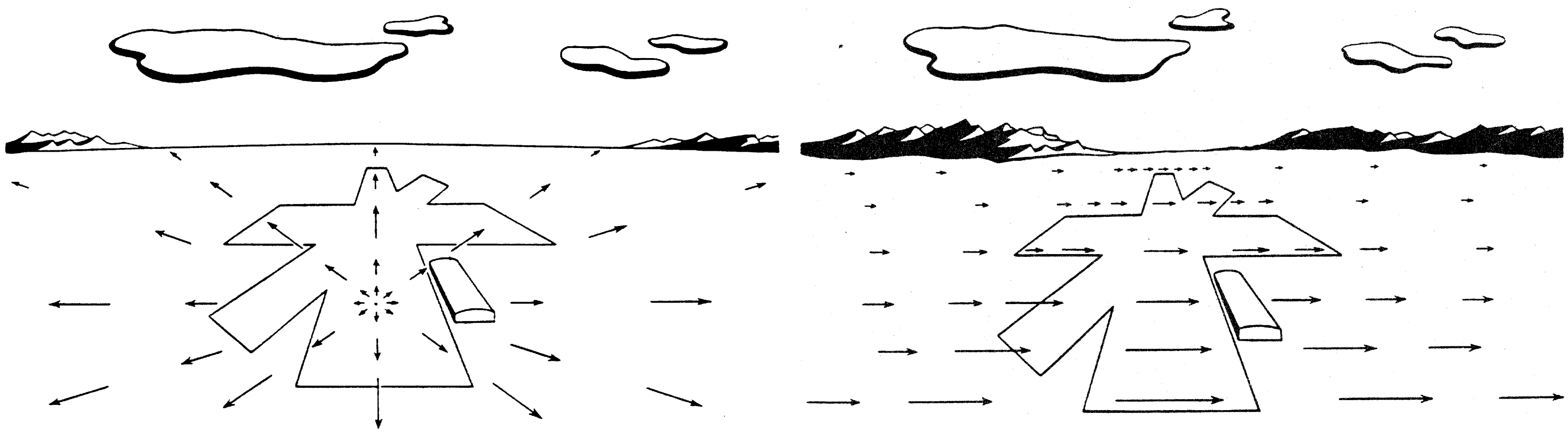
Motion and Flow

The Information from Image Motion

- 3D motion between observer and scene + structure of the scene
  - Wallach O'Connell (1953): Kinetic depth effect
  - Motion parallax: two static points close by in the image with different image motion; the larger translational motion corresponds to the point closer by (smaller depth)
- Recognition
  - Johansson (1975): Light bulbs on joints

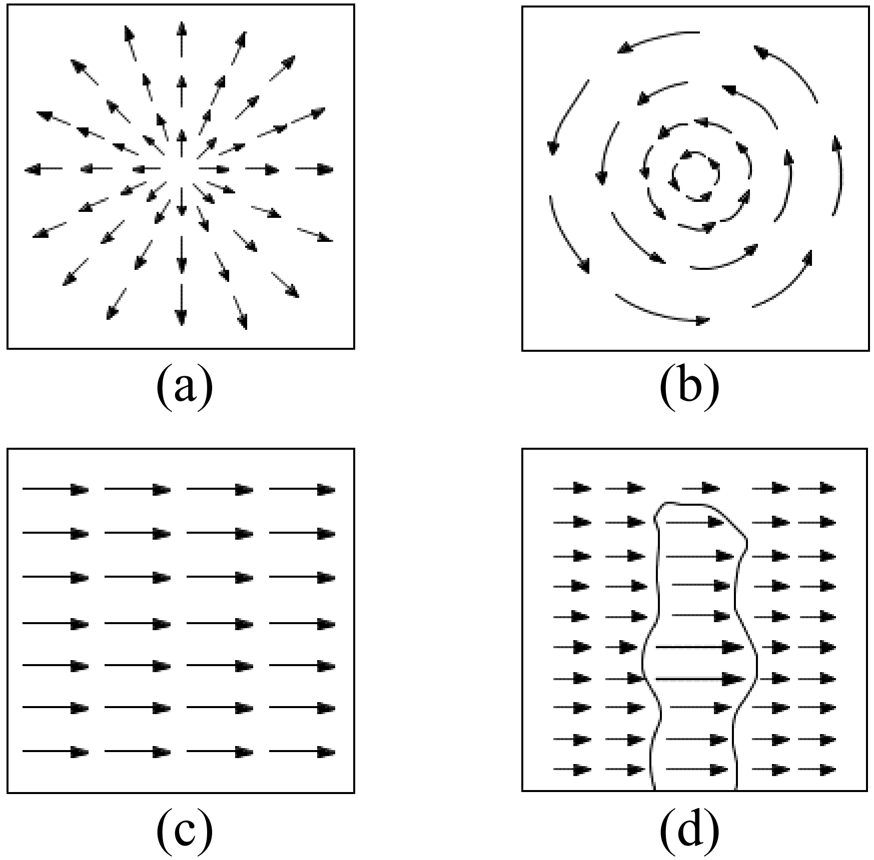
Examples of Motion Fields I



(a) (b)

(a) Motion field of a pilot looking straight ahead while approaching a fixed point on a landing strip. (b) Pilot is looking to the right in level flight.

