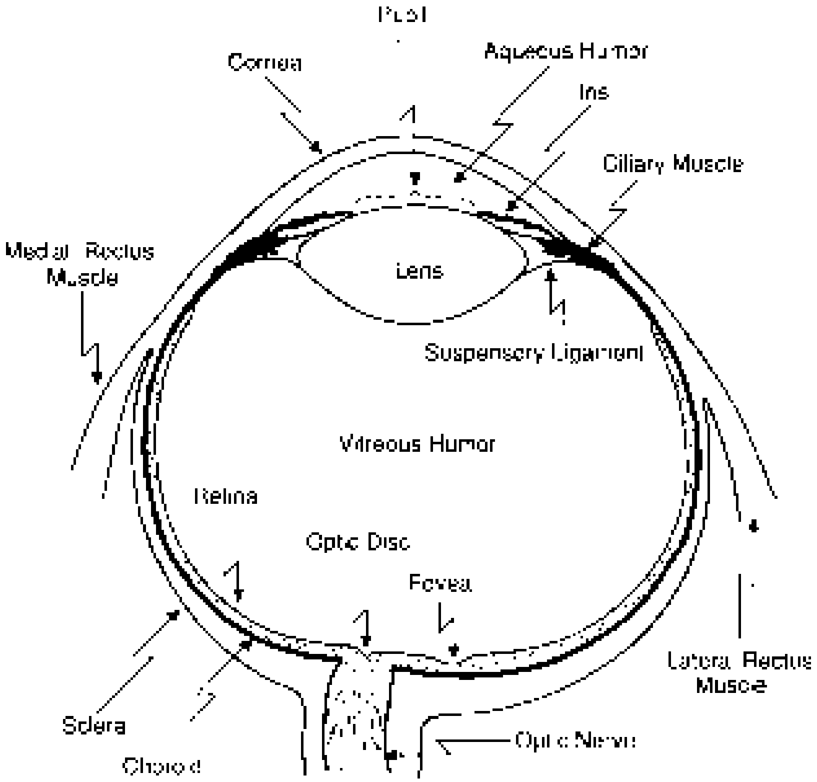
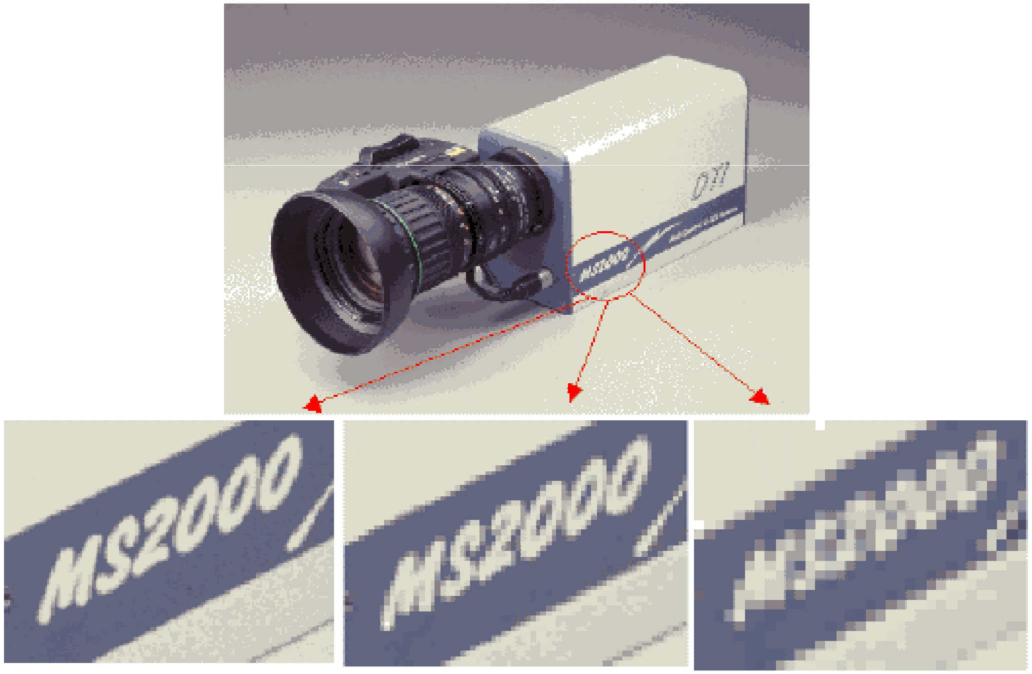
Image Formation in Man and Machines





High Resolution (1280 x 1024)

Standard Resolution (640 x 480)

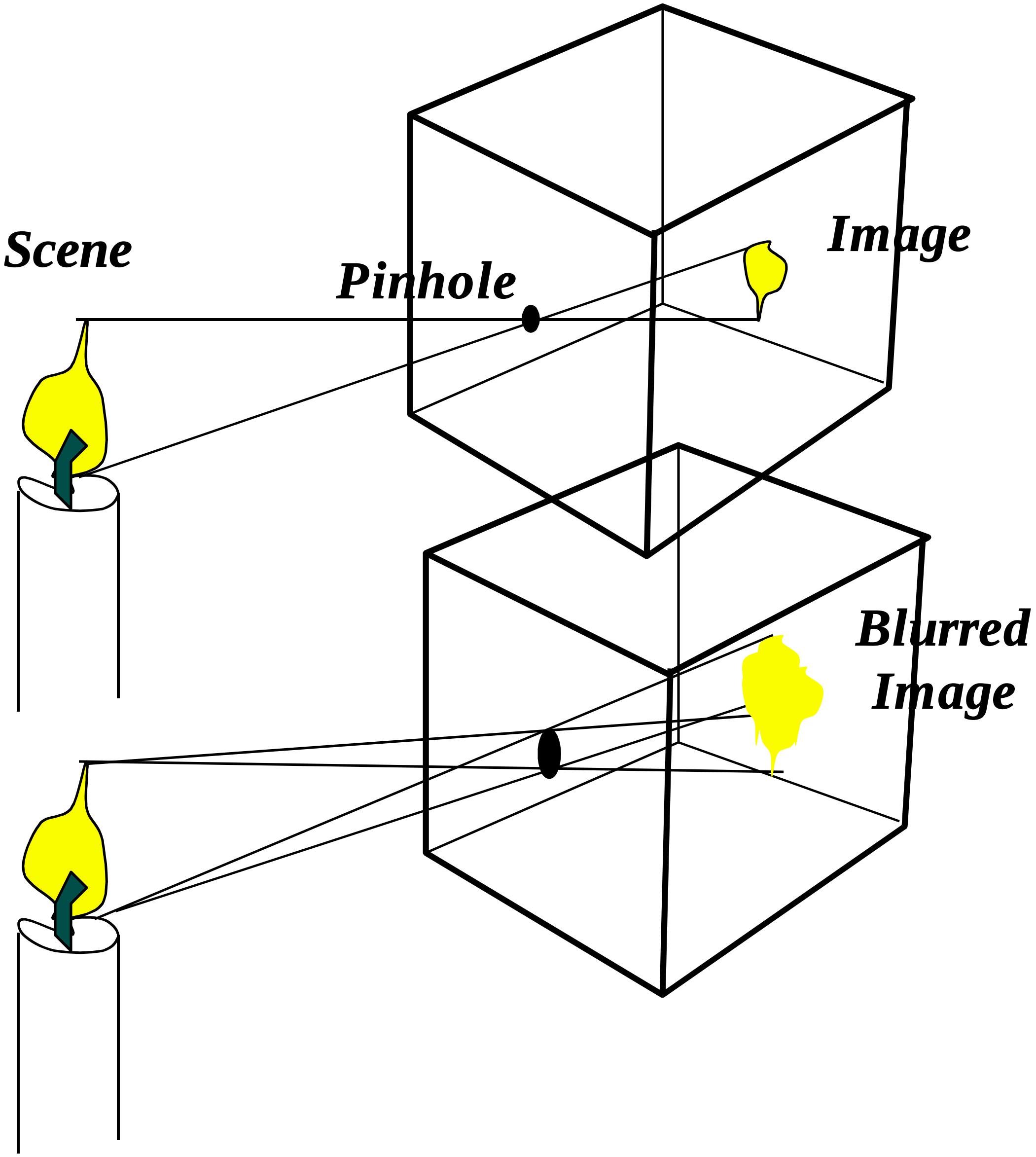
Approximate VCR Resolution

Overview

- Pinhole camera
- Refraction of light
- Thin-lens equation
- Optical power and accommodation
- Image irradiance and scene radiance
- Human eye
- Geometry of perspective imaging

Lens-less Imaging Systems - Pinhole Optics

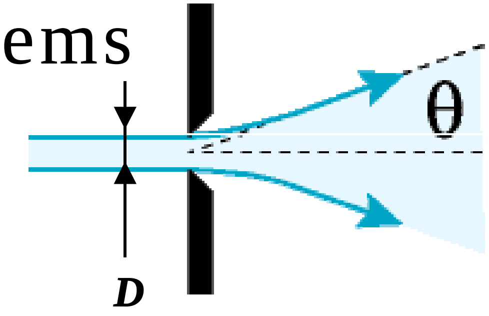
- Pinhole optics projects images
  - without lens
  - with infinite depth of field
- Smaller the pinhole
  - better the focus
  - less the light energy from any single point
- Good for tracking solar eclipses



Diffraction

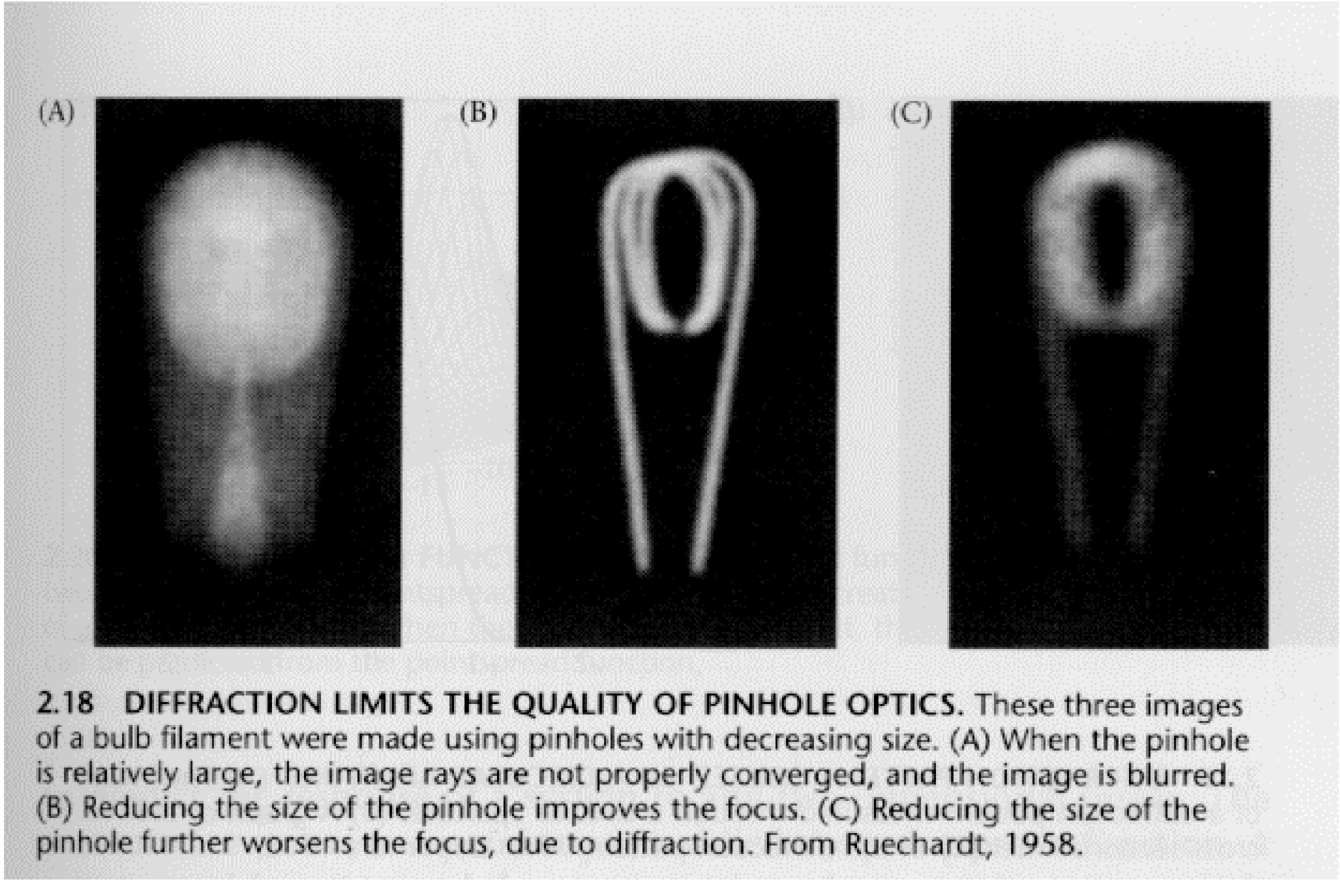
- Two disadvantages to pinhole systems
  - Low light collecting power
  - diffraction
- Diffraction
  - Light bends as it passes by the edge of a narrow aperture
- Human vision
  - at high light levels, pupil (aperture) is small and blurring is due to diffraction
  - at low light levels, pupil is open and blurring is due to lens imperfections

$$\theta = \frac{\lambda}{D}$$



Diffraction.

