



Classification: Logistic Regression

Machine Learning: Jordan Boyd-Graber
University of Maryland

LECTURE 1A

Slides adapted from Hinrich Schütze and Lauren Hannah

What are we talking about?

- Statistical classification: $p(y|x)$
- Classification uses: ad placement, spam detection
- Building block of other machine learning methods

Logistic Regression: Definition

- Weight vector β_i
- Observations X_i
- “Bias” β_0 (like intercept in linear regression)

$$P(Y = 0|X) = \frac{1}{1 + \exp[\beta_0 + \sum_i \beta_i X_i]} \quad (1)$$

$$P(Y = 1|X) = \frac{\exp[\beta_0 + \sum_i \beta_i X_i]}{1 + \exp[\beta_0 + \sum_i \beta_i X_i]} \quad (2)$$

- For shorthand, we'll say that

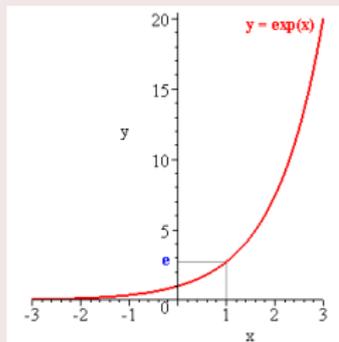
$$P(Y = 0|X) = \sigma(-(\beta_0 + \sum_i \beta_i X_i)) \quad (3)$$

$$P(Y = 1|X) = 1 - \sigma(-(\beta_0 + \sum_i \beta_i X_i)) \quad (4)$$

- Where $\sigma(z) = \frac{1}{1 + \exp[-z]}$

What's this “exp” doing?

Exponential



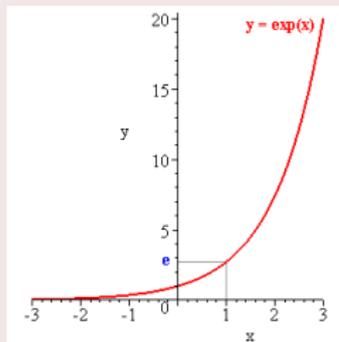
- $\exp[x]$ is shorthand for e^x
- e is a special number, about 2.71828
 - e^x is the limit of compound interest formula as compounds become infinitely small
 - It's the function whose derivative is itself
- The “logistic” function is $\sigma(z) = \frac{1}{1+e^{-z}}$
- Looks like an “S”
- Always between 0 and 1.

Logistic



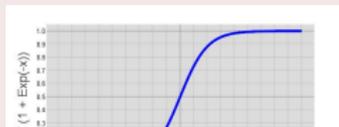
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- Looks like an "S"
- Always between 0 and 1.
 - Allows us to model probabilities
 - Different from **linear** regression

Logistic



Logistic Regression Example

feature	coefficient	weight
bias	β_0	0.1
“viagra”	β_1	2.0
“mother”	β_2	-1.0
“work”	β_3	-0.5
“nigeria”	β_4	3.0

Example 1: Empty Document?

$$X = \{\}$$

- What does $Y = 1$ mean?

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Example 1: Empty Document?

$$X = \{\}$$

- $P(Y = 0) = \frac{1}{1 + \exp[0.1]} = 0.48$
- $P(Y = 1) = \frac{\exp[0.1]}{1 + \exp[0.1]} = 0.52$
- Bias β_0 encodes the prior probability of a class

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Example 2

$X = \{\text{Mother, Nigeria}\}$

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$X = \{\text{Mother, Nigeria}\}$

- $$P(Y = 0) = \frac{1}{1 + \exp[0.1 - 1.0 + 3.0]} =$$
- $$P(Y = 1) = \frac{\exp[0.1 - 1.0 + 3.0]}{1 + \exp[0.1 - 1.0 + 3.0]} =$$
- Include bias, and sum the other weights

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$X = \{\text{Mother, Nigeria}\}$

- $$P(Y = 0) = \frac{1}{1 + \exp[0.1 - 1.0 + 3.0]} = 0.11$$
- $$P(Y = 1) = \frac{\exp[0.1 - 1.0 + 3.0]}{1 + \exp[0.1 - 1.0 + 3.0]} = 0.88$$
- Include bias, and sum the other weights

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$X = \{\text{Mother, Work, Viagra, Mother}\}$

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- Multiply feature presence by weight

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Example 3

$X = \{\text{Mother, Work, Viagra, Mother}\}$

- $$P(Y = 0) = \frac{1}{1 + \exp[0.1 - 1.0 - 0.5 + 2.0 - 1.0]} = 0.60$$
- $$P(Y = 1) = \frac{\exp[0.1 - 1.0 - 0.5 + 2.0 - 1.0]}{1 + \exp[0.1 - 1.0 - 0.5 + 2.0 - 1.0]} = 0.30$$
- Multiply feature presence by weight

How is Logistic Regression Used?

- Given a set of weights $\vec{\beta}$, we know how to compute the conditional likelihood $P(y|\beta, x)$
- Find the set of weights $\vec{\beta}$ that maximize the conditional likelihood on training data (next week)
- **Intuition:** higher weights mean that this feature implies that this feature is a good this is the class you want for this observation

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- Naïve Bayes is a special case of logistic regression that uses Bayes rule and conditional probabilities to set these weights

$$\arg \max_{c_j \in \mathcal{C}} [\ln \hat{P}(c_j) + \sum_{1 \leq i \leq n_d} \ln \hat{P}(w_i | c_j)]$$

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Contrasting Naïve Bayes and Logistic Regression

- Naïve Bayes easier
- Naïve Bayes better on smaller datasets
- Logistic regression better on medium-sized datasets
- On huge datasets, it doesn't really matter (data always win)
 - Optional reading by Ng and Jordan has proofs and experiments
- Logistic regression allows arbitrary features (biggest difference!)

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 - Optional reading by Ng and Jordan has proofs and experiments
- Logistic regression allows arbitrary features (biggest difference!)
- Don't need to memorize (or work through) previous slide—just understand that naïve Bayes is a special case of logistic regression

Next time ...

- How to learn the best setting of weights
- Regularizing logistic regression to encourage sparse vectors
- Extracting features