Examples of Motion Fields II



(a) Translation perpendicular to a surface. (b) Rotation about axis perpendicular to image plane. (c) Translation parallel to a surface at a constant distance. (d) Translation parallel to an obstacle in front of a more distant background.

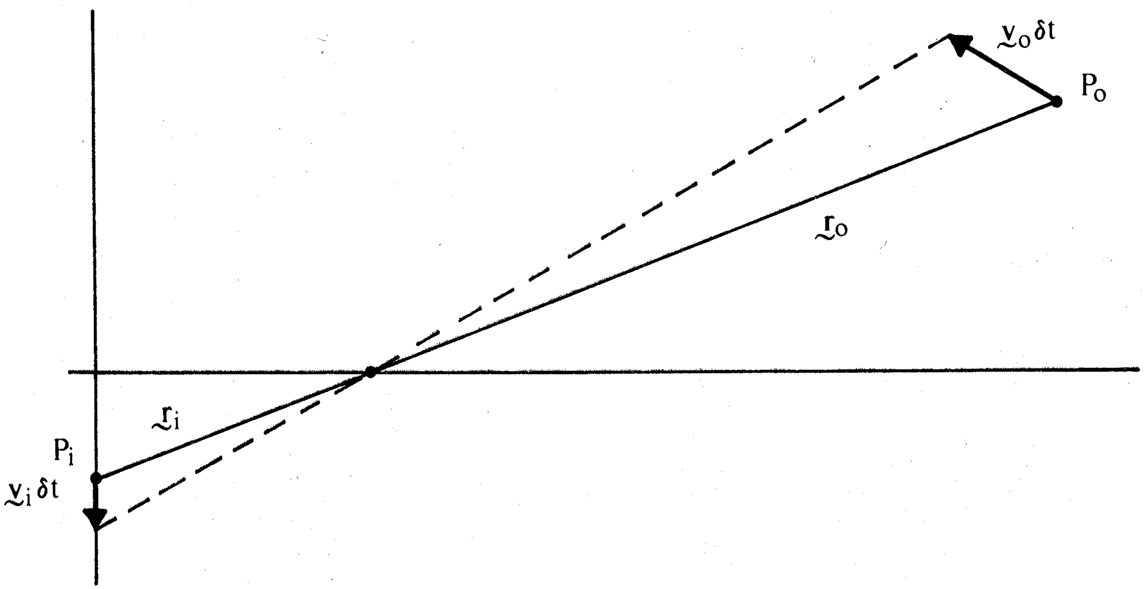
Motion Field and Optical Flow Field

- Motion field: projection of 3D motion vectors on image plane

Object point P_0 has velocity $\mathbf{v}_0$, induces $\mathbf{v}_i$ in image

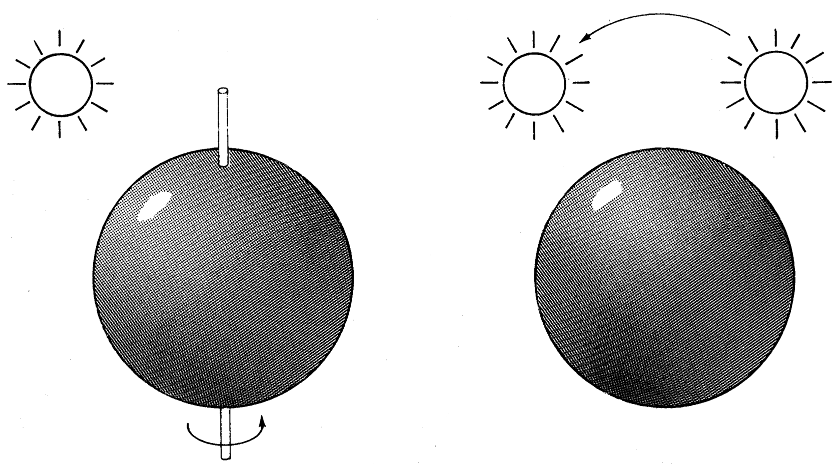
$$\mathbf{v}_0 = \frac{d\mathbf{r}_0}{dt} \qquad \mathbf{v}_1 = \frac{d\mathbf{r}_i}{dt}$$

