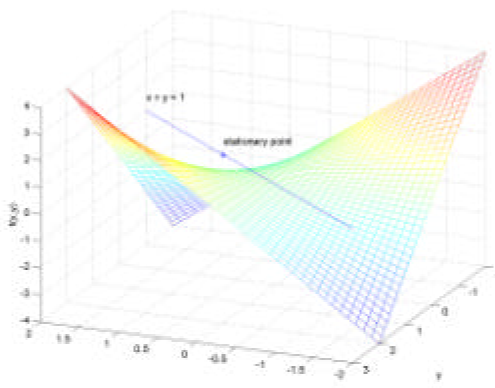
CMSC 828D: Fundamentals of Computer Vision
Homework 12

Instructors: Larry Davis, Ramani Duraiswami,
Daniel DeMenthon, and Yiannis Aloimonos
Solution based on homework submitted by Haiying Liu

1. **Find the stationary point of the curve $f(x, y) = xy$ subject to the constraint $x + y = 1$ using the method of Lagrange multipliers.**

Solution: Define $g(x, y, l) = f(x, y) + l(x + y - 1)$. Let its partial derivatives respect to x, y, l to be zeros:

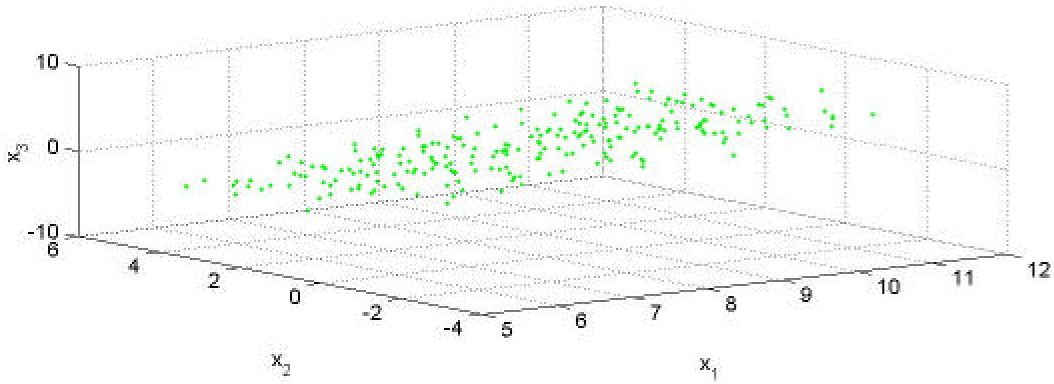


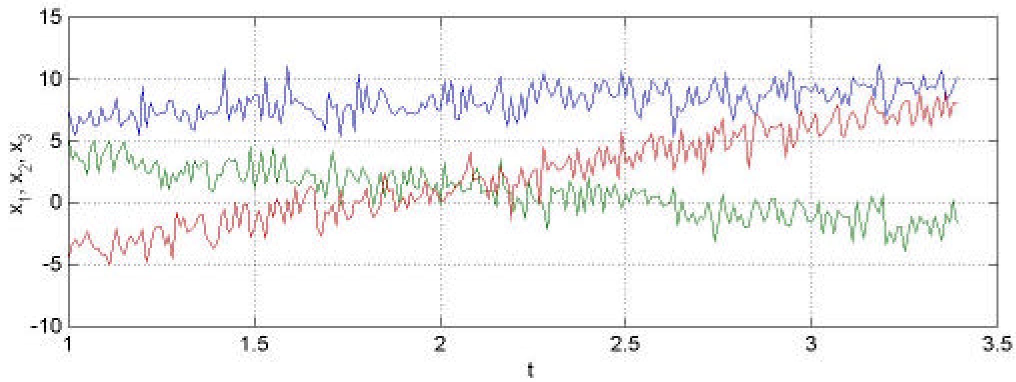
$$\left.\begin{array}{l} \frac{\partial g}{x} = y + l = 0 \\ \frac{\partial g}{y} = x + l = 0 \\ \frac{\partial g}{l} = x + y - 1 = 0 \end{array}\right\} \Rightarrow \begin{cases} x = \frac{1}{2} \\ y = \frac{1}{2} \\ l = -\frac{1}{2} \end{cases}$$

The stationary point of the curve subject to the constraint is at $(1/2, 1/2)$. The above plot shows relationships between the curve and the constraint. See Matlab script `hw12_1.m` in appendix for detail.

