# Time: 70 minutes. Closed books. Closed notes.

**You are required to shut down and put all electronic devices away. Make sure that cell phones don’t make any noise during the exam.**

**Your algorithms should be as efficient as possible. Your algorithms and answers, in general, should also be as simple as possible.**

**Explain your answers.**

**All the questions have equal weight.**

# Good Luck!

# Problem 1

Is each of the statement below true or false? Explain your answers.

(a) T(n)=T(9n/10)+T(n/10)+cn implies that T(n)=Θ(n2).

(b) A red-black tree with n internal nodes has height at most 2lg(n+1).

# Problem 2

Apply loop invariant to prove the correctness of Partition algorithm used for Quicksort. The pseudo code follows.

PARTITION (A, p, r)

1. x <- A[r]
2. i <- p-1
3. for j <- p to r-1
4. do if A[j]<=x
5. then i <- i +1
6. Exchange A[i] <-> A[j]
7. Exchange A[i+1] <-> A[r]
8. Return i+1

Write down your loop invariant, and use it to prove correctness.

# Problem 3

(a) When is a collection of hash function *universal*? Provide a formal definition.

(b) H is a set of two hash functions h1 and h2, defined below, from the universe U = {1, 2, 3, 4} to {0, 1}.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| U | 1 | 2 | 3 | 4 |
| h1 | 0 | 1 | 1 | 0 |
| h2 | 0 | 0 | 1 | 1 |

 Is H universal? Explain your answer.

# Problem 4

(a) Given the set of frequencies a :26 b :5 c : 9 d :13 e : 40 f : 7 show all the stages in HUFFMAN’s algorithm to generate optimal code.

 (b) What is the time complexity of HUFFMAN’s algorithm?

# Problem 5

5. (a) Given are two strings X=ABCABA, Y=BABCDA, show all stages of the dynamic programming algorithm for finding the length of their longest common subsequence (LCS).

(b) What are the time and space requirements of this LCS algorithm?

# Problem 6

6. (a) Given the weighted, undirected graph below, apply Prim’s algorithm to generate the minimum spanning tree(MST). Show step by step how each of the edges in the MST gets added. Use D as the starting vertex.

A

B

E

D

C

F

5

2

4

3

4

2

3

2

6

1

4

(b) Suppose the min-priority queue Q used by Prim’s algorithm is implemented by binary min-heap. Analyze the total time for Prim’s algorithm for a graph with n vertices and m edges.