$\mathbf{r}_0$ related to $\mathbf{r}_i$ by $\dfrac{\mathbf{r}_i}{f} = \dfrac{\mathbf{r}_0}{\mathbf{r}_0 \cdot \hat{\mathbf{z}}_0}$



- Optical flow field: apparent motion of brightness patterns
- We equate motion field with optical flow field

2 Cases Where this Assumption Clearly is not Valid

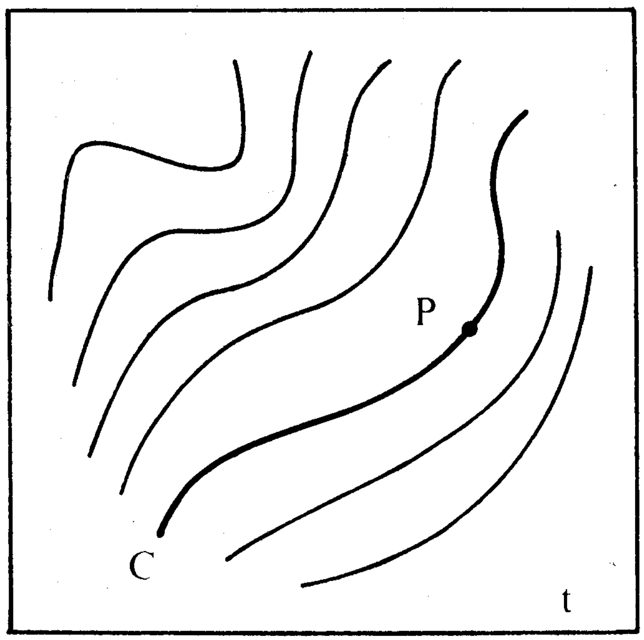


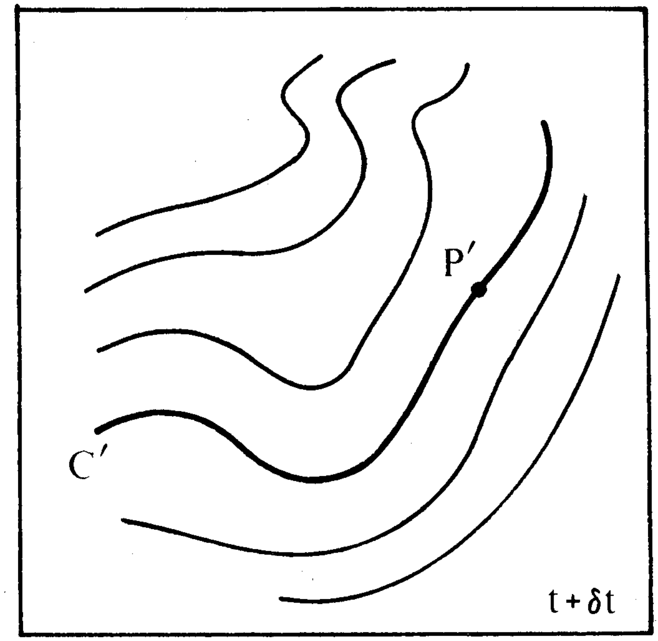
(a) (b)

(a) A smooth sphere is rotating under constant illumination. Thus the optical flow field is zero, but the motion field is not.

(b) A fixed sphere is illuminated by a moving source—the shading of the image changes. Thus the motion field is zero, but the optical flow field is not.

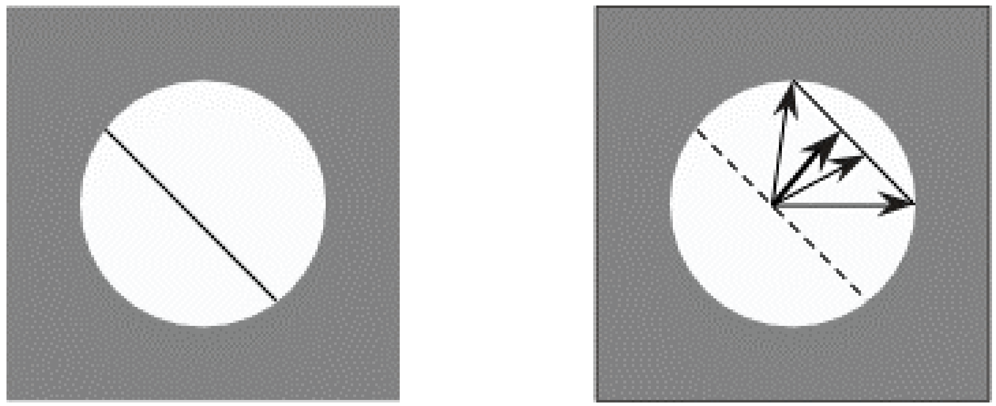
What is Meant by Apparent Motion of Brightness Pattern?