2. **The goal of this problem is to find position estimates for a missile moving in 3D. …**
   a. Plot the 3 coordinates of the missile position in space …

Solution: It is plotted below. Matlab script `hw12_2.m` is listed in appendix.





b. Write the state equation, …

Solution: The state equation is written as:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}^{(i)} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}^{(i-1)} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}^{(i-1)}$$

where $w_k \sim N(0,0.1)$, $\Delta t = 0.01$ second is the time step. Rewrite the above equation in a simpler form: $\mathbf{a}^{(i)} = \mathbf{F}\,\mathbf{a}^{(i-1)} + \mathbf{w}^{(i-1)}$.

c. Write the measurement equation, …

Solution: The measurement equation is written as:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^{(i)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^{(i)} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}^{(i)}$$

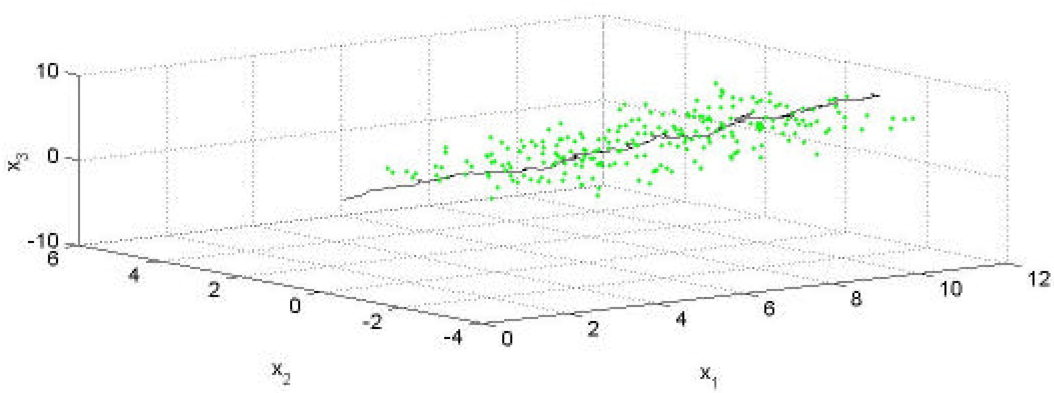
where $v_k \sim N(0,3)$. Rewrite the above equation in a simpler form: $\mathbf{x}^{(i)} = \mathbf{H}\mathbf{a}^{(i)} + \mathbf{v}^{(i)}$.

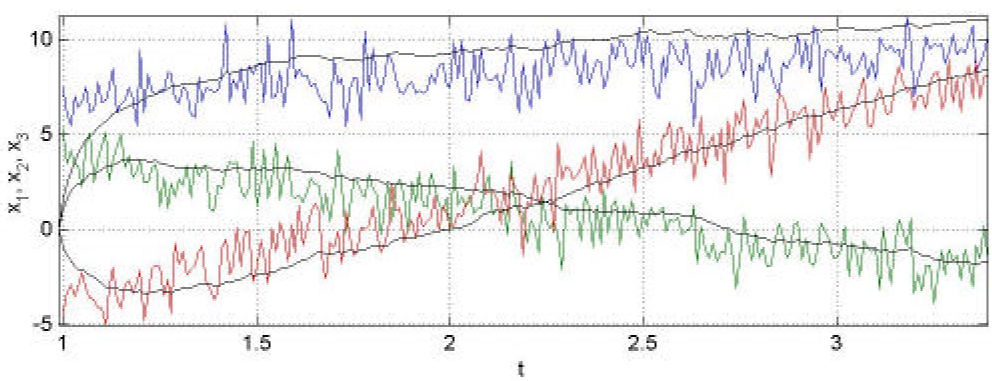
d. Write a Matlab Kalman filter function, …

Solution: See Matlab script `hw12_2.m` in appendix in detail.

e. Run the Kalman filter …

Solution: See Matlab script `hw12_2.m` in appendix in detail. The estimations are plotted below:





3. Read Chapter 19.4 of the book by …

Solution: Done.

Appendix:

- hw12_1.m:

```
function hw12_1
% Syntax: hw12_1
%
% Description: CMSC828D HW12_1
%
% Author: Haiying Liu
%   Date: Dec. 2, 2000
%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

dbstop if error

msg = nargchk(0, 0, nargin);
if (~isempty(msg))
  error(strcat('ERROR:', msg));
end

clear msg;

%========================================================================

figure;

meshX  = -2:0.1:2;
meshY  = -2:0.1:2;
[x, y] = meshgrid(meshX, meshY);

f_xy   = x .* y;

mesh(x, y, f_xy);
grid on;

lx = -2:0.1:2;
ly = 1 - lx;
lz = ones(length(lx), 1);

hold on;
plot3(lx, ly, lz);

plot3(.5, .5, 1, '*');
text(.5, .5, 1.5, 'stationary point');
text(2, -1, 1.5, 'x + y = 1');

xlabel('x');
ylabel('y');
zlabel('f(x,y)');
view(-157, 26);

print -djpeg hw12_1;
```

