Handed out September 13, 2000
Due back September 18, 2000

Perspective projection

1. Show that in a properly focused imaging system the distance f' from the lens to the image plane is equal to $(1+m) f$, where f is the focal length and m is the magnification. This distance is called the effective focal length. Show that the distance between the image plane and an object must be $(m + 2 + 1/m) f$
   How far must the object be from the lens for unit magnification?

2. What shapes can the perspective image of a sphere have? (Hint: Draw the pinhole camera geometry as in class, and check the shapes. Use drawings rather than algebra.)

3. Perspective effects can be significant when a wide-angle lens is used, while images obtained with a telephoto lens tends to approximate orthographic projection. Explain why these are only rough rules of thumb.

4. You see a first image of a cube with a pinhole camera with focal length f. The whole cube is visible. Then you rotate the camera around its center of projection to obtain new images.

(a) Do the lines of sight change? Can you get any new information about the cube by this camera motion? Can you see a new facet that you were not seeing in the first image?

(b) Assume that a vertex V of the cube is seen as a point with coordinates (x, y) in the first image. The camera is then rotated around an axis of rotation passing through the center of projection and parallel to the y axis of the image plane, so that the unit vector k perpendicular to the image plane becomes a new vector k'. The unit vector of the y axis is a known vector j. What are the new coordinates of the image of V after the rotation, as functions of x, y, j, k and k'.

5. Using Matlab, and the results of problem 4, create the image you would obtain if the camera that took the image `'flowers.tif'` (converted to grayscale) were to be rotated by 10^o to the left around its center of projection. (Set any pixels that were not visible in the first image to white. Set the focal length to 250 pixels.)
6. Estimate the number of operations it takes to compute the determinant of an N dimensional matrix using the recursive algorithm explained in class. Compare the numbers of operations it would take the recursive algorithm and an algorithm that requires N^3 operations.
7. In Matlab explore the following matrix decompositions and write two sentences about each of them. LU (`LU`), Cholesky (`Chol`), eigenvalue (`eig`), `QR` and the singular value decomposition (`SVD`). Show results for a 5 by 5 random matrix (make the matrix symmetric when required to do so).
8. The eigenvalues of a matrix **A** can be obtained by solving the equation $|\mathbf{A}-\lambda\mathbf{I}| = 0$ (where $|\mathbf{A}|$ denotes the determinant of **A**). Calculate the eigenvalues of an arbitrary 2 by 2 matrix analytically. Modify your quadratic equation Matlab function (homework 1) to return the eigenvalues of any 2 by 2 matrix.
9. Read chapter 2.0 through chapter 2.3 of Numerical recipes. To those who do not have the book, you can find a copy on the web at http://lib-www.lanl.gov/numerical/bookcpdf.html