{8} \tag{9}$$

- For ham:

$$\tag{10}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{dog} \mid c) = \frac{T_{cw} + 1}{\left(\sum_{w' \in V} T_{cw'}\right) + |V|} \tag{7}$$

$$= \frac{1 + 1}{8 + 8} \tag{8}$$

$$= \frac{1}{8} \tag{9}$$

- For ham:

$$\hat{P}(w = \text{dog} \mid c) = \frac{T_{cw} + 1}{\left(\sum_{w' \in V} T_{cw'}\right) + |V|} \tag{10}$$

$$\tag{11}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{dog} \,|\, c) = \frac{T_{cw} + 1}{\left(\sum_{w' \in V} T_{cw'}\right) + |V|} \tag{7}$$

$$= \frac{1 + 1}{8 + 8} \tag{8}$$

$$= \frac{1}{8} \tag{9}$$

- For ham:

$$\hat{P}(w = \text{dog} \,|\, c) = \frac{T_{cw} + 1}{\left(\sum_{w' \in V} T_{cw'}\right) + |V|} \tag{10}$$

$$= \frac{1 + 1}{8 + 8} \tag{11}$$

$$\tag{12}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{dog} \mid c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{7}$$

$$= \frac{1+1}{8+8} \tag{8}$$

$$= \frac{1}{8} \tag{9}$$

- For ham:

$$\hat{P}(w = \text{dog} \mid c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{10}$$

$$= \frac{1+1}{8+8} \tag{11}$$

$$= \frac{1}{8} \tag{12}$$

Prediction

What if you saw a document with the word "dog"?

Prediction

What if you saw a document with the word "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{13}$$

$$\tag{14}$$

Prediction

What if you saw a document with the word "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{13}$$

$$= \frac{5}{9} \cdot \frac{1}{8} \tag{14}$$

$$\tag{15}$$

Prediction

What if you saw a document with the word "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{13}$$

$$= \frac{5}{9} \cdot \frac{1}{8} \tag{14}$$

$$= 0.07 \tag{15}$$

Prediction

What if you saw a document with the word "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{13}$$

$$= 0.07 \tag{14}$$

- For ham:

$$\tag{15}$$

Prediction

What if you saw a document with the word "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \tag{13}$$

$$= 0.07 \tag{14}$$

- For ham:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \tag{15}$$

$$\tag{16}$$

Prediction

What if you saw a document with the word "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \tag{13}$$

$$= 0.07 \tag{14}$$

- For ham:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \tag{15}$$

$$= \frac{4}{9} \cdot \frac{1}{8} \tag{16}$$

$$\tag{17}$$

Prediction

What if you saw a document with the word "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{13}$$

$$= 0.07 \tag{14}$$

- For ham:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{15}$$

$$= \frac{4}{9} \cdot \frac{1}{8} \tag{16}$$

$$= 0.06 \tag{17}$$

Prediction

What if you saw a document with the word "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \qquad (13)$$

$$= 0.07 \qquad (14)$$

- For ham:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \qquad (15)$$

$$= 0.06 \qquad (16)$$

These aren't probabilities? What if we wanted the real probabilities?

Conditional Probabilities

- For spam:

$$\text{(17)}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{logistic} \,|\, c) = \frac{T_{cw} + 1}{\left(\sum_{w' \in V} T_{cw'}\right) + |V|} \tag{17}$$

$$\tag{18}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{logistic} \,|\, c) = \frac{T_{cw} + 1}{\left(\sum_{w' \in V} T_{cw'}\right) + |V|} \tag{17}$$

$$= \frac{0 + 1}{8 + 8} \tag{18}$$

$$= \frac{1}{16} \tag{19}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{logistic} \,|\, c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{17}$$

$$= \frac{0 + 1}{8 + 8} \tag{18}$$

$$= \frac{1}{16} \tag{19}$$

- For ham:

$$\tag{20}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{logistic} \,|\, c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{17}$$

$$= \frac{0 + 1}{8 + 8} \tag{18}$$

$$= \frac{1}{16} \tag{19}$$

- For ham:

$$\hat{P}(w = \text{logistic} \,|\, c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{20}$$

$$\tag{21}$$

Conditional Probabilities

- For spam:

$$\hat{P}(w = \text{logistic} \mid c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{17}$$

$$= \frac{0 + 1}{8 + 8} \tag{18}$$

$$= \frac{1}{16} \tag{19}$$

- For ham:

$$\hat{P}(w = \text{logistic} \mid c) = \frac{T_{cw} + 1}{(\sum_{w' \in V} T_{cw'}) + |V|} \tag{20}$$

$$= \frac{2 + 1}{8 + 8} \tag{21}$$

$$= \frac{3}{16} \tag{22}$$

Prediction

What if you saw a document with the words "logistic" "logistic" "dog"?

Prediction

What if you saw a document with the words "logistic" "logistic" "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{23}$$

$$\tag{24}$$

Prediction

What if you saw a document with the words "logistic" "logistic" "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \tag{23}$$

$$= \frac{5}{9} \cdot \frac{1}{8} \cdot \frac{1}{16} \cdot \frac{1}{16} \tag{24}$$

$$\tag{25}$$

Prediction

What if you saw a document with the words "logistic" "logistic" "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \qquad (23)$$

$$= \frac{5}{9} \cdot \frac{1}{8} \cdot \frac{1}{16} \cdot \frac{1}{16} \qquad (24)$$

$$= 0.0002 \qquad (25)$$

Prediction

What if you saw a document with the words "logistic" "logistic" "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{23}$$

$$= 0.0002 \tag{24}$$

- For ham:

$$\tag{25}$$

Prediction

What if you saw a document with the words "logistic" "logistic" "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \tag{23}$$

$$= 0.0002 \tag{24}$$

- For ham:

$$P(c|d) \propto P(c) \prod_{1 \leq i \leq n_d} P(w_i|c) \tag{25}$$

$$\tag{26}$$

Prediction

What if you saw a document with the words "logistic" "logistic" "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{23}$$

$$= 0.0002 \tag{24}$$

- For ham:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{25}$$

$$= \frac{4}{9} \cdot \frac{1}{8} \cdot \frac{3}{16} \cdot \frac{3}{16} \tag{26}$$

$$\tag{27}$$

Prediction

What if you saw a document with the words "logistic" "logistic" "dog"?

- For spam:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{23}$$

$$= 0.0002 \tag{24}$$

- For ham:

$$P(c|d) \propto P(c) \prod_{1 \le i \le n_d} P(w_i|c) \tag{25}$$

$$= \frac{4}{9} \cdot \frac{1}{8} \cdot \frac{3}{16} \cdot \frac{3}{16} \tag{26}$$

$$= 0.002 \tag{27}$$

HW2

- Posted this weekend
- Logistic regression w/ stochastic gradient
- Helpful to look at it before next week (very similar to in-class exercise)