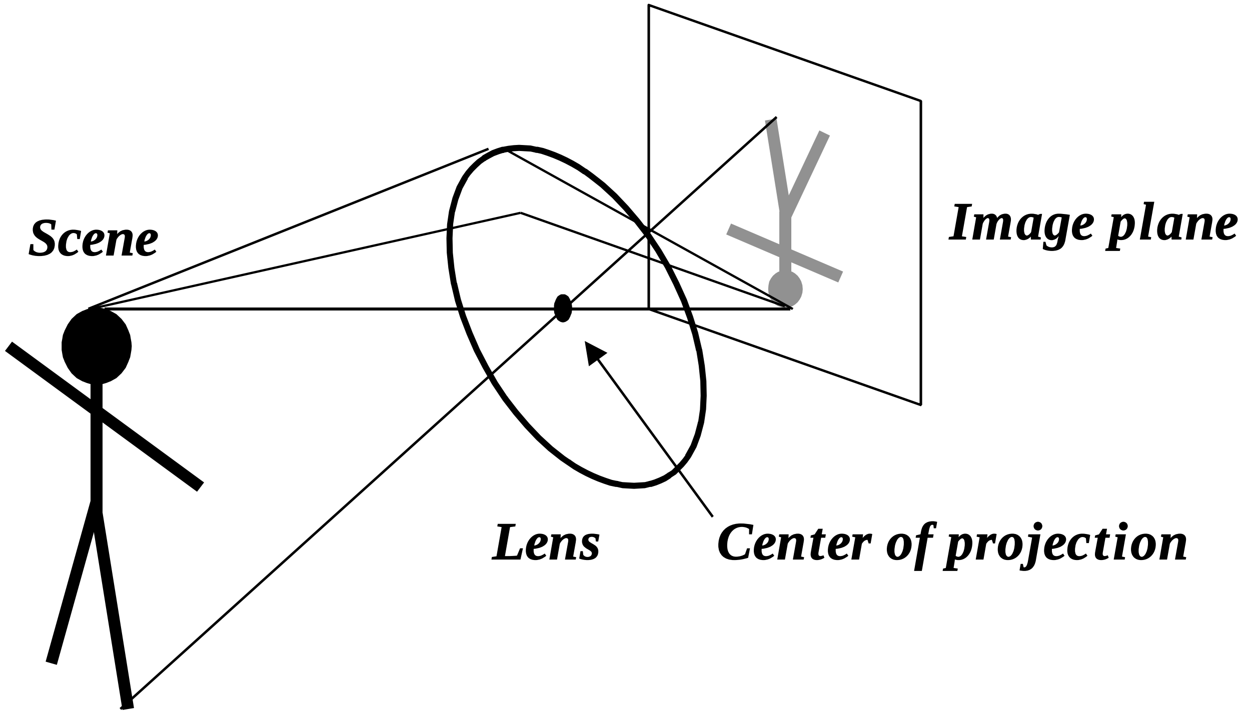
Diffraction and pinhole optics



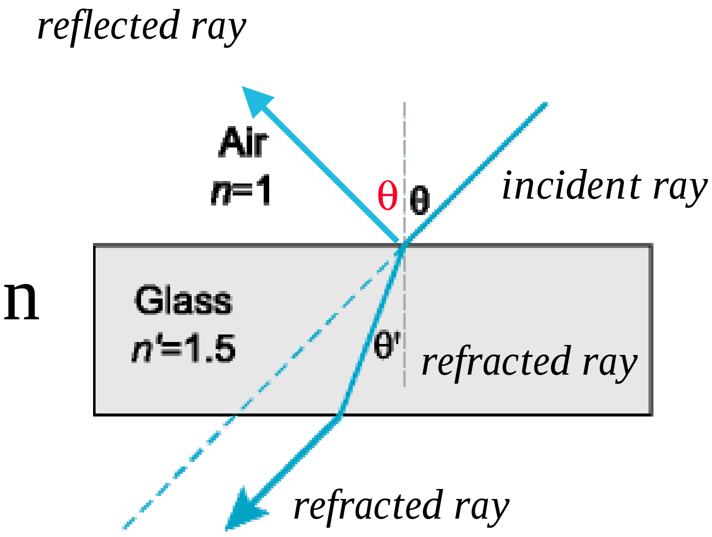
2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

Lenses Collect More Light

- With a lens, diverging rays from a scene point are converged back to an image point



Refraction: Snell's law



- If θ is the angle of incidence and θ ' is the angle of refraction then

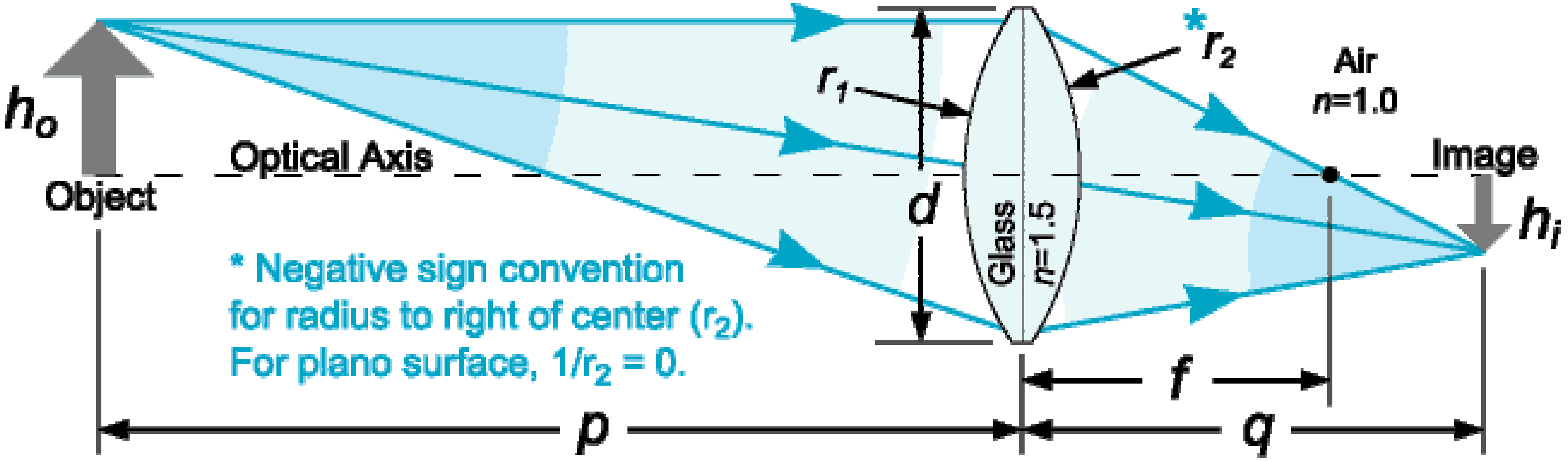
$$n \sin\theta = n' \sin\theta'$$

  where *n* and *n*' are the refractive indices of the two media

- Refractive index is the ratio of speed of light in a vacuum to speed of light in the medium

Refractive indices
  glass - 1.50
  water - 1.333
  air - 1.000

Lens Equations



* Negative sign convention for radius to right of center (r_2). For plano surface, $1/r_2 = 0$.

Lens Equation:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$$

Lens Maker's Equation:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Magnification:

$$m=\frac{h_i}{h_o}=-\frac{q}{p}$$

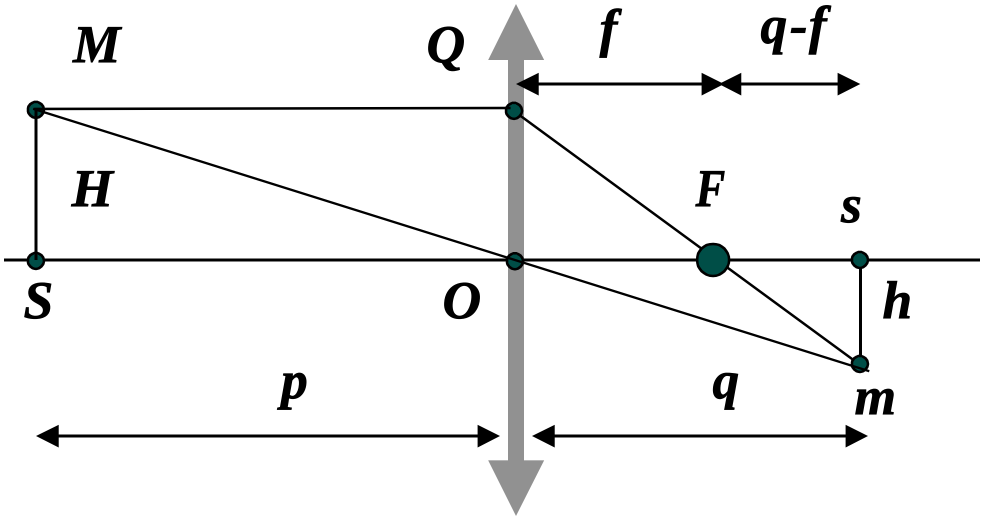
F-number:

$$f/\#=\frac{f}{d}$$

Some useful lens equations.

Thin-Lens Equation

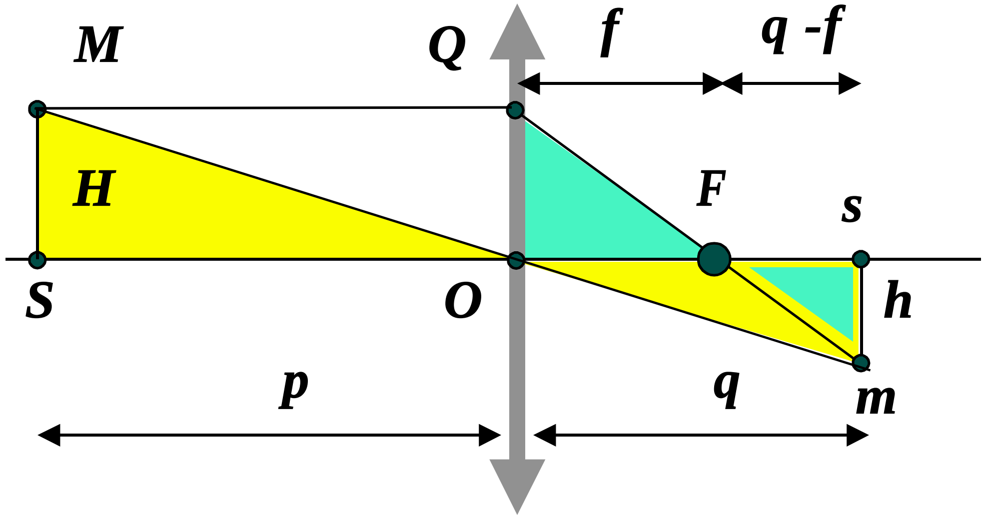
- Thin-lens equation
  - relates the distance between the scene point being viewed and the lens to the distance between the lens and the point's image (where the rays from that point are brought into focus by the lens)
  - Let ***M*** be a point being viewed
    - ***p*** is the distance of ***M*** from the lens along the optical axis
    - The thin lens focuses all the rays from ***M*** onto the same point, the image point ***m*** at distance ***q*** from the lens.