- hw12_2.m:

```
function hw12_2
% Syntax: hw12_2
%
% Description: CMSC828D HW12_2
%
% Author: Haiying Liu
%   Date: Dec. 2, 2000
%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

dbstop if error

msg = nargchk(0, 0, nargin);
if (~isempty(msg))
  error(strcat('ERROR:', msg));
end

clear msg;

%=======================================================================
%= a. Plot the 3 coordinates of the missile position in space as
%=    a function of time on a single figure.

% Read the raw data.
rawData = textread('KF_meas.dat', '%f');
x       = reshape(rawData, 3, length(rawData) / 3)';
nX      = size(x, 1);

figure;

subplot(1, 2, 1);
plot3(x(:, 1), x(:, 2), x(:, 3), 'g.');

xlabel('x_1');
ylabel('x_2');
zlabel('x_3');
grid on;
view(3);

subplot(1, 2, 2);
dt = 0.01;
t  = 1:dt:(nX - 1)* dt + 1;
plot(t, x(:, 1), t, x(:, 2), t, x(:, 3));
xlabel('t');
ylabel('x_1, x_2, x_3');
grid on;

print -djpeg hw12_2a;

%=======================================================================
%= d. Write a Matlab Kalman filter function ...

% See the function 'KalmanFilter' below.
```

```
%==========================================================================
%= e. Run the Kalman filter. Plot the estimates for the missile ...

% Initialization.
phi = [ ...
    1 0 0 dt 0  0 ; ...
    0 1 0 0  dt 0 ; ...
    0 0 1 0  0  dt; ...
    1 0 0 0  0  0 ; ...
    0 1 0 0  0  0 ; ...
    0 0 1 0  0  0 ; ...
  ];

H   = [ ...
    1 0 0 0 0 0; ...
    0 1 0 0 0 0; ...
    0 0 1 0 0 0; ...
  ];

Q   = 0.01 * eye(6);

R   = 4 * eye(3);

state.a       = zeros(6, 1);
state.phi     = phi;
state.Q       = Q;
state.K       = [];
state.P       = eye(6);
state.PP      = eye(6);

mesaurement.x = [];
measure.H = H;
measure.R = R;

% Estimate for the dyanamic system.
est_a = zeros(nX, 6);
for index = 1:nX
  measure.x = x(index, :)';
  state = KalmanFilter(state, measure);
  est_a(index, :) = state.a';
end

subplot(1, 2, 1);
hold on;
plot3(est_a(:, 1), est_a(:, 2), est_a(:, 3), 'k');

subplot(1, 2, 2);
hold on;
t = [1 - dt, t];
est_a = [zeros(6, 1)'; est_a];
plot(t, est_a(:, 1), 'k', t, est_a(:, 2), 'k', t, est_a(:, 3), 'k');
axis tight;

print -djpeg hw12_2f;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
function newState = KalmanFilter(state, measure)
% Syntax: newState = KalmanFilter(state, measure)
%
%         state        - structure for state equation including
%                          state.a  : state variable;
%                          state.phi: system matrix;
%                          state.Q  : covariance for state Gaussian noise
%                                     at time i-1;
%                          state.K  : gain;
%                          state.P  : covairance matrix for prediction
%                                     error, i.e. P'_i;
%                          state.PP : covariance for estimation error,
%                                     i.e. P_i-1.
%         measure  - structure for measure equation including
%                          measure.x: measure variable;
%                          measure.H: system transfer matrix;
%                          measure.R: covariance for measure
%                                         Gaussian noise
%
% Description: Kalman filter
%
% Author: Haiying Liu
%   Date: Dec. 2, 2000
%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

dbstop if error

msg = nargchk(2, 2, nargin);
if (~isempty(msg))
  error(strcat('ERROR:', msg));
end

clear msg;

%======================================================================

% Initialize the new estimation structure by old one.
newState    = state;

% Compute the covariance matrix for prediction error.
newState.P  = state.phi * state.PP * (state.phi)' + state.Q;

% Compute Gain.
newState.K  = state.P * (measure.H)' * ...
  inv(measure.H * state.P * (measure.H)' + measure.R);

% Predict the new state.
newState.a  = state.phi * state.a + newState.K * ...
  (measure.x - measure.H * state.phi * state.a);

% Compute covariance for estimation error used for next round recursion.
newState.PP = (eye(size(newState.K, 1), size(measure.H, 2)) - ...
  newState.K * measure.H) * newState.P;
```