# CMSC 828D: Solutions for Questions 1, 2 and 3 of Midterm Exam

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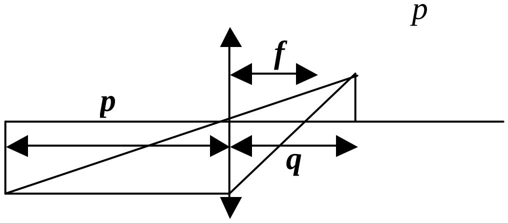
1. Suppose we are viewing an object 2 meters in front of a camera with a focal length of 50 mm. How far behind the lens will the image of this point be brought into focus? (2 points)

The distance we are looking for is q. We have

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \Rightarrow q = \frac{f\,p}{p - f} = \frac{f}{1 - \frac{f}{p}}$$

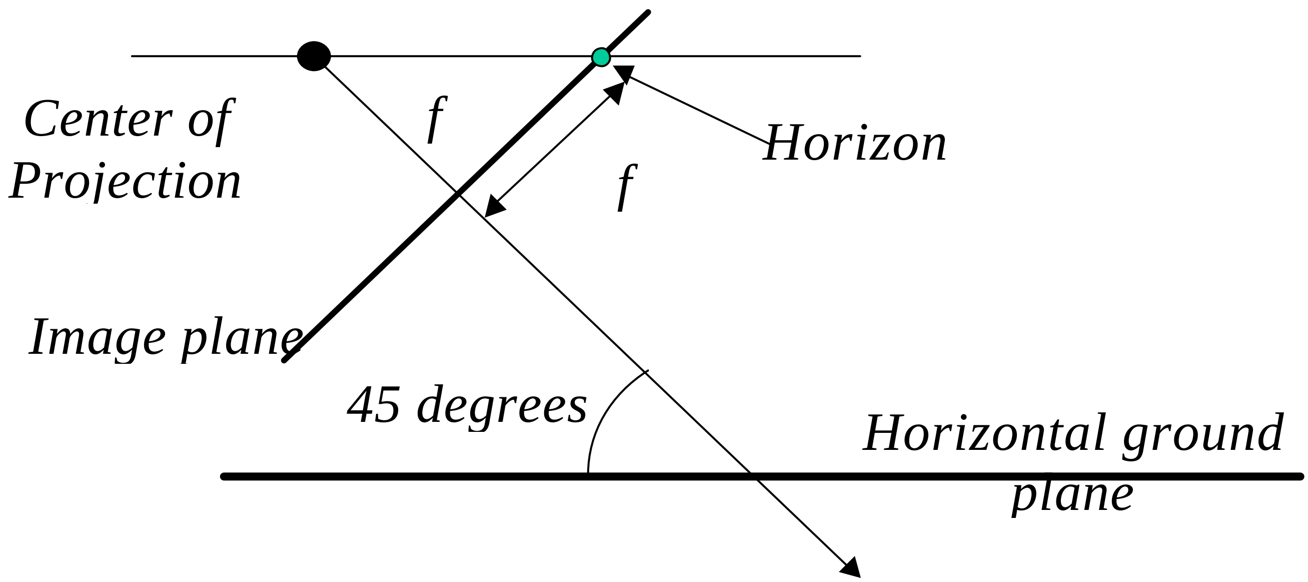



Since $f/p$ is small, we can approximate $q$ as $q \approx f(1 + \frac{f}{p}) = f + \frac{f^2}{p}$. For $f$ =50 mm and $p$ =2000 mm, the term added to the focal length is equal to 2500/2000, i.e. 1.25 mm.

Therefore q is approximately 51.25 mm.

2. A camera with focal length $f$ = 250 pixels has horizontal pixel rows and is pointing down at 45 degrees toward a horizontal plane. Where does the camera see the horizon of that plane? (in other words, where is the vanishing line for that plane located in the image plane of the camera?). Explain your conclusion. . (4 points)

Consider lines on the horizontal plane. Vanishing points of these lines are obtained as images of points at infinity on these lines. Therefore they are image points obtained by lines of sight parallel to these lines. The horizon is the locus of all the vanishing points of the lines of the horizontal plane. It is the intersection of all the lines of sight parallel to the horizontal plane with the image plane, i.e. the intersection of a horizontal plane through the center of projection with the image plane. This intersection is a row, since the intersection of a horizontal plane with any plane is a horizontal line, and since pixel rows were specified to be horizontal. This row is at a distance from the image center equal to the focal length (250 pixels) because of the assumption that the camera is pointing down at 45 degrees.




3. Given a stereo pair of images who optical axes intersect at some 3D point, we say that the stereo pair is verged on that 3D point (see figure below). Consider the special case in which the two cameras share a common $x$ and $y$ axis, but are then both rotated by the same (absolute) angle, $q$, around the $y$ axis to create the verged pair. Disparity for the 3D point at which the optical axes intersect is then obviously 0.

a) What is the formula for the $z$- coordinate of the 3D point given its conjugate pair in this configuration. (2 points)

We assume that the 3D points we consider are in the plane (O1, O2, M). Consider such a point Q. It projects on the baseline in H. We want to express distance HQ in terms of $q$, $x_1$ and $x_2$ (see Figure next page).

$$\frac{HQ}{\tan(p/2 - q_1 - a_1)} = O_1H, \quad \frac{HQ}{\tan(p/2 - q_2 - a_2)} = O_2H$$

$$\Rightarrow HQ(\tan(q_1 + a_1) - \tan(q_2 + a_2)) = O_1H + HO_2 = d$$

Now we use the notation $t = \tan q$ . Also note that $\tan q = \tan q_2 = -\tan q_1$.We have $\tan a_1 = x_1 / f$, $\tan a_2 = x_2 / f$ . Expanding the tangent terms in the equation above we obtain

$$HQ = \frac{d(f + t x_1)(f - t x_2)}{f(t^2 - 1)(x_2 - x_1) - 2t(f^2 + x_1 x_2)}$$

b) For what 3D points is disparity positive? negative?

The angle ($QO_1$, $QO_2$) is equal to

$\pi$ - $(\pi/2 - q + a_1)$- $(\pi/2 - q - a_2)$, i.e. $2q - (a_1 - a_2)$. The disparity is zero when $x_1 = x_2$, **i.e.** $a_1 = a_2$. In this case the angle ($QO_1$, $QO_2$) is equal to $2q$. Therefore when the disparity is zero, the point Q sees the segment under a constant angle $2q$. Note that the 3D point

M at the intersection of the optical axes of the camera is such a point with disparity zero. Thus the locus of the points Q with zero disparity is an arc of circle circumscribing the triangle $MO_1O_2$. For the points P inside the region defined by $O_1O_2$ and the arc of circle, the angle ($QO_1$, $QO_2$) is larger than $2q$, therefore $(a_1 - a_2)$ must be negative and the disparity is negative. With the same reasoning, the points outside the region defined by $O_1O_2$ and the arc of circle have a positive disparity.