Thin-Lens Equation

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

- *m* can be determined by intersecting two known rays
  - ***MQ*** is parallel to the optical axis, so it must be refracted to pass through F.
  - MO passes through the lens center, so it is not bent.
- Note two pairs of similar triangles
  - MSO and Osm (yellow)
  - OQF and Fsm (green)



$$\frac{H}{p}=\frac{h}{q}=\frac{H+h}{p+q}$$

$$\frac{H}{f}=\frac{h}{q-f}=\frac{H+h}{q}$$

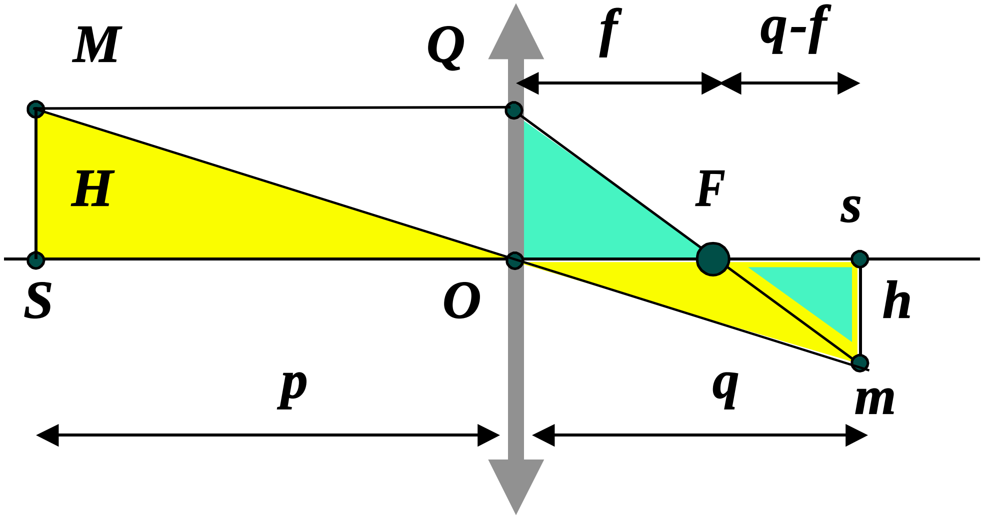
Divide 2 equations:

$$\frac{p}{f}=\frac{p+q}{q}$$

$$\frac{1}{f}=\frac{p+q}{p\,q}$$

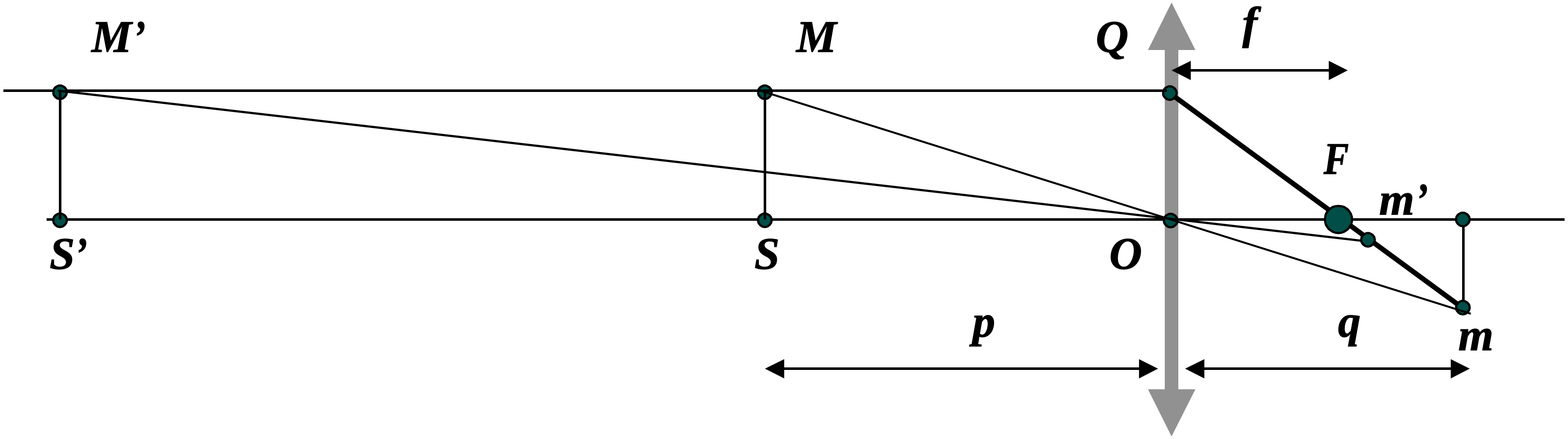
Thin-Lens Equation $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

- Notice that the distance behind the lens, ***q***, at which a point, ***M***, is brought into focus depends on ***p***, the distance of that point from the lens
  - familiar to us from rotating the focus ring of any camera



Thin-Lens Equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$



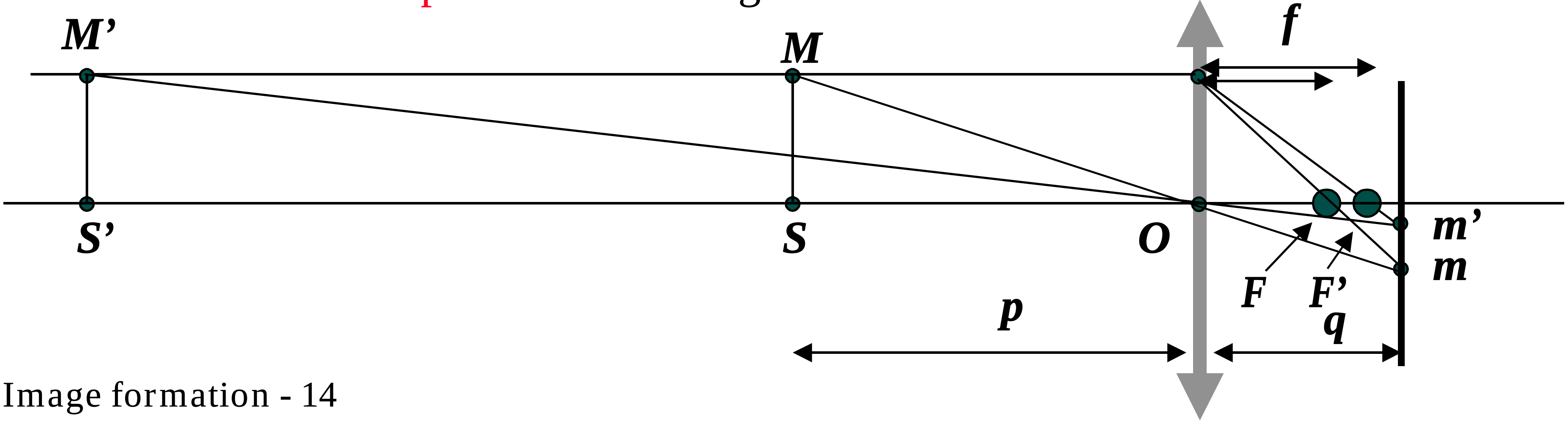
- As ***p*** gets large, ***q*** approaches ***f***
- As ***q*** approaches ***f***, ***p*** approaches infinity

Optical Power and Accommodation

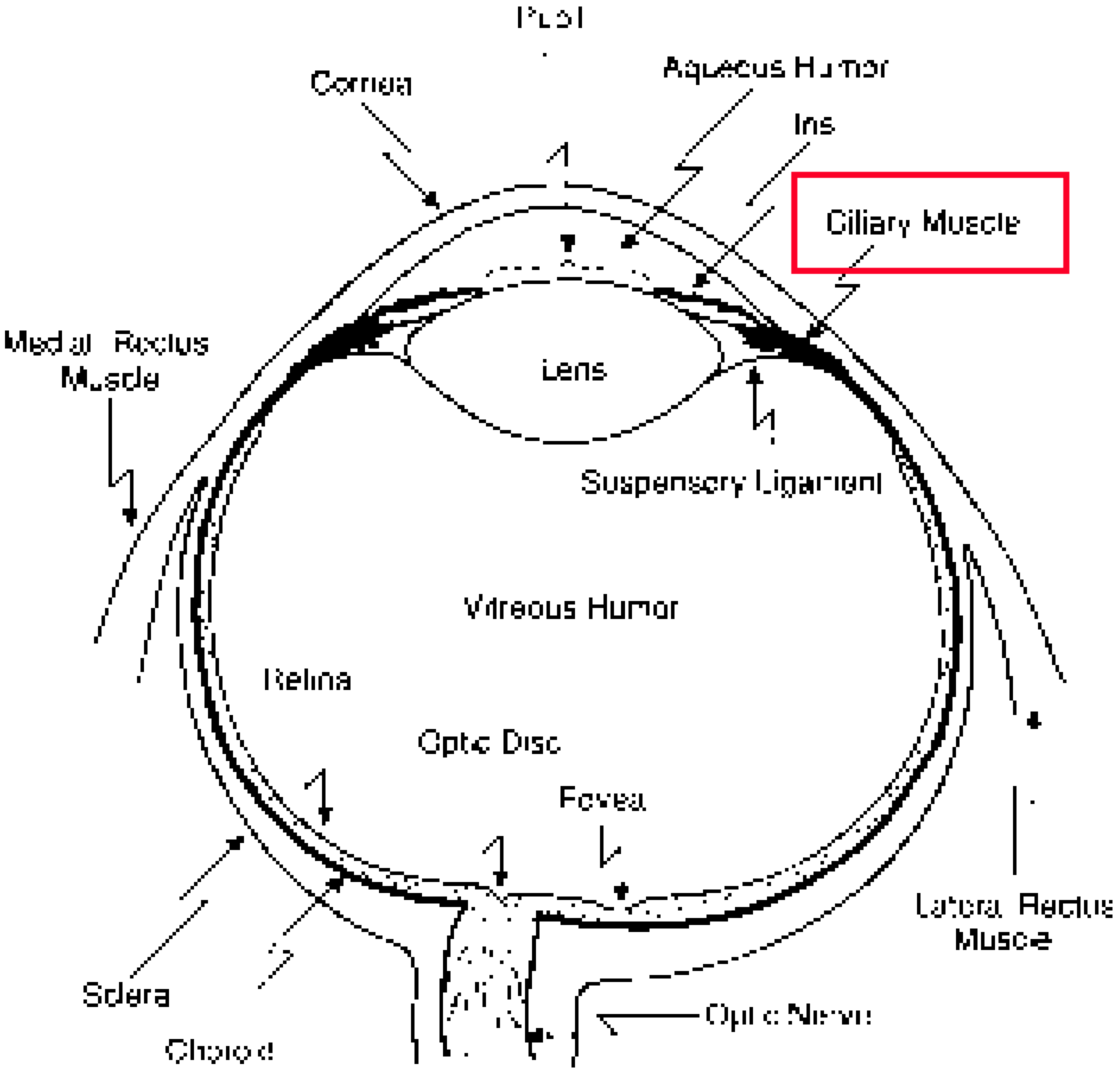
- Optical power of a lens - how strongly the lens bends the incoming rays
  - Short focal length lens bends rays significantly
  - It images a point source at infinity (large ***p***) at distance ***f*** behind the lens. The smaller ***f***, the more the rays must be bent to bring them into focus sooner.
  - Optical power is 1/***f***, with ***f*** measured in meters. The unit is called the diopter
  - Human vision: when viewing faraway objects the distance from the lens to the retina is 0.017m. So the optical power of the eye is 58.8 diopters

Accommodation

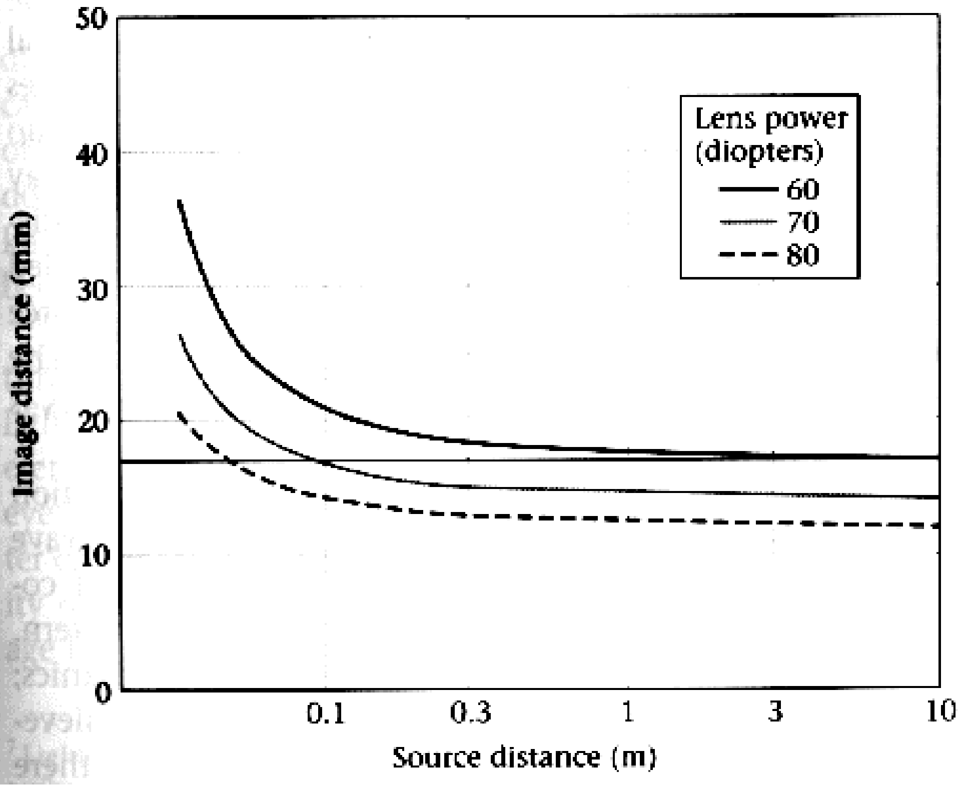
- How does the human eye bring nearby points into focus on the retina?
  - by increasing the power of the lens
  - muscles attached to the lens change its shape to change the lens power
  - accommodation: adjusting the focal length of the lens
  - bringing points that are nearby into focus causes faraway points to go out of focus
  - depth-of-field: range of distances in focus