The apparent motion of brightness patterns is an awkward concept. It is not easy to decide which point P' on a contour C' of constant brightness in the second image corresponds to a particular point P on the corresponding contour C in the first image.

Aperture Problem



(a) (b)

(a) Line feature observed through a small aperture at time *t*.

(b) At time $t+\delta t$ the feature has moved to a new position. It is not possible to determine exactly where each point has moved. From local image measurements only the flow component perpendicular to the line feature can be computed.

Normal flow: Component of flow perpendicular to line feature.

The Optical Flow Constraint Equation

Let $E(x,y,t)$ be the irradiance and $u(x,y), v(x,y)$ the components of optical flow.

$$E(x+u\delta t, y+v\delta t, t+\delta t) = E(x,y,t)$$

Taylor expansion

$$E(x,y,t)+\delta x\frac{\partial E}{\partial x}+\delta y\frac{\partial E}{\partial y}+\delta t\frac{\partial E}{\partial t}+e = E(x,y,t)$$

dividing by δt and taking limit $\delta t \rightarrow 0$

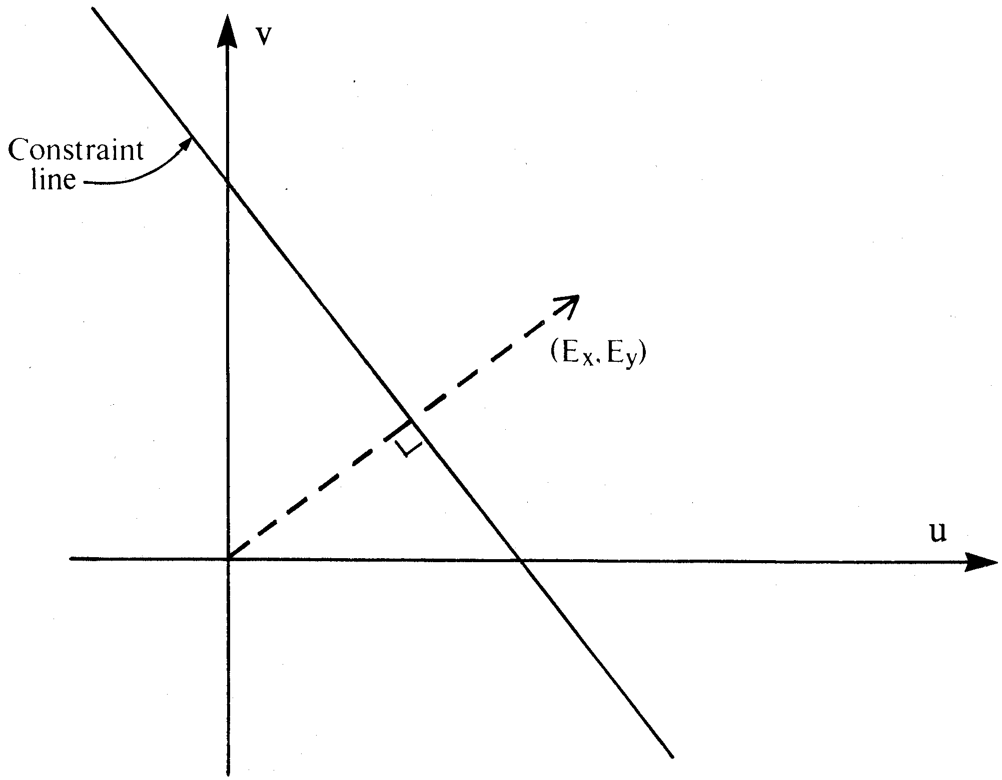
$$\frac{\partial E}{\partial x}\frac{dx}{dt}+\frac{\partial E}{\partial y}\frac{dy}{dt}+\frac{\partial E}{\partial t}=0$$

which is the expansion of the total derivative

$$\frac{dE}{dt}=0$$

short: $\boxed{E_x u + E_y v + E_t = 0}$

Interpretation



Values of (u, v) satisfying the constraint equation lie on a straight line in velocity space. A local measurement only provides this constraint line (aperture problem).

