

Mid term exam: CMSC/AMSC/MAPL 460 Computational Methods 10/18/2005
75 minutes.

1. ...
2. For a matrix A of size 40×30 a matrix B of size 30×30 and a vector x of size 30, write down the number of operations needed to perform $A(Bx)$ and $(AB)x$. Will the answers be the same in each case? (10 points)

A matrix vector product between a $M \times N$ matrix and a N vector takes MN multiplications and additions. In the case of a matrix matrix product between a $M \times N$ matrix and a $N \times L$ matrix vector takes MLN multiplications and additions and we end up with a $M \times L$ matrix.

Doing the operations the first way we first form the product $y=Bx$ at a cost of $30^2 = 900$ multiplications and 30×29 additions, and end up with a vector of size 30. We next perform a matrix vector product Ay which takes 1200 multiplications and 40×29 additions. The total cost is 2100 multiplications and 70×29 additions.

Doing the operations the second way we first form the product matrix $C=AB$ at a cost of $40 \times 30 \times 30 = 36000$ multiplications and $40 \times 30 \times 29$ additions, and end up with a matrix of size 40×30 . We next perform a matrix vector product Cx which takes 1200 multiplications and 40×29 additions. The total cost is 37200 multiplications and $40 \times 31 \times 29$ additions.

The two ways, in full precision should give the same answer.

3. Write brief descriptions of the following. In each case provide any pertinent details such as formulae, but be brief: (5 points each)
 - a. well posed and well-conditioned problems
 - b. golden-search for minimization
 - c. Why does the order of accessing matrix elements affect the efficiency of software?
 - d. Cramer's rule
 - e. Why is solving a system of equations by computing the inverse and then multiplying it by the right hand side usually a bad idea?
 - f. The differences between cubic Hermite splines, cubic natural splines, and the shape-preserving cubic splines.
 - g. Permutation matrix

Refer to class notes for these answers

4. Write a Matlab script that will use Horner's rule to evaluate:
$$a_1 + a_2 (x-x_1) + a_3 (x-x_1) (x-x_2) + a_4 (x-x_1) (x-x_2) (x-x_3) + \dots$$
 (15 points)

In Horner's rule we evaluate the factored product from the highest power down by performing one multiplication and one addition each step.

$$a_1 + (x-x_1) (a_2 + a_3 (x-x_2) (a_3 + a_4 (x-x_3)))$$

The following script evaluates the rule for $x=3$

Given the arrays $a(1:N)$ and $x(1:N-1)$

`xval=3`

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s=1
for i=n-1:1
    s=a(i+1)*(xval-x(i))*s+a(i);
end

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5. Write down the Lagrange form of the interpolating polynomial for the data $(x; f(x)) = (4, -5), (7, 2), (9, 3)$. Evaluate this polynomial at $x=5$. (10 points)

$$f(x) = \frac{(x-7)(x-9)}{(4-7)(4-9)}(-5) + \frac{(x-4)(x-9)}{(7-4)(7-9)}(2) + \frac{(x-4)(x-7)}{(9-4)(9-7)}(3)$$

$$= -(x-7)(x-9)/3 - (x-4)(x-9)/3 + 3(x-4)(x-7)/10$$

Evaluating at $x = 5$

$$f(5) = -(5-7)(5-9)/3 - (5-4)(5-9)/3 + 3(5-4)(5-7)/10$$

$$= -8/3 + 4/3 - 6/10 = -58/30 = -29/15 = -1.9333$$

6. Let $f(x) = x^3 - 27 = 0$. Perform two iterations each using the (15 points)

- a. Bisection method, with a starting interval of $[0, 5]$

$$f(0) = -27$$

$f(5) = 98$. Root is bracketed. So next evaluation point is midpoint 2.5

$f(2.5) = (2.5)^3 - 27 = 15.625 - 27 = -11.375$. So root is between 2.5 and 5. Next evaluation point is 3.75

$f(3.75) = 3.75^3 - 27 = 25.7344$. So root is between 2.5 and 3.75. Next evaluation point is 3.125.

- b. Secant method with starting values of $x_1 = 1$ and $x_2 = 2$.

$$x_k = x_{k-1} - f(x_{k-1}) \frac{(x_{k-1} - x_{k-2})}{(f(x_{k-1}) - f(x_{k-2}))}$$

$$f(1) = -26$$

$$f(2) = -19$$

$$x_1 = 2 - (-19)(1)/(7) = 2 + 19/7 = 4.7143; f(x_1) = 77.7726$$

$$x_2 = 4.7143 - (77.7726)(2.7143)/(77.7726 - (-19))$$

$$= 4.7143 - (77.7726)(2.7143)/(77.7726 - (-19))$$

$$= 2.5329$$

- c. Newton's method with a guess of $x = 2$

$$f'(x) = 3x^2$$

Newton method formula is $x_{k+1} = x_k - f(x)/f'(x)$

$$1^{st} \text{ iteration: } x_1 = 2 - (-19)/(12) = 3 \frac{7}{12} = 3.5833$$

$$2^{nd} \text{ iteration } x_2 = 3.5833 - 19.011/28.5208 = 2.9167$$