Accommodation



Accommodation

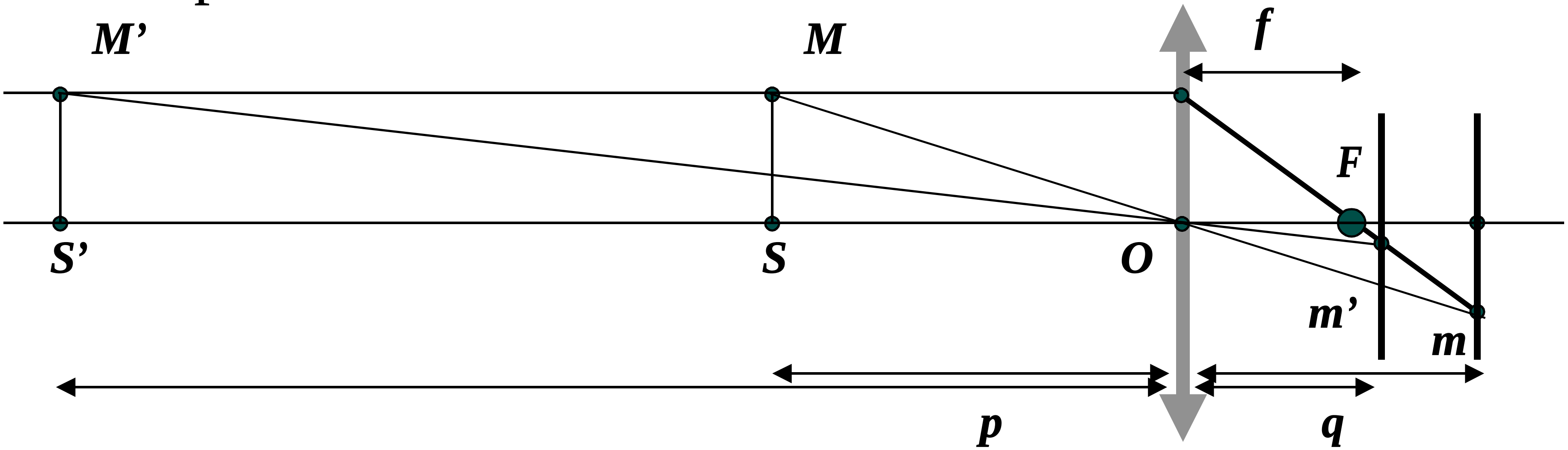


2.16 DEPTH OF FIELD OF THE HUMAN EYE. Image distance is shown as a function of source distance. The solid horizontal line shows the distance of the retina from the lens center. A lens power of 60 diopters brings distant objects into focus, but not nearby objects; to bring nearby objects into focus the power of the lens must increase. The depth of field (the distance over which objects will continue to be in reasonable focus) can be estimated from the slope of the curve.

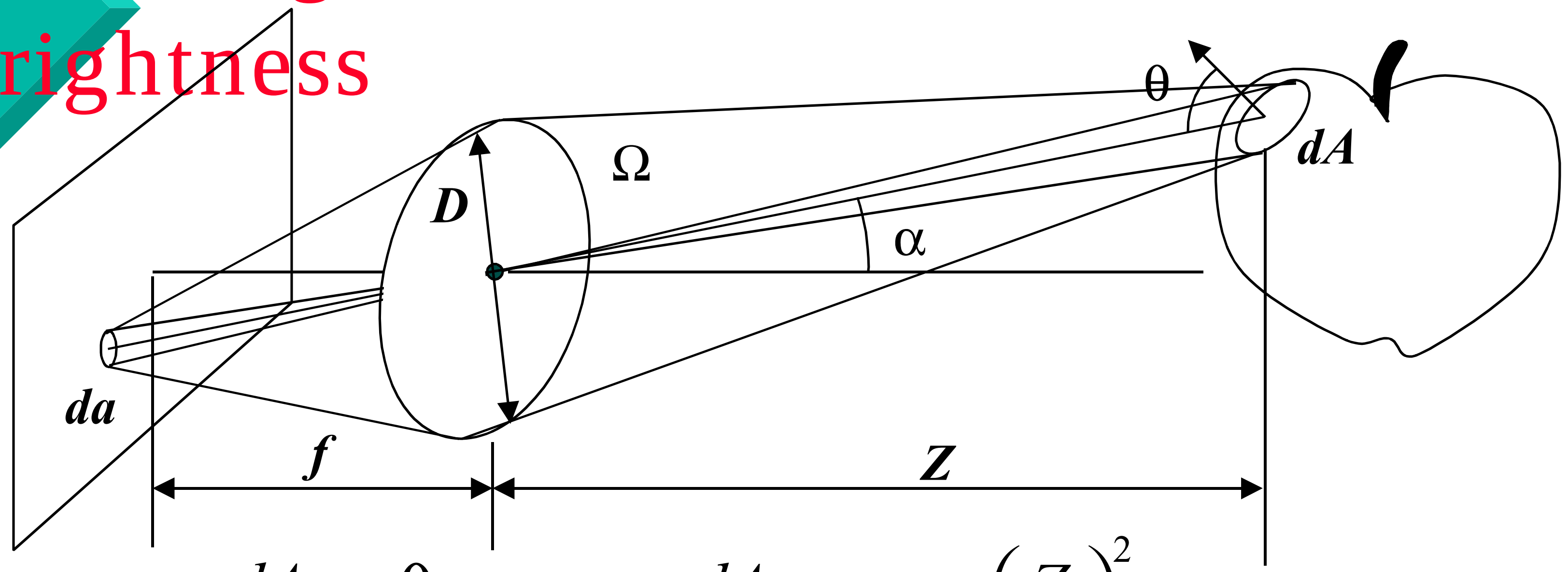
Sources at > 1 meter are imaged at same distance
Sources closer than 1 m are imaged at different distances

Accommodation

- Physical cameras - mechanically change the distance between the lens and the image plane



Pixel Brightness and Scene Brightness



$$\frac{da\cos\alpha}{(f/\cos\alpha)^2} = \frac{dA\cos\theta}{(Z/\cos\alpha)^2} \Longrightarrow \frac{dA}{da} = \frac{\cos\alpha}{\cos\theta}\left(\frac{Z}{f}\right)^2$$

$$dP = L\,dA\,\Omega\cos\theta \Longrightarrow dP = L\,dA\,\frac{\pi}{4}\left(\frac{D}{Z}\right)^2\cos^3\alpha\cos\theta$$

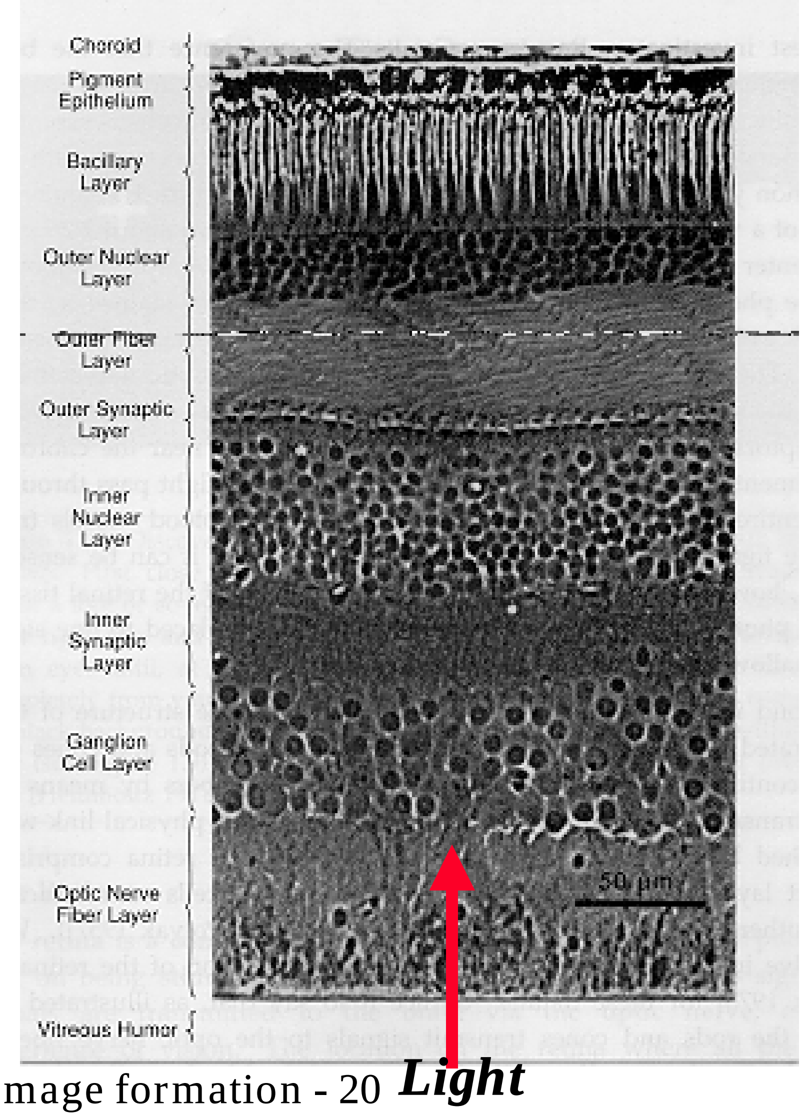
$$E = \frac{dP}{da} = L\frac{dA}{da}\frac{\pi}{4}\left(\frac{D}{Z}\right)^2\cos^3\alpha\,\cos\theta \Longrightarrow E = \frac{\pi}{4}\left(\frac{D}{f}\right)^2\cos^4\alpha\;L$$

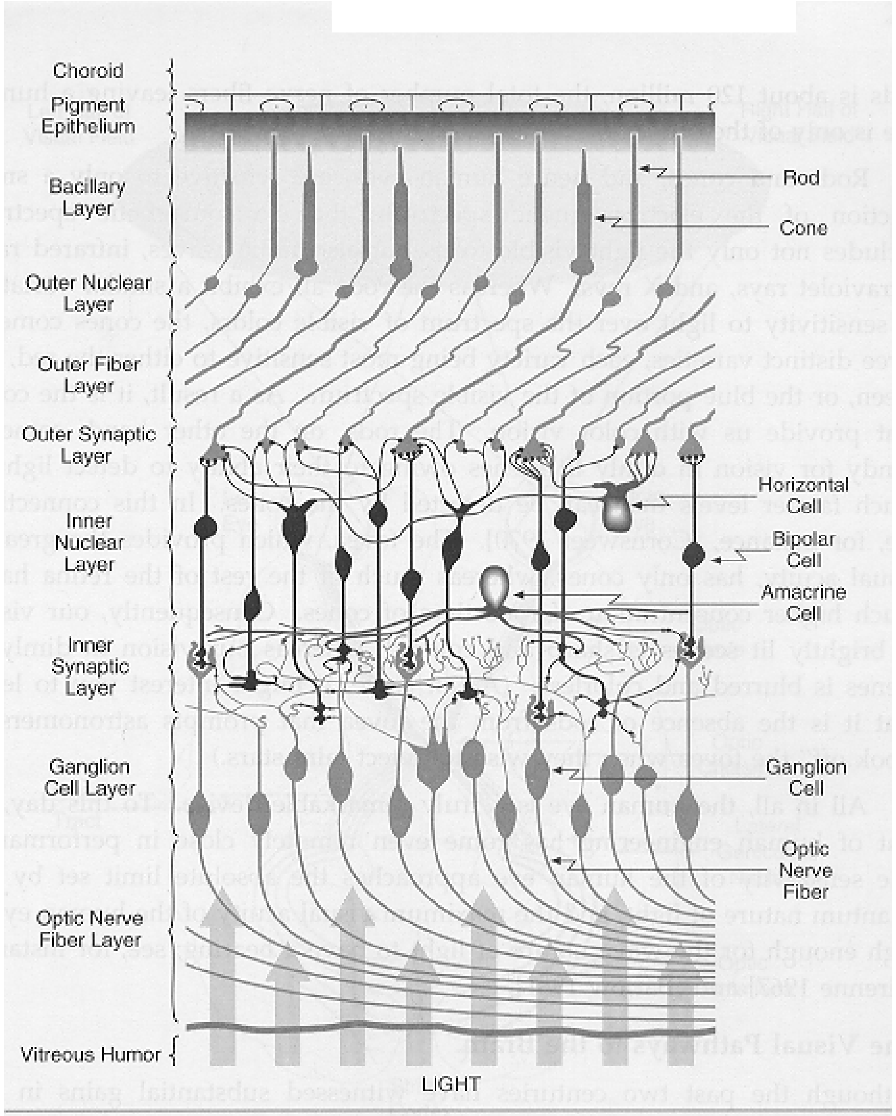
Image Irradiance and Scene Radiance

$$E = \frac{\pi}{4}\left(\frac{D}{f}\right)^2 \cos^4\alpha \;\; L$$

- Image irradiance E is proportional to scene radiance
- ***Brighter scene points produce brighter pixels***
- Image irradiance is proportional to inverse of square of f-number (*f*/*D*), is larger for small f-number