Normal flow $\mathbf{u}_n$

$$\left(E_x, E_y\right)\cdot\left(u, v\right) = -E_t$$

Let $\mathbf{n} = \dfrac{\left(E_x E_y\right)^T}{\left\|\left(E_x, E_y\right)^T\right\|}$

$$\mathbf{u}_n = (\mathbf{u}\cdot\mathbf{n})\mathbf{n} = \left(\frac{-E_x E_t}{\sqrt{E_x{}^2 + E_y{}^2}}, \frac{-E_y E_t}{\sqrt{E_x{}^2 + E_y{}^2}}\right)^T$$

Additional Constraints

- Additional constraints are necessary to estimate optical flow, for example, constraints on size of derivatives, or parametric models of the velocity field.
- Horn and Schunck (1981): global smoothness term

$$e_s = \iint_D \left(u_x{}^2 + u_y{}^2\right) + \left(v_x{}^2 + v_y{}^2\right) dx\,dy : \text{departure from smoothness}$$

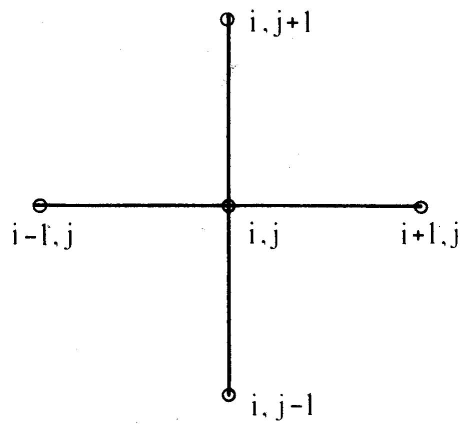
$$e_c = \iint_D \left(E_x u + E_y v + E_t\right)^2 dx\,dy : \text{error in optical flow constraint equation}$$

Let $\nabla A = \left(A_x, A_y\right)^T$ denote the gradient of A

$$\iint (\nabla E \cdot \mathbf{u} + E_t)^2 + \lambda\left(\|\nabla u\|_2{}^2 + \|\nabla v\|_2{}^2\right) dx\,dy \rightarrow \min$$

- This approach is called regularization.
- Solve by means of calculus of variation.

Discrete implementation leads to iterative equations

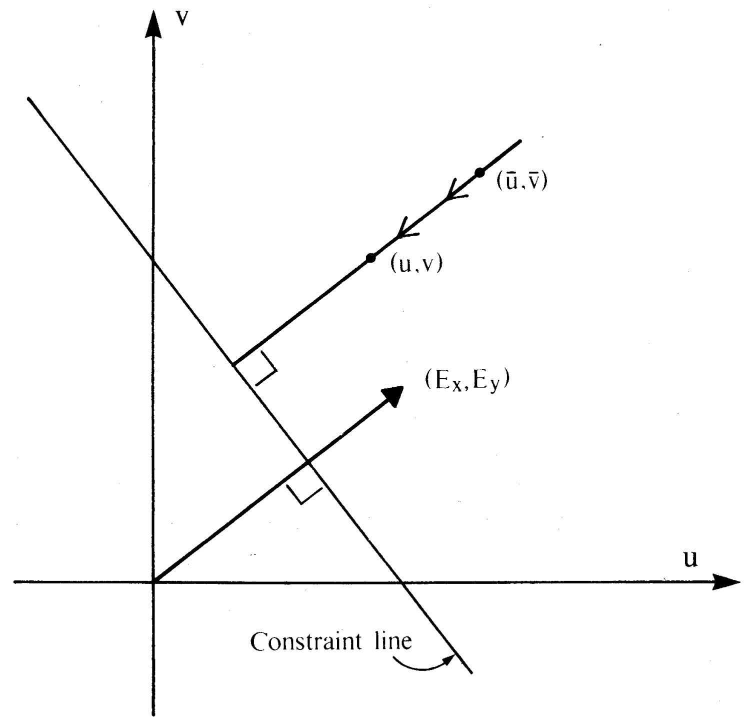


$\bar{u}, \bar{v}$ denotes local averages of u and v

$$u^{n+1} = \bar{u}^n - \frac{\left(E_x \bar{u}^n + E_y \bar{v}^n + E_t\right)}{\frac{1}{\lambda} + E_x^{\ 2} + E_y^{\ 2}} E_x$$

$$v^{n+1} = \bar{v}^n - \frac{\left(E_x \bar{u}^n + E_y \bar{v}^n + E_t\right)}{\frac{1}{\lambda} + E_x^{\ 2} + E_y^{\ 2}} E_y$$

Geometric interpretation



In the iterative scheme for estimating the optical flow, the new value (u, v) at a point is the average of the values of the neighbors $(\bar{u}, \bar{v})$, minus an adjustment in the direction toward the constraint line.

