Thermal modeling on rough surfaces

- Airless bodies have strong horizontal temperature gradients due to shadows cast by rugged topography.
- Lunar cold traps exist because of terrain shadowing and are defined by surface temperature.
- Thermal models must incorporate shadows, but also long- and short-wave radiation between surface elements [3].
- Element-to-element radiation dominates runtime.

Contribution

- We develop a fast algorithm for solving the equations governing scattering of long- and short-wave radiation.
- Solve two discretized radiosity equations to compute temp.
- Offline, we precompute a compressed low-rank version of the discretized kernel matrix by compressing low-rank off-diagonal blocks using sparse SVDs.
- Online, multiplication requires nearly $O(N)$ time, where $N$ is the number of triangular elements.
- The matrix only depends on the geometry of the planet so it can be used for simulations spanning a long time.

Physical model [5]

- Energy balance on the surface:
  \[ \omega T^4 + (1 - \rho)E + Q = \rho_0 Q \] (3)

  where \( \rho \) = albedo
  \( E \) = incoming solar radiation (insolation) [W m\(^{-2}\)]
  \( Q \) = reflected sunlight [W m\(^{-2}\)]
  \( \rho_0 \) = thermal emission [W m\(^{-2}\)]
  \( T \) = temperature [K]
  \( \sigma \) = emissivity
  \( \sigma_T \) = Stefan-Boltzmann constant [W m\(^{-2}\) K\(^{-4}\)]

- Governing equations for scattering:
  \[ Q(\vec{x}) = \frac{1}{2} \int_{S} \int_{S'} \rho_0(\vec{y}) \rho(\vec{y'}) Q(\vec{y'}) (\vec{F}(\vec{y}) \cdot \vec{F}(\vec{y'})) dA dA' \] (2)

  \[ Q(\vec{x}) = \frac{1}{2} \int_{S} \int_{S'} \rho_0(\vec{y}) (1 - \rho(\vec{y'})) dA \] (3)

  where \( S \) = surface of the planet or crater
  \( dA \) = surface area element [m\(^2\)]
  \( F(\vec{x}) \) = \( \int_{S} \rho(\vec{y}) (\vec{F}(\vec{y}) \cdot \vec{n}(\vec{y})) dA \)
  \( S_n \) = surface normal on \( S \)
  \( |x_n| \) = magnitude of \( \vec{n}(\vec{y}) \)
  \( \vec{F}(\vec{x}) = \frac{1}{2} [ f(\vec{x}) \vec{n}(\vec{x}) + x \vec{n}(\vec{x}) ] \)
  \( \vec{x} \) = visible field of view if \( \vec{r} \)

- We use this test problem to validate our numerical method.

The radiosity method

- Equations (2) and (3) are radiosity integral equations:
  \[ B(\vec{x}) = E(\vec{x}) + \rho(\vec{x}) (\vec{Q}(\vec{x}) + \rho(\vec{x}) \int_{V} \vec{Q}(\vec{y}) dV) \] (4)

  where \( B \) = radiosity [W m\(^{-2}\)]
  \( E \) = self-emitted radiosity [W m\(^{-2}\)]
  \( \rho \) = albedo
  \( F \) = geometric kernel

- Midpoint discretization concentration of (4) gives the system:
  \[ KB = (I - F) E + B = E \] (5)

  \[ F \] = \[ \begin{bmatrix} \begin{bmatrix} \rho(\vec{p}) & \ldots & \rho(\vec{p}_2) \end{bmatrix} & \begin{bmatrix} \rho(\vec{p}_2) & \rho(\vec{p}_3) \ldots \rho(\vec{p}_n) \end{bmatrix} & \ldots & \begin{bmatrix} \rho(\vec{p}_N-1) & \rho(\vec{p}_N) \end{bmatrix} \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \] (6)

  where \( F \) is the low-rank factor and:
  \( p_i \) = centroid of \( i \)-th triangle
  \( n_i \) = surface normal at \( p_i \)
  \( A_i \) = area of \( i \)-th triangle
  \( V_i \) = visibility between \( p_i \) and \( p_j \)

- Solving the discrete radiosity system:
  - The system (5) can be solved in a small number of Neumann or Jacobi iterations (typically 2 to 5).
  - Newly perfectly conditioned since it’s a discretized BIE.
  - Main challenge: fast multiplication by \( F \).

Low rank compression of form factor matrix \( F \)

- Spatial partitioning: use quadtrees or octrees to recursively partition triangular elements.
- Low-rank interactions: blocks of \( F \) that correspond to interactions between nonoverlapping cells in quadtrees or octrees are typically sparse with a dense low-rank subblock.
- SVV compression: compute SVD to find dense subblock and compress it within a given tolerance \( \epsilon \).
- Best low-rank approximation by SVD:
  \[ \min_{F, F_k} \| F - F_k \|_F \] (8)

  where \( F \) is a fixed rank, \( F_k \) = \( U_k \Sigma_k V_k \) is computed from the rank-\( k \) truncated SVD of \( F \), and \( \epsilon \) is the \( \ell_2 \)-singular value of \( F \).

- This approach is similar to the \( H \)-matrix format [1].

Examples of compressed form matrices \( F \)

- \( F \) = SVDblock, \( \epsilon \) = sparse block, \( \epsilon \) = dense block, \( \epsilon \) = zero block

\( F \) is used for simulations spanning a long time

- Equations (2) and (3) are used for simulations spanning a long time

Ingersoll test problem: numerical results

- Only part of the crater is illuminated (top-left).
- The steady state temperature in the shadow is constant (top-right).
- Pointwise error after one bounce is below \( \epsilon \) = 10\(^{-4}\) tolerance (bottom-left).
- Error between \( T \) and exact Ingersoll temperature is uniform (bottom-right).
- Errors are sensitive to the triangulation (bottom-right).

Ingersoll test problem: performance results

- We build the compressed \( F \) matrix with a tolerance of \( \epsilon = 10^{-4} \) and a maximum SVD rank of \( k = 60 \).
- We compare the exact steady state temperature in the shadow region given by (9) with the numerical steady state temperature vs. problem size (bottom-right).

References


Illumination and Temperature on Rough Terrain: Fast Methods for Solving the Radiosity Equation

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