Emailed November 15, 2000
Due back November 27, 2000

Homework 11

1. Investigate the following uniqueness issue. Consider an image motion field due to rigid motion of the observer.

   (a) Is it possible that there are different rigid motions and surfaces which produce the same flow field? If yes, what are the constraints on the surfaces in view?

   Hint: Suppose we have a motion $\{\mathbf{t}_1, \boldsymbol{\omega}_1\}$ and depth surface $Z_1(x,y)$ that yields the same motion field as a motion $\{\mathbf{t}_2, \boldsymbol{\omega}_2\}$ and a depth surface $Z_2(x,y)$.
   We equate the two motion fields and obtain

   $$\frac{1}{Z_1}\left(\hat{\mathbf{z}} \times (\mathbf{t}_1 \times \mathbf{r})\right) - \frac{1}{Z_2}\left(\hat{\mathbf{z}} \times (\mathbf{t}_2 \times \mathbf{r})\right) = \hat{\mathbf{z}} \times (\mathbf{r} \times (\delta\boldsymbol{\omega} \times \mathbf{r}))$$

   where $\delta\boldsymbol{\omega} = \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1$.
   Take the product of this vector equation with $(\mathbf{t}_1 \times \mathbf{r})$ and with $(\mathbf{t}_2 \times \mathbf{r})$ to obtain two scalar equations. From these equations the surfaces can be obtained. What are these ambiguous surfaces?

   (b) If we consider an infinitely large image plane do these surfaces have positive value everywhere (that is, are they in front of the camera)?

2. Consider an observer moving with rigid motion $\mathbf{t} = (U, V, W)$ and $\boldsymbol{\omega} = (\alpha, \beta, \gamma)$.

   (a) The surface being imaged is a plane described as $Z = Z_0 + pX + q$ where X, Y, Z are the 3D coordinates of points on the surface. Show that

   $$\frac{Z_0}{Z} = 1 - px - qy$$

   where x and y are image coordinates.

   (b) Develop the equations for the motion field (u, v) in terms of the motion and scene parameters.

   (c) For this case, the motion parameters are not determined uniquely. Consider two different sets of rigid motions and corresponding planar surfaces, $(\mathbf{t}_1, \boldsymbol{\omega}_1, Z_{0_1}, p_1, q_1)$ and $(\mathbf{t}_2, \boldsymbol{\omega}_2, Z_{0_2}, p_2, q_2)$. How are these sets of parameters related to each other?

3. Create the following synthetic scene: A plane in the background and a cube in the foreground. The observer is moving rigidly with translation $\mathbf{t} = (U, V, W)$ and rotation $\boldsymbol{\omega} = (\alpha, \beta, \gamma)$. Write a program that plots

   (a) the motion field

   (b) the normal motion field, where the gradient distribution is chosen by a random generator.

4. Compute the normal flow field for the Yosemite Sequence using the method of Lucas and Kanade as implemented in Barron et al. (1994), which consists of the following steps:

   (a) Filtering of the image sequence with a spatiotemporal Gaussian filter, with standard deviation $\sigma = 1.5$ and kernel size $11 \times 11 \times 11$.

   (b) Estimation of the spatial and temporal derivatives with a 5-point symmetric kernel $1/12 \times (-1, 8, 0, 8, -1)$ applied to the blurred image.

   (c) Evaluation of normal flow values at points with high spatial gradient.

   Thus to compute one flow field 15 images are required.