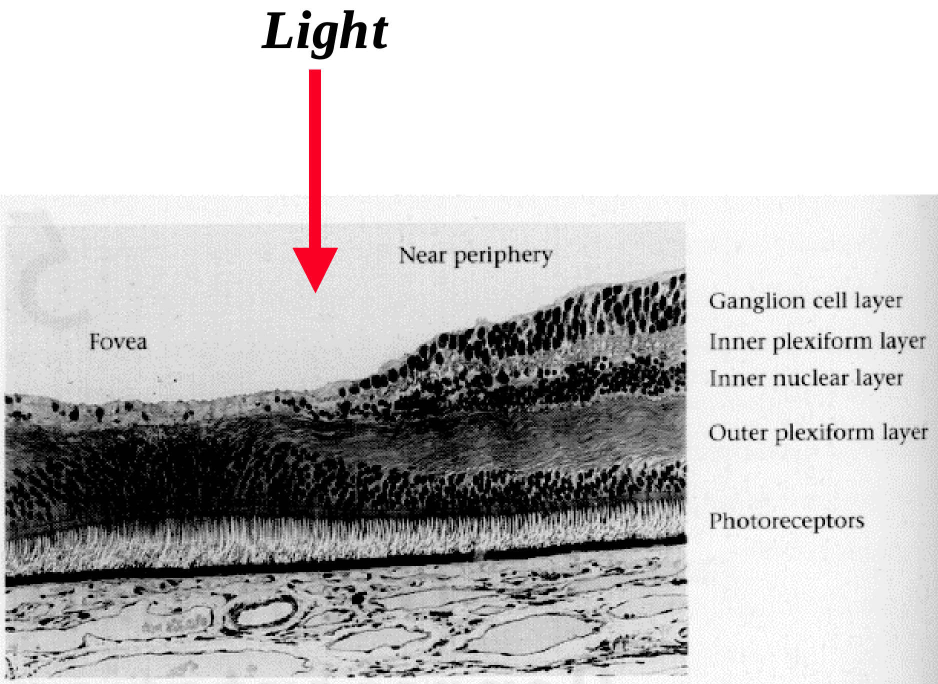
Retina


Choroid
Pigment Epithelium
Bacillary Layer
Outer Nuclear Layer
Outer Fiber Layer
Outer Synaptic Layer
Inner Nuclear Layer
Inner Synaptic Layer
Ganglion Cell Layer
Optic Nerve Fiber Layer
Vitreous Humor
50 µm
Light

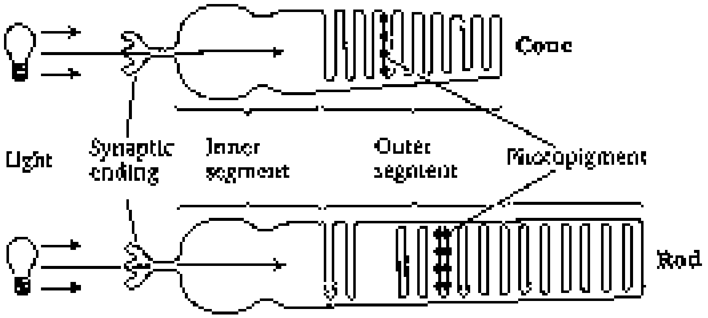

Choroid
Pigment Epithelium
Bacillary Layer
Outer Nuclear Layer
Outer Fiber Layer
Outer Synaptic Layer
Inner Nuclear Layer
Inner Synaptic Layer
Ganglion Cell Layer
Optic Nerve Fiber Layer
Vitreous Humor
Rod
Cone
Horizontal Cell
Bipolar Cell
Amacrine Cell
Ganglion Cell
Optic Nerve Fiber
LIGHT

Retina

- Limitations of human vision
  - Blood vessels and other cells in front of photoreceptors
  - shadows cast on photoreceptors
  - non-uniform brightness



5.1 **THE HUMAN RETINA,** seen here in cross-section, is a thin sheet of neural tissue that lines the back of the eye. In the periphery and near periphery, the retina is a multilayered structure. The cornea and lens would be at the top of this picture, so in the periphery light must pass through the several retinal layers before being absorbed by the photoreceptors. In the fovea, the retina consists of only a single layer of photoreceptors, as the neurons responsible for carrying the responses of the foveal cones are displaced to the side, out of the light path. Micrograph courtesy of Anita Hendrickson.

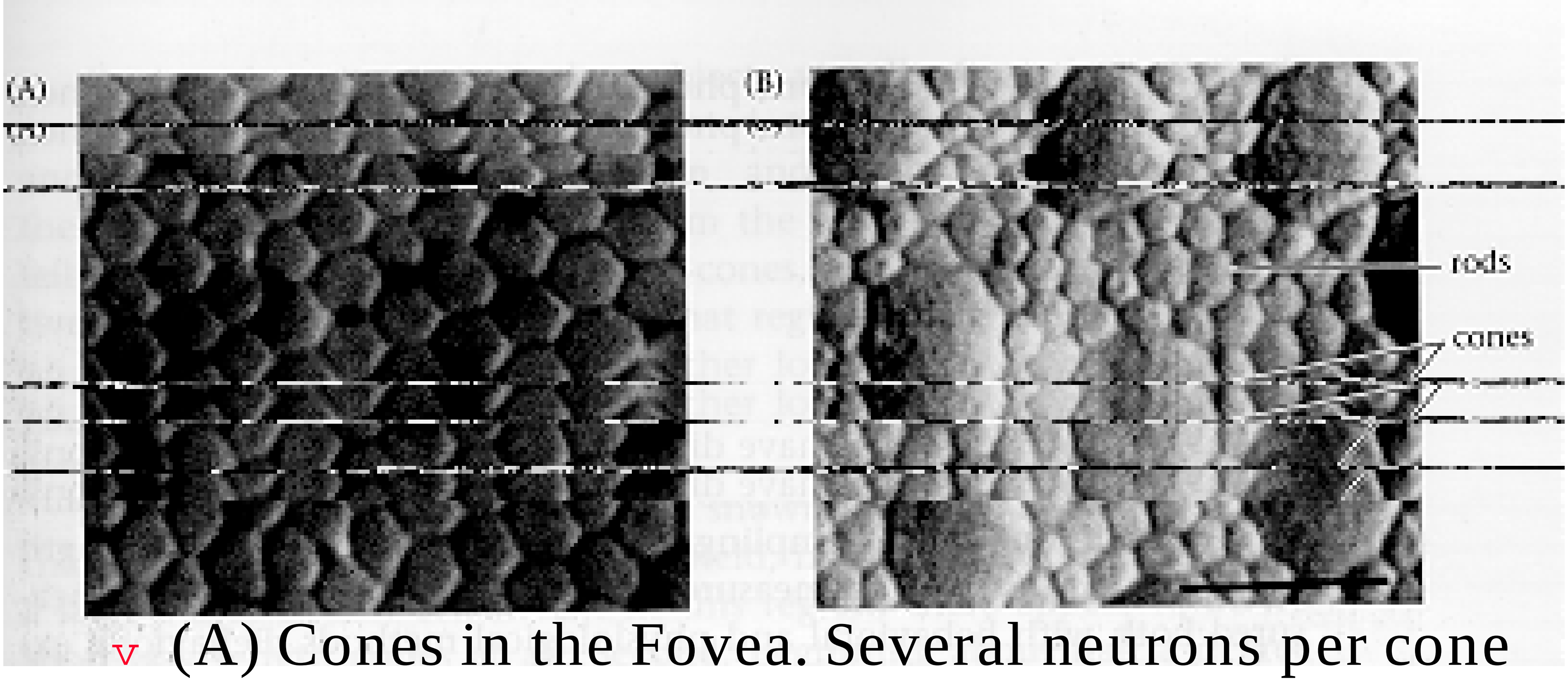


3.2 **MAMMALIAN ROD AND CONE PHOTORECEPTORS** contain the light-absorbing pigments that initiate vision. Light enters the photoreceptors through the inner segment and is funneled to the outer segment, which contains the photopigment.

Photoreceptor Mosaics

- The retina is covered with a mosaic of photoreceptors
- Two different types of photoreceptors
  - Rods - approximately 100,000,000
  - Cones - approximately 5,000,000
- Rods
  - Sensitive to low levels of light: scotopic light levels
- Cones
  - Sensitive to higher levels of light: photopic light levels
- Mesopic light levels - both rods and cones active

Photoreceptor Mosaics



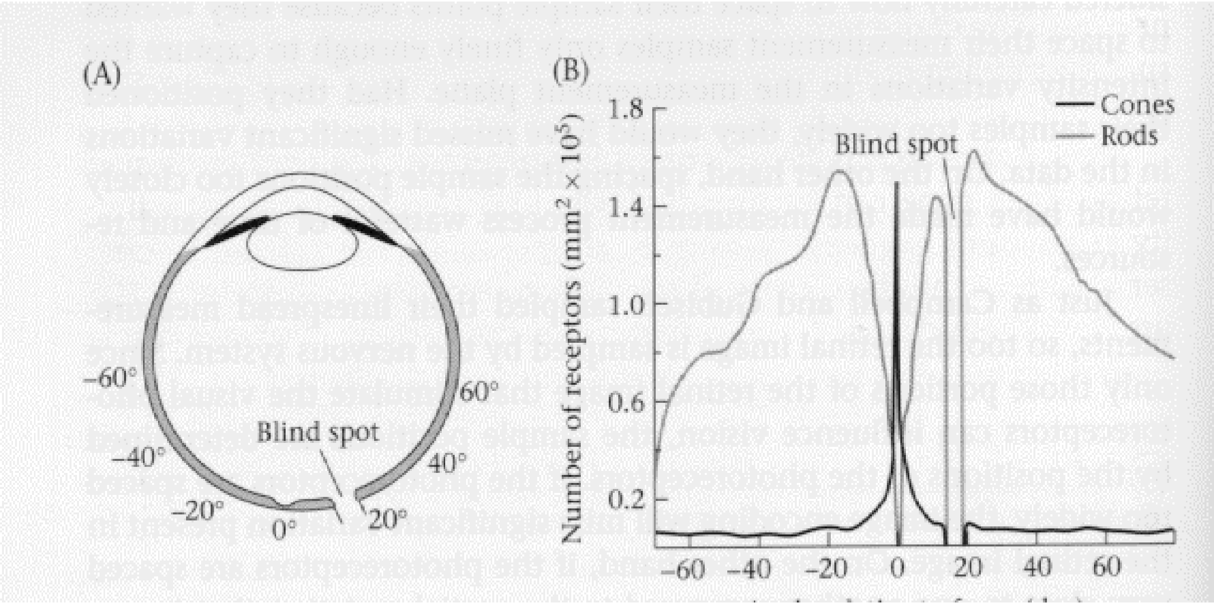
- (A) Cones in the Fovea. Several neurons per cone
- (B) Cones and Rods in the periphery
- Rods are small but several rods per neuron

