1 Problem (Homework 5)

Compute matrix-vector product

$$\mathbf{v} = \mathbf{\Phi u}, \tag{1}$$

or

$$v_j = \sum_{i=1}^{N} \Phi_{ji} u_i, \quad j = 1, ..., M, \tag{2}$$

with absolute error $\epsilon < 10^{-6}$.

where

$$\mathbf{\Phi} = \begin{pmatrix} \Phi_{11} & \Phi_{12} & ... & \Phi_{1N} \\ \Phi_{21} & \Phi_{22} & ... & \Phi_{2N} \\ ... & ... & ... & ... \\ \Phi_{M1} & \Phi_{M2} & ... & \Phi_{MN} \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ ... \\ u_N \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ ... \\ v_M \end{pmatrix}, \tag{3}$$

$$\Phi_{ji} = \frac{1}{y_j - x_i}, \quad i = 1, ..., N, \quad j = 1, ..., M.$$

and $x_1, ..., x_N$ are random points uniformly distributed on [0,10], $M = N - 1$, and each y_j is located between the closest x_i's on each side, $j = 1, ..., N - 1$.

Homework 5

1. Derive expressions for S|S and R|R operators and implement them. Check if your program works correctly.

2. Write a program that implements both straightforward multiplication based on Eq. (2) and Multi Level FMM (MLFMM).

3. Provide a graph of the absolute maximum error between the straightforward and the MLFMM method for $N = 10^3$, and grouping parameter s varying between 1 and 100, and several p ($p \sim 10$).

4. Provide a graph of the CPU time vs s at fixed p that insures that the required accuracy is achieved. Find optimum s for your implementation.

5. Provide a graph that compares the CPU time required by the straightforward method and the MLFMM for N varying between 10^2 and 10^3 for straightforward and N varying between 10^2 and 10^4 for the MLFMM (use the optimum s found).

6. Find the "break-even" point (i.e. N at which the "Fast" method requires the same CPU time as the straightforward method) for your implementation.

7. Provide a graph of actual error (between the standard and the fast methods) for N varying between 10^2 and 10^3 and the truncation numbers used.

Hints

Use your previous homework programs to compute S|R-translation operators, and straightforward solution.