# Time: 70 minutes. Closed books. Closed notes.

**You are required to shut down and put all electronic devices away. Make sure that cell phones don’t make any noise during the exam.**

**Your algorithms should be as efficient as possible. Your algorithms and answers, in general, should also be as simple as possible.**

**Explain your answers.**

**All the questions have equal weight.**

# Good Luck!

 Your Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Honor Pledge**

The university has a nationally recognized Honor Pledge, administered by the Student Honor Council. The Student Honor Council proposed and the university Senate approved an Honor Pledge. The University of Maryland Honor Pledge reads:

"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination."

 Please write the exact wording of the Pledge followed by your signature in the space below:

Pledge: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Your signature:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

# Problem 1

You are asked to radix-sort n integers whose range is 0 to k, k > n. How many rounds of count sort you need? You answer should be a simple formula.

# Problem 2

1. Apply loop invariants to prove the correctness of Bubble Sort algorithm. The pseudo code follows.

BUBBLESORT.A
1 for i =1 to A.length - 1
2 for j = A.length downto i + 1
3 if A[j] < A[j-1]
4 exchange A[j] with A[j- 1]

Write down a loop invariant for the internal loop and another for the external loop, and use these invariants to prove correctness.

1. What is the running time of Bubble Sort?

# Problem 3

1. Given the Binary Search Tree below, there are some keys that, if inserted, will increase the height of the tree. Find an integer key (distinct from those already in the tree) for which this is true.

 95

 / \

 40 113

 / \ / \

 37 42 111 125

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 39 112

1. If the tree in the previous part was a Red Black Tree, show the changes that would happen if the same element is inserted into the tree. Does the height of the tree change? Why or why not?

 95

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 40 113

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 37 42 111 125

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 39 112

**Problem 4**

Given a rooted binary tree with weighted vertices, provide an efficient algorithm to find the maximum weight of any subset of vertices for which no two vertices are connected by an edge. Assume the tree is stored in an array like a heap.

The following discussion may help you solve the above problem. Denote the rooted tree T. Let T' be a subtree of T, v be the root of T', u be the parent of v in T and T'' be the subtree rooted at u.  Observation: Vertex v can either be in the optimal solution for T'  or not. Hint: Consider the optimal solution for the subtree T''; both options (noted in the observation) are of interest.

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# Problem 5

Solve the following recurrences:

*T(1) = 1*

 T(n) = T(n/2) + T(n/4) + T(n/8) + n

*T(1) = 1*

*T(n) = T(n-1) + n2*

# Problem 6

There are n white dots and n black dots, equally spaced, in a line. You want to connect each white dot with one black dot, with a minimum total length of “wire”.

1. Provide an efficient algorithm for connecting the dots using minimum length of wire.
2. Show the result of your algorithm for the following formation.
3. Prove that your algorithm uses the minimum total length of wire.