Photoreceptor Mosaics

- Fovea is area of highest concentration of photoreceptors
  - fovea contains no rods, just cones
  - approximately 50,000 cones in the fovea
  - cannot see dim light sources (like stars) when we look straight at them!
- TV camera photoreceptor mosaics
  - nearly square mosaic of approximately 800X640 elements for complete field of view

The Human Eye

- Limitations of human vision
  - the image is upside-down!
  - high resolution vision only in the fovea
    - only one small fovea in man
    - other animals (birds, cheetas) have different foveal organizations
  - blind spot

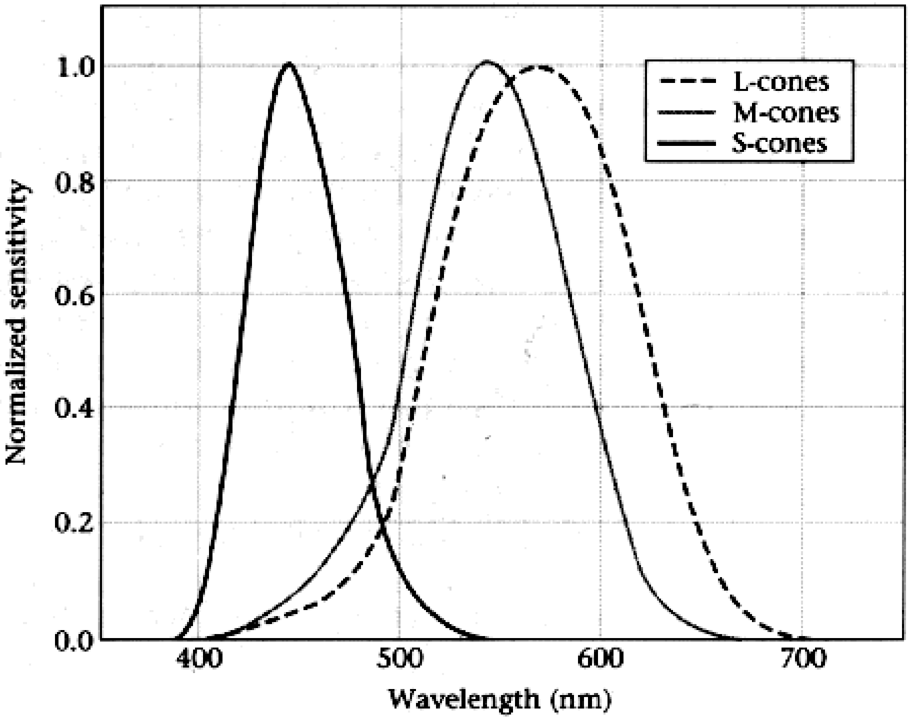


Blind Spot

- Close left eye
- Look steadily at white cross
- Move head slowly toward and away from figure
- At a particular head position, the white disk disappears completely from view.

Cones and color

- There are three different types of cones
  - they differ in their sensitivity to different wavelengths of light

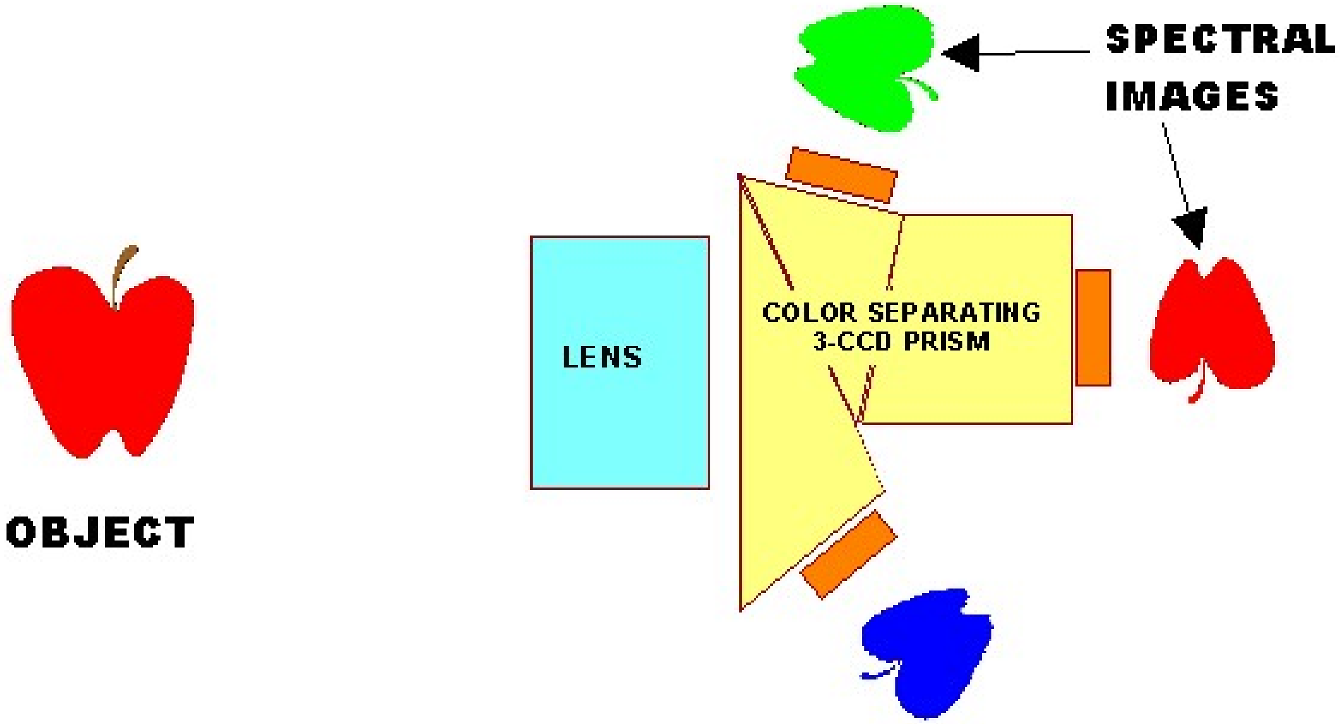


3.3 SPECTRAL SENSITIVITIES OF THE L-, M-, AND S-CONES in the human eye. The measurements are based on a light source at the cornea, so that the wavelength loss due to the cornea, lens, and other inert pigments of the eye plays a role in determining the sensitivity. Source: Stockman and MacLeod, 1993.

Color Cameras

- Two types of color cameras
  - Single CCD array
    - in front of each CCD element is a filter - red, green or blue
    - color values at each pixel are obtained by hardware interpolation
      - subject to artifacts
      - lower intensity quality than a monochromatic camera
  - 3 CCD arrays packed together, each sensitive to different wavelengths of light

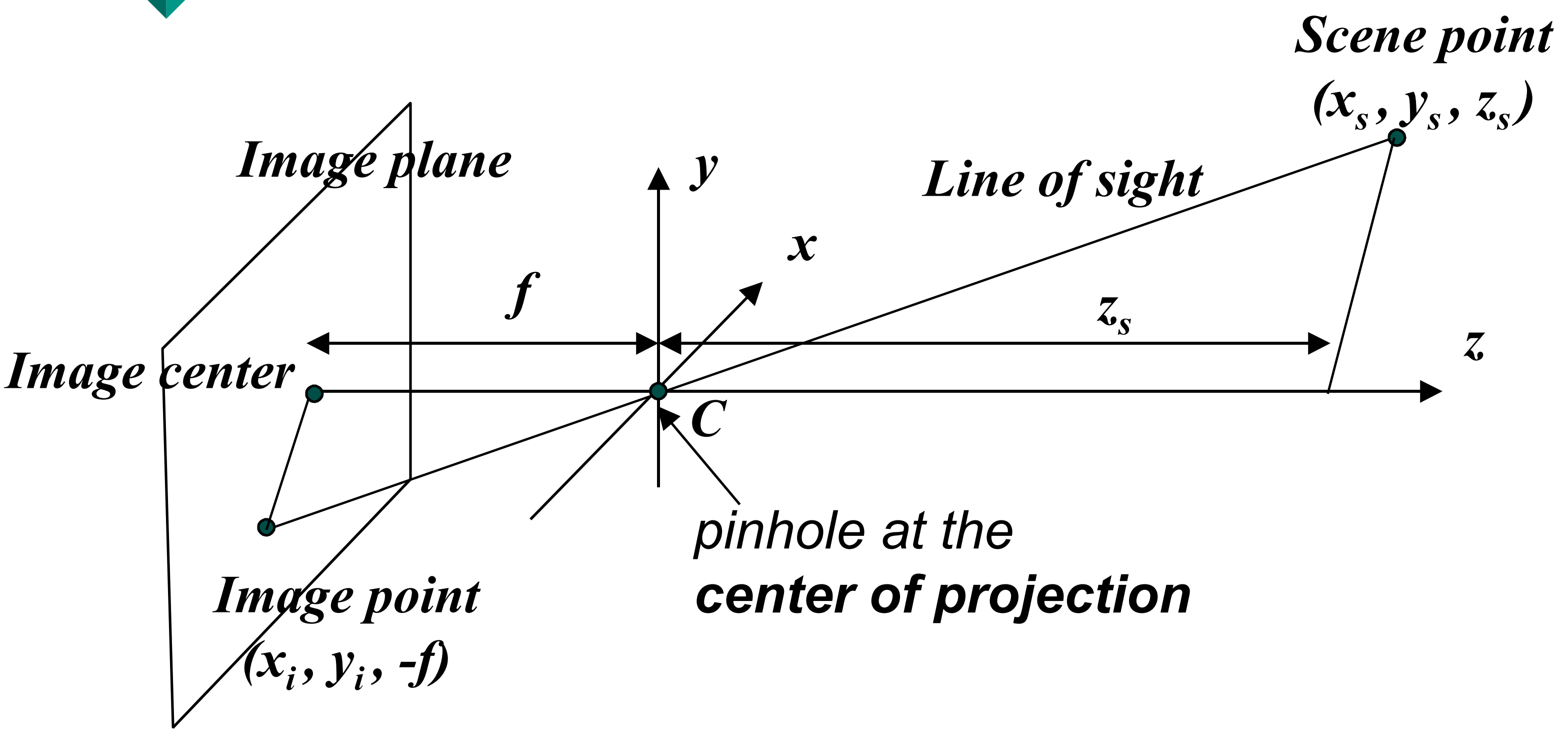
3-CCD Camera



Cones, CCD's and space

- How much of the world does a cone see?
  - measured in terms of visual angle
  - the eye lens collects light over a total field of view of about 100^o
  - each cone collects light over a visual angle of about 8.5×10^{-3} degrees (about 30 seconds)
- How much of the world does a single camera CCD see
  - example: 30^o lens
  - 30/500 gives about 6×10^{-2} degrees per CCD
  - Eye's acuity is 10 times higher.