Other Differential Techniques

- Lucas Kanade (1984): Weighted least-squares (LS) fit to a constant model of **u** in a small neighborhood Ω;

$$\sum_{\mathbf{x}\in\Omega} W^2(\mathbf{x})\left(\nabla E(\mathbf{x},t)\cdot\mathbf{u} + E_t(\mathbf{x},t)\right)^2 \rightarrow \min$$

Denote $A = \left(\nabla E(\mathbf{x}_1),\ldots,\nabla E(\mathbf{x}_n)\right)^T$, $W = \mathrm{diag}\left(W(\mathbf{x}_1),\ldots,W(\mathbf{x}_n)\right)^T$,

$$\mathbf{b} = -\left(E_t(\mathbf{x}_1),\ldots,E_t(\mathbf{x}_n)\right)^T$$

$$\mathbf{u} = \left(A^T W^2 A\right)^{-1} A^T W^2 \mathbf{b}$$

- Nagel (1983,87): Oriented smoothness constraint; smoothness is not imposed across edges

$$\iint \left(\nabla E^T \mathbf{u} + E_t\right)^2 + \frac{\alpha^2}{\|\nabla E\|_2^{\,2} + 2\delta} \times \left[\left(u_x E_y - u_y E_x\right)^2 + \left(v_x E_y - v_y E_x\right)^2 + \delta\left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2\right)\right]$$

- Uras et al. (1988): Use constraints on second-order derivatives

$$\frac{d\nabla E(\mathbf{x},t)}{dt} = 0 \qquad \begin{bmatrix} E_{xx}(\mathbf{x},t) & E_{xy}(\mathbf{x},t) \\ E_{xy}(\mathbf{x},t) & E_{yy}(\mathbf{x},t) \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = -\begin{pmatrix} E_{tx}(\mathbf{x},t) \\ E_{ty}(\mathbf{x},t) \end{pmatrix}$$

Classification of Optical Flow Techniques

- Gradient-based methods
- Frequency-domain methods
- Correlation methods

3 Computational Stages

1. Prefiltering or smoothing with low-pass/band-pass filters to enhance signal-to-noise ratio
2. Extraction of basic measurements (e.g., spatiotemporal derivatives, spatiotemporal frequencies, local correlation surfaces)
3. Integration of these measurements, to produce 2D image flow using smoothness assumptions

Energy-based Methods

- Adelson Berger (1985), Watson Ahumada (1985), Heeger (1988):
  Fourier transform of a translating 2D pattern:

  $$F\,E(w_x, w_y, w_t) = F\,E(w_x, w_y, 0)\,\delta(w_x u + w_y v + w_t)$$

  All the energy lies on a plane through the origin in frequency space

  Local energy is extracted using velocity-tuned filters (for example, Gabor-energy filters)

  Motion is found by fitting the best plane in frequency space
- Fleet Jepson (1990): Phase-based Technique
  - Assumption that phase is preserved (as opposed to amplitude)
  - Velocity tuned band pass filters have complex-valued outputs

  $$R(x,t,w) = \rho(x,t,w)e^{i\phi(x,t,w)}$$

  with ρ the amplitude and ϕ the phase

  $$\frac{d\phi}{dt} = 0 \quad \text{or} \quad \phi_x u + \phi_y v + \phi_t = 0$$

Correlation-based Methods

Anandan (1987), Singh (1990)

1. Find displacement (dx, dy) which maximizes cross correlation

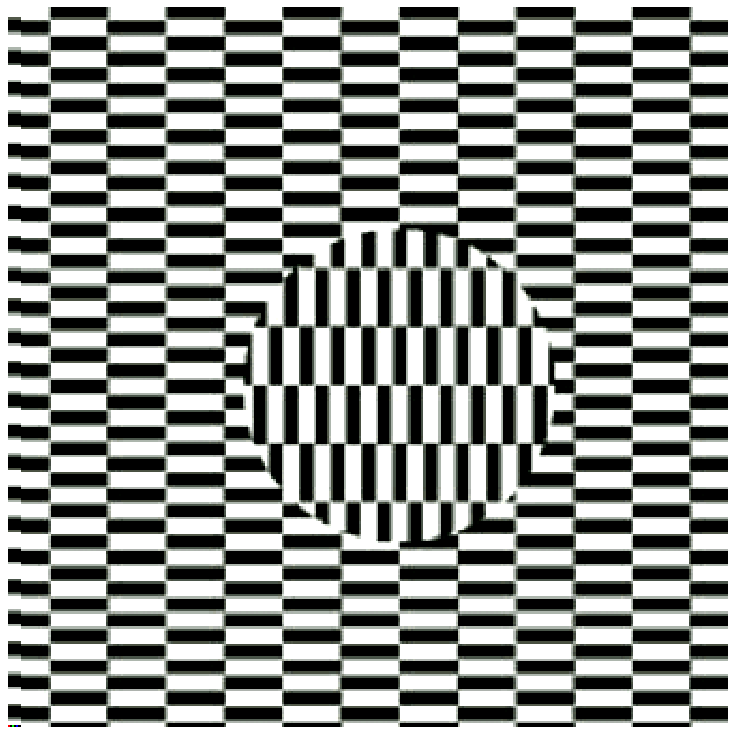
$$CC(dx,dy) = \sum_{j=-n}^{+n} \sum_{i=-n}^{+n} W(i,j) \cdot E_1(i,j) \cdot E_2(i-dx, j-dy)$$

or minimizes sum of squared differences (SSD)

$$SSD(dx,dy) = \sum_{j=-n}^{+n} \sum_{i=-n}^{+n} W(i,j) \cdot \left(E_1(i,j) - E_2(i-dx, j-dy)\right)^2$$

