# Time: 70 minutes. Closed books. Closed notes.

**You are required to shut down and put all electronic devices away. Make sure that cell phones don’t make any noise during the exam.**

**Your algorithms should be as efficient as possible. Your algorithms and answers, in general, should also be as simple as possible.**

**Explain your answers.**

**All the questions have equal weight.**

# Good Luck!

 Your Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Honor Pledge**

The university has a nationally recognized Honor Pledge, administered by the Student Honor Council. The Student Honor Council proposed and the university Senate approved an Honor Pledge. The University of Maryland Honor Pledge reads:

"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination."

 Please write the exact wording of the Pledge followed by your signature in the space below:

Pledge: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Your signature:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

# Problem 1

Solve the following recurrence. For simplicity, confine your analysis to exact power of 2.

T(n) = 2T(n/2) + n lg n

# Problem 2

Let G(V,E) be a connected, undirected graph. For every e in E, w(e) is its (real-valued) ‘weight’. Let (S,V-S) be any cut of G.Let (u,v) be a light edge crossing (S,V-S) *[w(u,v) is minimal among the edges crossing S,V-S]*. Suppose that no two edge weights are equal. First, consider the following algorithm for building the graph F. Include in F a light edge for every cut (S,V-S) of G; in case the same edge is picked for F more than once, include in F only one copy of that edge.

 (i) What is the complexity of this algorithm?

(ii) How many edges will F include? Prove you answer.

(iii) Second, provide the most efficient algorithm you can for building F. What is its complexity?

(iv) Run your (efficient) algorithm on the following graph.

A

B

E

D

C

F

6

9

11

10

8

5

3

2

7

1

4

# Problem 3

Given are two text strings A of length n and B of length m. You want to transform A into B with a minimum number of operations of the following types: delete a character from A, insert a character into A, or change some character in A into a new character. The minimal number of such operations required to transform A into B is called the edit distance between A and B.

(i) Provide the most efficient algorithm you can for computing the edit distance between A and B?

(ii) What is its complexity?

(iii) Given two strings X=ACGCTAC, Y=CTGACA, show all stages of your algorithm for finding their edit distance.

**Problem 4**

We can generalize Huffman's algorithm to ternary codewords (i.e., codewords using the symbols 0,1, and 2).

(i) Given the set of frequencies a:1, b:1, c:2, d:3, e:5, f:8, g:13, h:21, i:22 show all stages in Huffman's algorithm to generate optimal ternary code.

(ii) What is the complexity of the ternary Huffman's algorithm?

# Problem 5

Apply loop invariant to prove the correctness of Iterative Tree Search algorithm. The pseudocode follows.

**Iterative-Tree-Search**(x,k)

 **While** x ≠ NIL and k ≠ x.key **do**

 **if** k < x.key

 **then** x = x.left

 **else** x = x.right

 **end if**

 **end while**

 **return** x

State your loop invariant, and use it to prove correctness.