Perspective Imaging - Pinhole Camera Model

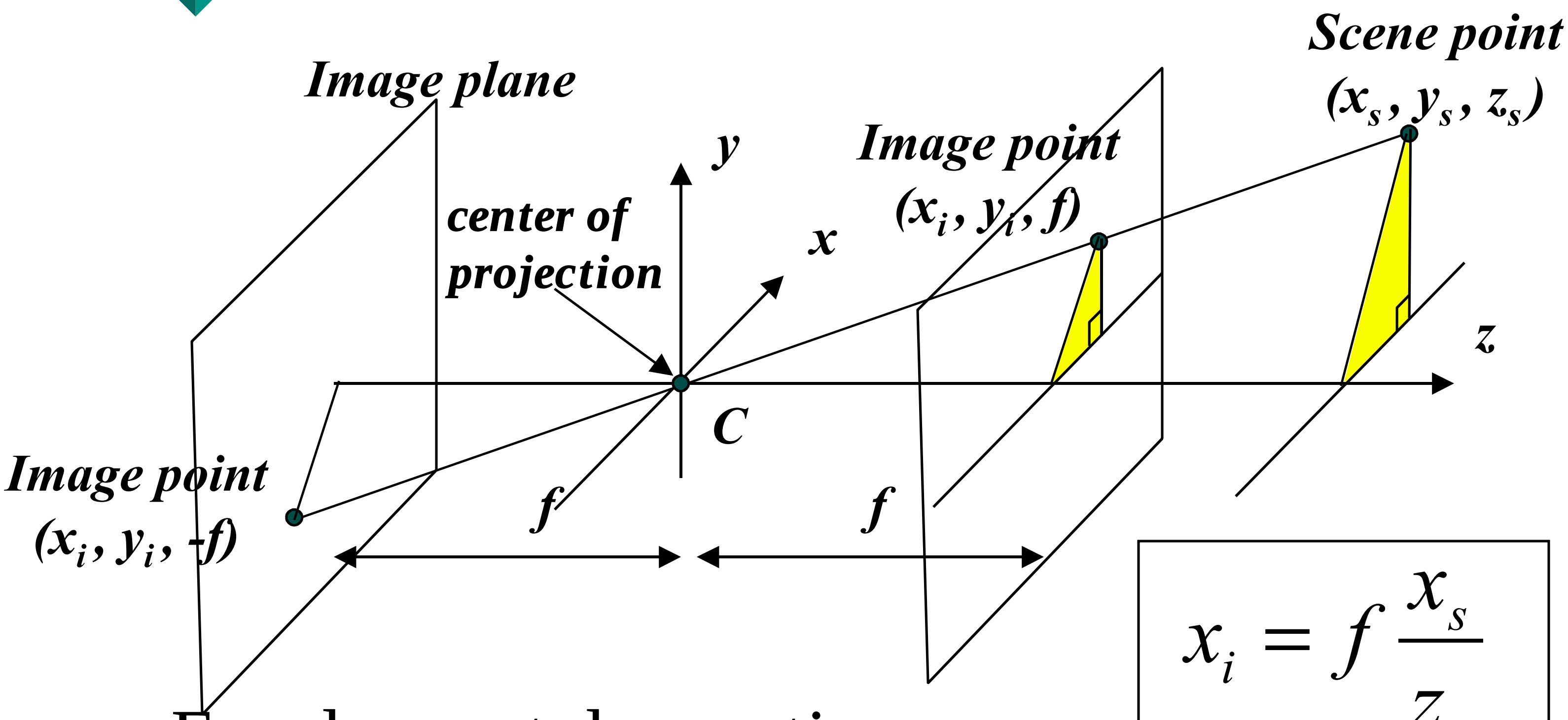


- The point on the image plane that corresponds to a particular point in the scene is found by following the line that passes through the scene point and the center of projection

Perspective Imaging=Central Projection

- ***Line of sight*** to a point in the scene is the line through the center of projection to that point
- Image plane is parallel to the x-y plane
  - distance to image plane is ***f*** - focal length
  - this inverts the image
  - move the image plane in front of the center of projection

Perspective Imaging



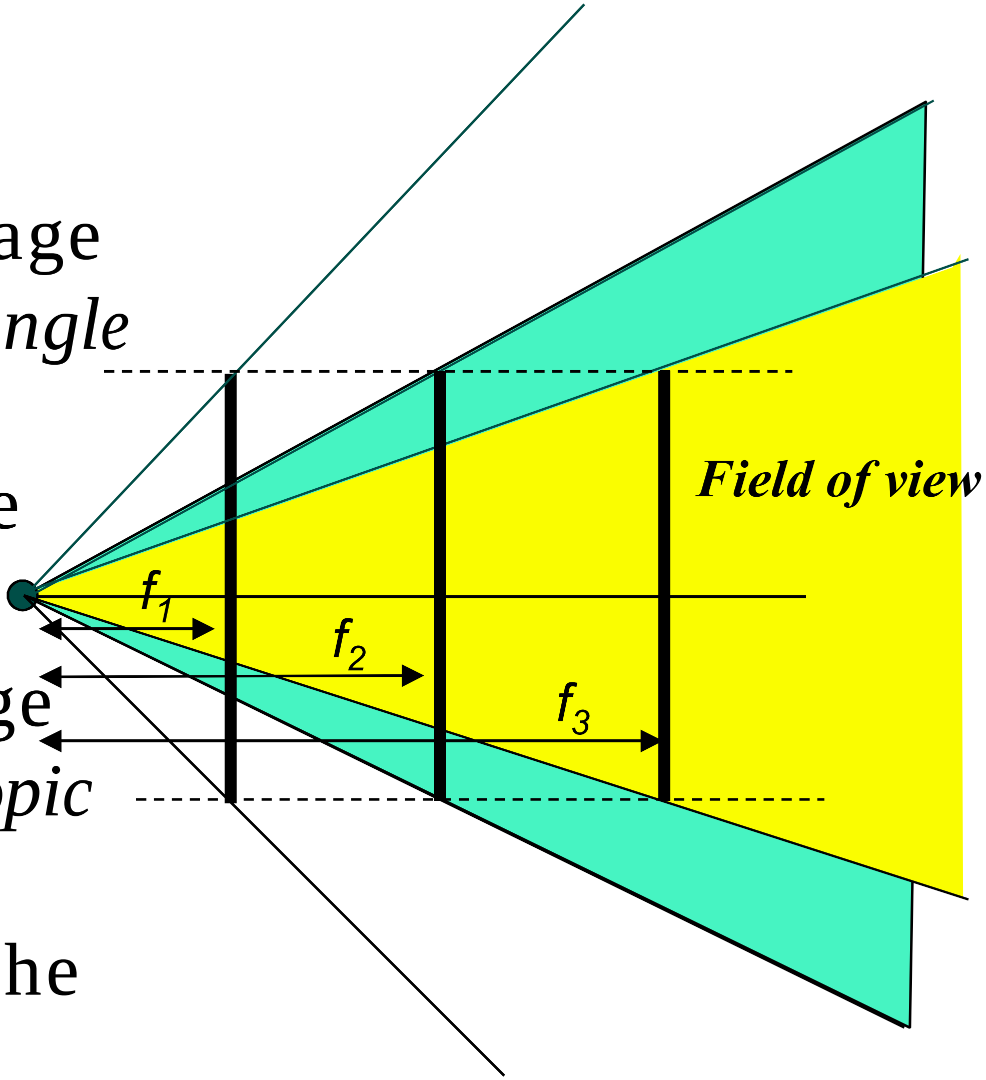
- Fundamental equations for perspective projection onto a plane

$$x_i = f\frac{x_s}{z_s}$$

$$y_i = f\frac{y_s}{z_s}$$

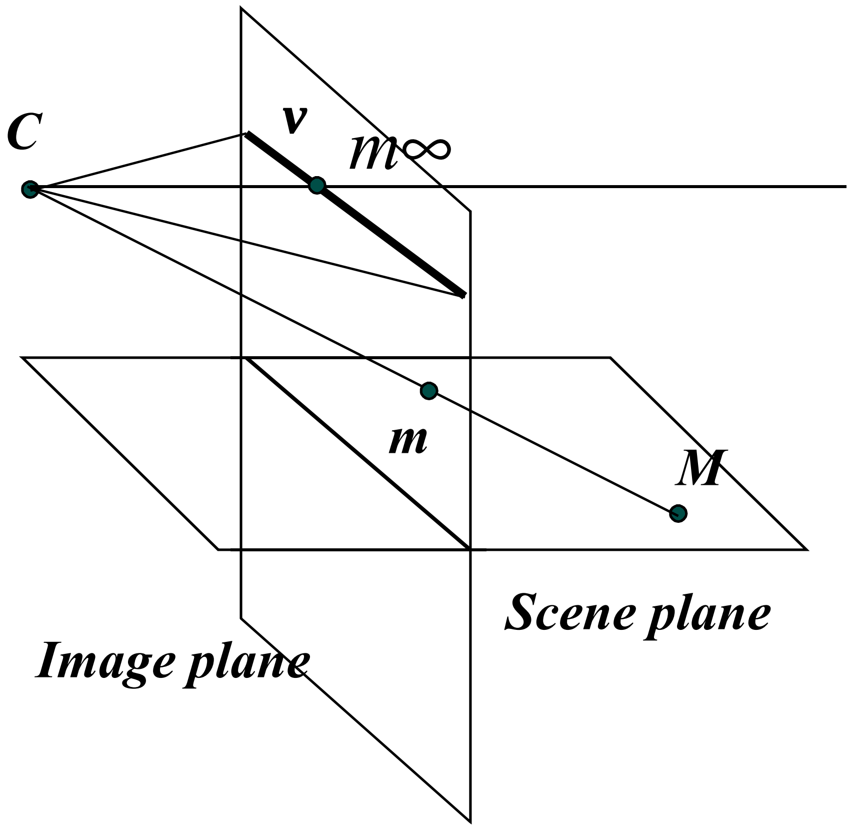
Field of View

- As ***f*** gets smaller, image becomes more *wide angle* (more world points project onto the finite image plane)
- As ***f*** gets larger, image becomes more *telescopic* (smaller part of the world projects onto the finite image plane)



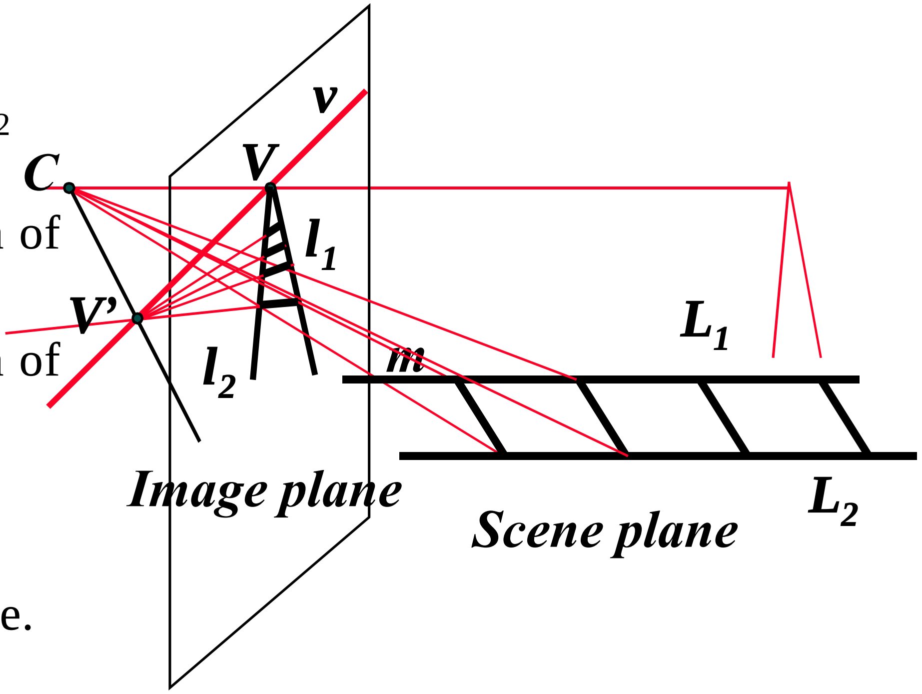
Vanishing Points and Lines

- We are looking at a scene on a plane
- To each point ***M*** on the scene plane we associate an image point ***m***
- The line, ***v***, of the image plane that belongs to the plane through ***C*** parallel to the scene plane is called the **vanishing line**, or the **horizon line**

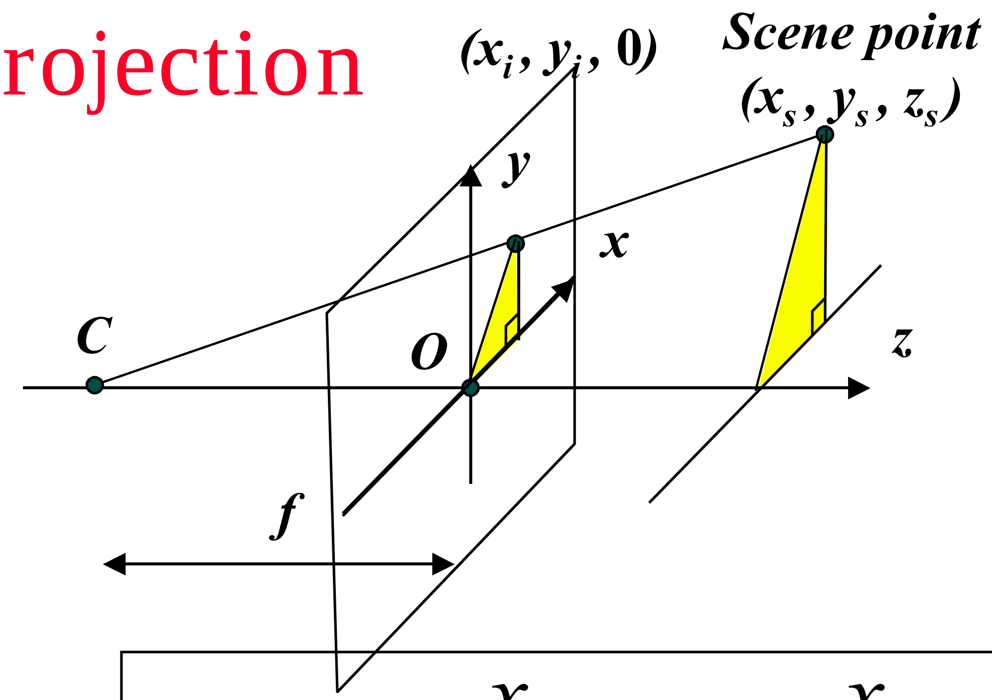


Vanishing Points and Lines

- The images of the points on rail **L_1** belong to a plane **P_1** defined by **C** and **L_1**
  - The lines of sight for each point of the track **L_1** lie on this plane
- The image of rail **L_2** belongs to a plane **P_2** defined by **C** and track
- The image **l_1** of **L_1** belongs to intersection of plane **P_1** and image plane
- The image **l_2** of **L_2** belongs to intersection of plane **P_1** and image plane
- Planes **P_1** and **P_2** intersect along line **CV** parallel to **L_1** and **L_2**
- Point **V** belongs to **P_1**, **P_2** and image plane.
- Therefore, **l_1** and **l_2** intersect in **V**
- **V** is image of a scene point of **L_1** and a scene point of **L_2**. Since **CV** is parallel to **L_1** and **L_2**, these points are at infinity.