2. Smooth the correlation outputs

A Pattern of Hajime Ouchi



Bias in Flow Estimation

Symmetric noise in spatial and temporal derivatives

Notation: $\delta A = A - A'$, where A is the estimate, A' the actual value and δA the error

$$\left(E_{x_i} - \delta E_{x_i}\right)u + \left(E_{y_i} - \delta E_{y_i}\right)v = E_{t_i} - \delta E_{t_i}$$

in matrix form $(E - \delta E)\mathbf{u} = \mathbf{b}$

LS solution $\mathbf{u} = \left(E^T E\right)^{-1} E^T \mathbf{b}$

expected value of $\mathbf{u}$ $E(\mathbf{u}) = \mathbf{u}' - n\sigma_s^{\,2}\left(E'^T E'\right)^{-1}\mathbf{u}'$

n number of measurements

σ_s standard deviation of spatial noise

- Underestimation in length
- Bias in direction: more underestimation in direction of fewer measurements

Epipolar Constraint for Discrete Motions

C'-C, $m-C$, $m'-C'$ coplanar, or $\mathbf{t}$, $\mathbf{m}$ and $R\mathbf{m}'$ coplanar

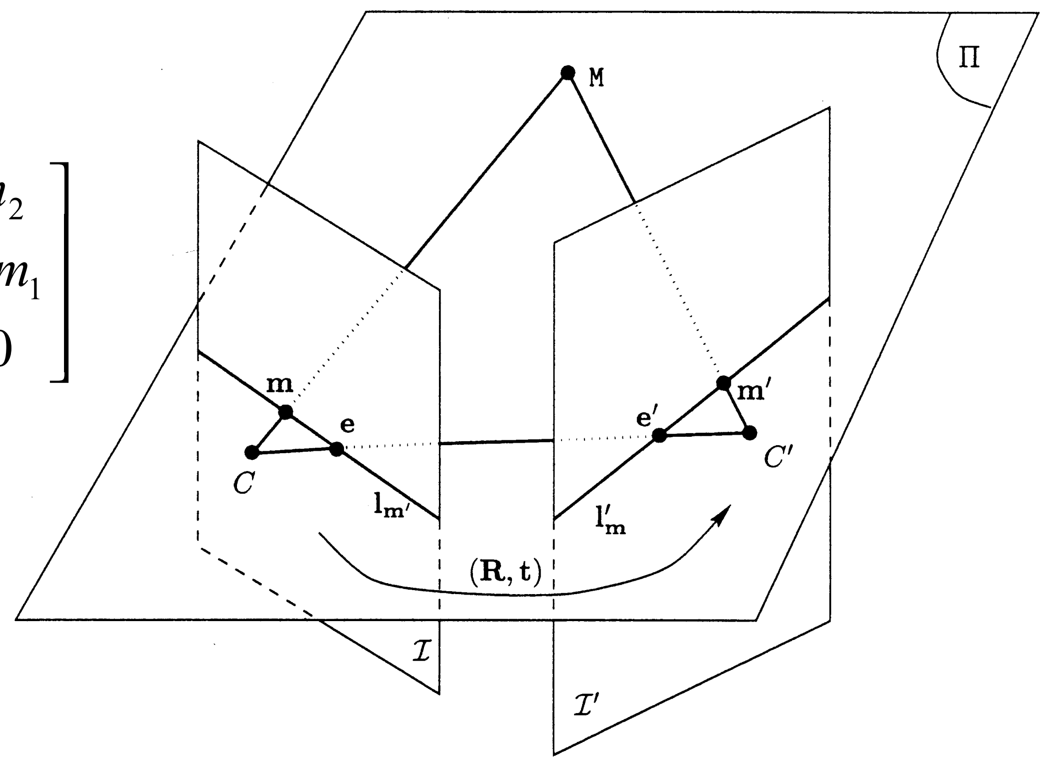
$(\mathbf{t}\times\mathbf{m})^T \cdot R\mathbf{m}' = 0$ epipolar constraint

$([\mathbf{t}]_\times \mathbf{m})^T \cdot R\mathbf{m}' = 0$

$\mathbf{m}^T E\mathbf{m}' = 0,\ E = [\mathbf{t}]_\times R$

$$\text{Def}: \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}_\times = \begin{bmatrix} 0 & -m_3 & m_2 \\ m_3 & 0 & -m_1 \\ -m_2 & m_1 & 0 \end{bmatrix}$$

$$\mathbf{t}\times\mathbf{m} = [\mathbf{t}]_\times \mathbf{m}$$

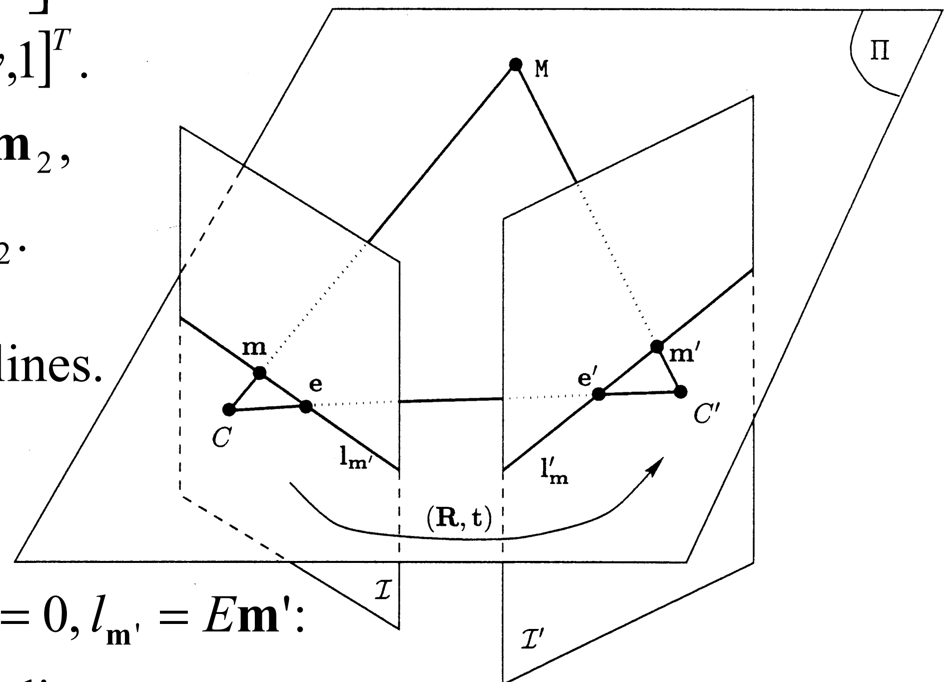


Consider a line $ax + by + c = 0$ or $[a, b, c]\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$,

or $\mathbf{m}^T l = 0$ with $l = [a, b, c]^T$ and $\mathbf{m} = [x, y, 1]^T$.

If a line goes through two points $\mathbf{m}_1$ and $\mathbf{m}_2$,

then $\mathbf{m}_1{}^T l = 0$ and $\mathbf{m}_2{}^T l = 0$ or $l = \mathbf{m}_1 \times \mathbf{m}_2$.



$l_{\mathbf{m}'}$ and $l'_{\mathbf{m}}$ are the corresponding epipolar lines.

$l_{\mathbf{m}'} = \mathbf{e} \times R\mathbf{m}' = \mathbf{t} \times R\mathbf{m}' = E\mathbf{m}'$

$l_{\mathbf{m}'} = E\mathbf{m}$

Epipolar constraint : $\mathbf{m}^T E\mathbf{m}' = 0$ or $\mathbf{m}^T l_{\mathbf{m}'} = 0$, $l_{\mathbf{m}'} = E\mathbf{m}'$:

points lie on their corresponding epipolar lines.

The epipole lies on all epipolar lines

$\mathbf{e}^T E\mathbf{m}' = 0 \; \forall \mathbf{m}'$, or $\mathbf{e}^T E = 0$.

$$e_1 = E_{32}E_{21} - E_{22}E_{31}$$

$$e_2 = E_{31}E_{12} - E_{11}E_{32}$$

$$e_3 = E_{22}E_{11} - E_{21}E_{12}$$

Sources:

- Horn (1986)
- J. L. Barron, D. J. Fleet, S. S. Beauchemin (1994). Systems and Experiment. Performance of Optical Flow Techniques. *IJCV* 12(1):43–77. Available at http://www.cs.queesu.ca/home/fleet/ research/Projects/flowCompare.html
- http://www.cfar.umd.edu/~fer/postscript/ouchipapernew.ps.gz (paper on Ouchi illusion)
- http://www.cfar.umd.edu./ftp/TRs/CVL-Reports-1999/TR4080-fermueller.ps.gz (paper on statistical bias)
- http://www.cis.upenn.edu/~beau/home.html

  http://www.isi.uu.nl/people/michael/of.html (code for optical flow estimation techniques)