From Perspective Projection to Orthographic Projection

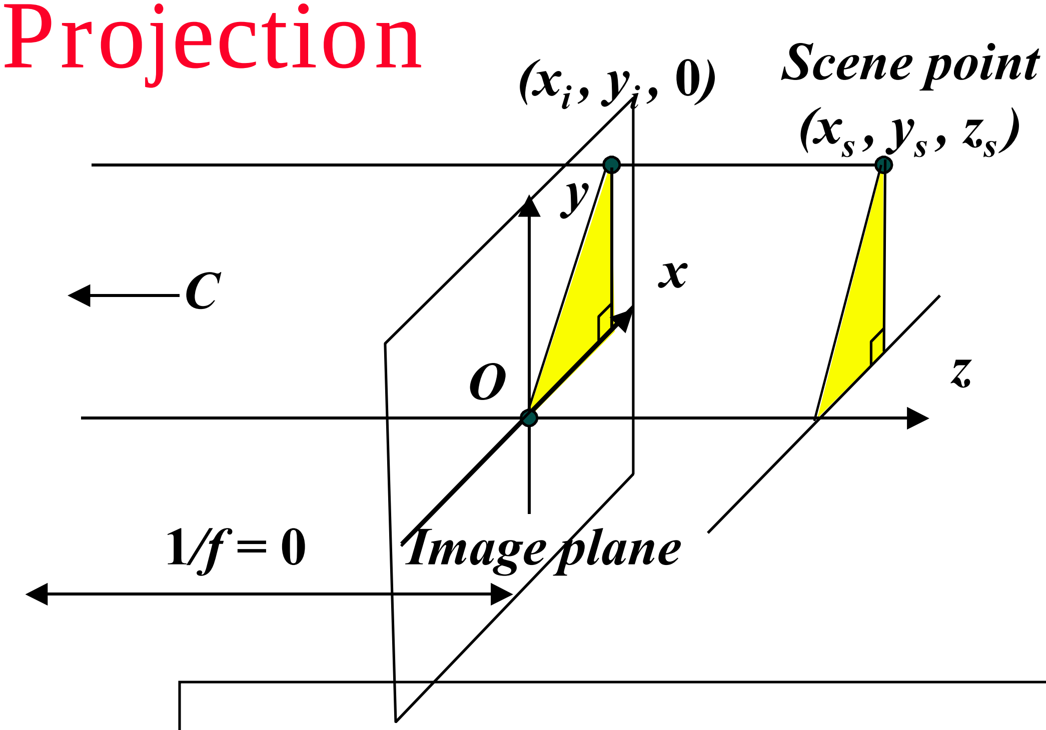


- Select origin of coordinate system at image center
- World coordinates are independent of focal length
- $\beta = 1/\boldsymbol{f}$
- **When $\beta = 0$, orthographic projection**

$$x_i = f\frac{x_s}{f+z_s} = \frac{x_s}{1+\beta\ z_s}$$

$$y_i = f\frac{y_s}{f+z_s} = \frac{y_s}{1+\beta\ z_s}$$

Orthographic Projection

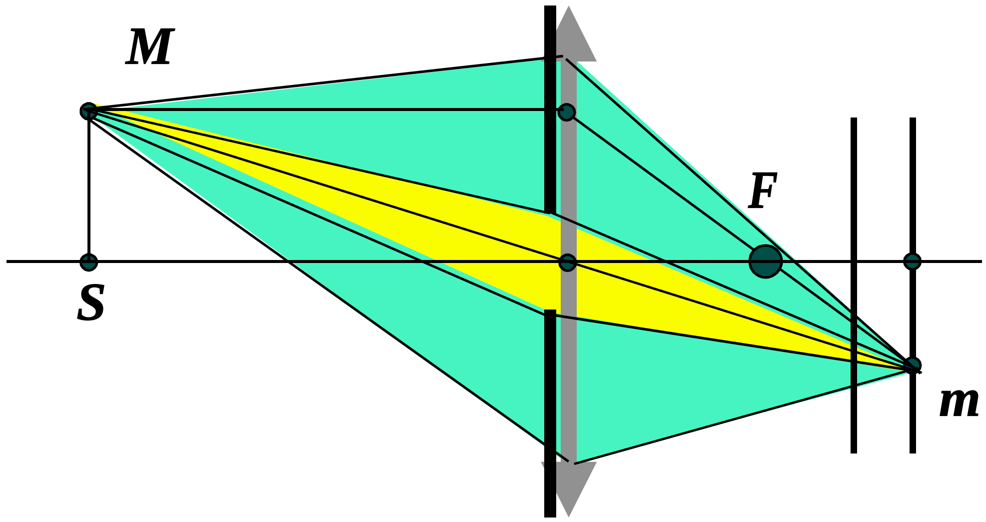


$$x_i = x_s$$

$$y_i = y_s$$

Depth of Field and f-number

- Depth of field is smaller for small f-number

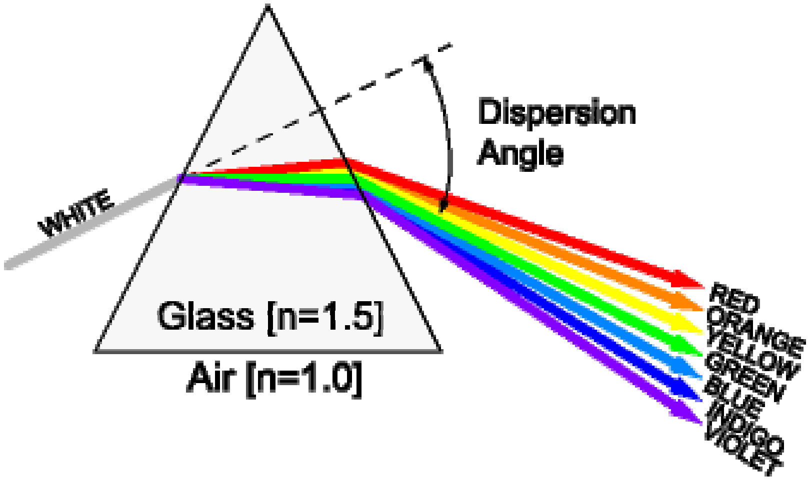


Lens Imperfections

- Lens imperfections might cause rays not to intersect at a point
  - Deviations in shape from the ideal lens
  - Material imperfections that might cause the refractive index to vary within the lens

Refraction of Color

- Why does the prism separate the light into its spectral components?
  - Prism bends different wavelengths of light by different amounts
    - Refractive index is a function of wavelength
    - Shorter wavelengths are refracted more strongly than longer wavelengths

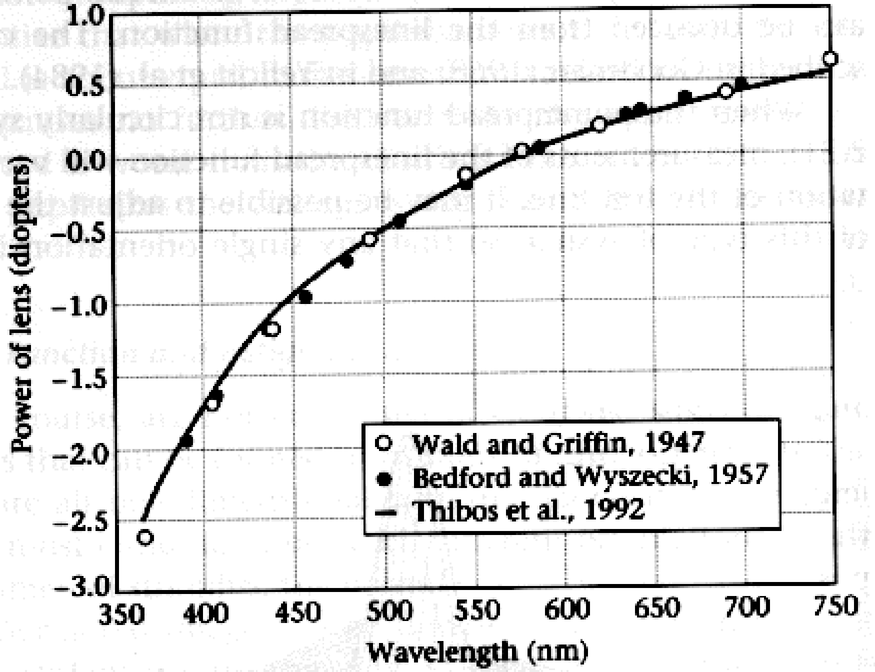


Refraction through a prism.

Chromatic Aberration

- Chromatic aberration
  - Different wavelengths of light from the same point source are focused at different distances behind the lens
  - When incident light is a mixture of wavelengths, we can observe a chromatic fringe at edges
  - Accommodation can bring any wavelength into good focus, but not all simultaneously
  - Human visual system has other mechanisms for reducing chromatic aberration
  - Color cameras have similar problems

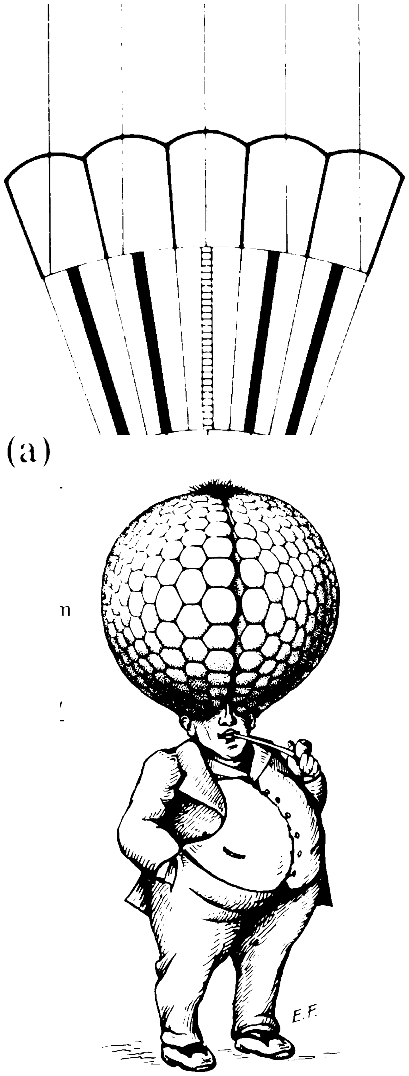
Chromatic Aberration



2.22 CHROMATIC ABERRATION OF THE HUMAN EYE. The data points measure the optical power one must add to the human eye in order to bring different wavelengths into a common focus with a 578-nm light. The smooth curve plots a formula created by Thibos et al. (1992) that predicts the measurements and interpolates smoothly between them. The formula is $D(\lambda) = p - q/(\lambda - c)$, where λ is wavelength in micrometers, $D(\lambda)$ is the defocus in diopters, $p = 1.7312$, $q = 0.63346$, and $c = 0.21410$. After Marimont and Wandell, 1993.

Compound Eyes

- Many (small) animals have compound eyes
  - each photoreceptor has its own lens
  - images seen by these eyes are equally sharp in all directions
  - examples: flies and other insects
- But these eyes do not “scale” well biologically



References

- Foundations of Vision, Brian Wandell, Sinauer Associates, Sunderland MA, 1995
- Introductory Techniques for 3-D Computer Vision, E. Trucco and A. Verri, Prentice-Hall, pp. 18-28
- Computer Vision, Ballard & Brown, Prentice-Hall, pp. 19-22, pp.206-208
- Robot Vision, B.K.P. Horn, MIT Press, pp. 18-27
- A Guided Tour of Computer Vision, V. Nalwa, AT&T Press, pp. 3-49