Image formation
Unfinished business

• Grader/TA: Justin Domke: [domke@cs.umd.edu]
• Web site for more illusions, and how illusions arise
  
  http://www.cfar.umd.edu/~fer/optical/index.html
• Homework for this week will be given next class
  – Meanwhile get access to Matlab
  – Homework problems posted on the class web site
Image Formation

• Vision infers world properties form images.
• How do images depend on these properties?
• Two key elements
  – Geometry
    • Geometry of images captured by cameras (Chapter 1)
  – Radiometry
    • What do the intensities in the image tell us about the light in the real world? (Chapter 4)
  – We consider only simple models of these
Light emits photons

Photons travel in a straight line

When they hit an object they:
• bounce off in a new direction
• or are absorbed
• (exceptions later).

And then some reach the eye/camera.
Basic fact: Light is linear

- Double intensity of sources, double photons reaching eye.
- Turn on two lights, and photons reaching eye are same as sum of number when each light is on separately.
- Next class will deal with light – today will discuss image formation by cameras and mammalian eyes.
"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays".

*Leonardo Da Vinci*

• Camera in Latin means “chamber” or “room”
• Even now this use of the word in a couple of places
  – Bicameral (both the senate and the house of representatives … both rooms)
  – In camera (closed-room court proceedings)
• Used to observe eclipses (eg., Bacon, 1214-1294)
• By artists (eg., Vermeer).
Camera Obscura

POPULAR 19TH CENTURY ATTRACTION

Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)
Cameras

• First photograph due to Niepce
• First on record shown in the book - 1822
**Pinhole cameras**

- Abstract camera model - box with a small hole in it
- Pinhole cameras work in practice
- Are a good model of images taken by modern cameras and of images formed on the retina
- Form an inverted image
  - To help us think and do geometry easily, we use
Properties of images: Distant objects are smaller
Parallel lines meet

Common to draw image plane *in front* of the focal point.
Moving the image plane merely scales the image.

(Forsyth & Ponce)
Vanishing points

• Each set of parallel lines meets at a different point
  – The *vanishing point* for this direction
• Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  – The line is called the *horizon* for that plane
Properties of Projection

• Points project to points
• Lines project to lines
• Planes project to the whole image or a half image
• Angles are not preserved
• Degenerate cases
  – Line through focal point projects to a point.
  – Plane through focal point projects to line
  – Plane perpendicular to image plane projects to part of the image (with horizon).
Take out paper and pencil
1. **Draw a horizon line.**

2. **Make a vanishing point.**

3. **Draw a square or rectangle.**

4. **Draw orthogonal lines from shape corners to vanishing point.**

5. **Draw a horizontal line to end your form.**

6. **Draw orthogonals from shape corners to vanishing point.**

7. **Erase the orthogonals.**

8. **Draw another form!**

9. **Try stacking.**

10. **Add windows and doors.**

11. **Draw a vertical line to make the form’s side.**
Add windows and doors.
The equation of projection

(Forsyth & Ponce)
The equation of projection

• Cartesian coordinates:
  – We have, by similar triangles, that
    \[(x, y, z) \rightarrow (f' \frac{x}{z}, f' \frac{y}{z}, f')\]
  – Ignore the third coordinate, and get
    \[(x, y, z) \rightarrow (f' \frac{x}{z}, f' \frac{y}{z})\]

• Equations are non-linear

\[x' = f' \frac{x}{z}\]
\[y' = f' \frac{y}{z}\]
Orthographic projection

\[ x' = x \]
\[ y' = y \]
Weak perspective (scaled orthographic projection)

• Issue
  – perspective effects, but not over the scale of individual objects
  – collect points into a group at about the same depth, then divide each point by the depth of its group

(Forsyth & Ponce)
Weak perspective (scaled orthographic projection)

\[(x, y, z) \rightarrow s(x, y)\]

• s is constant for all points.
• Parallel lines no longer converge, they remain parallel.
• Essentially we are assuming that the world is flat far-away
• Next step up: assume there are a finite number of such groups
Pros and Cons of These Models

• Weak perspective much simpler math.
  – Accurate when object is small and distant.
  – Most useful for recognition.
• Pinhole perspective much more accurate for scenes.
  – Used in structure from motion.
• When accuracy really matters, must model real cameras.
Pinhole camera image issues

- Not enough light is captured by the camera
- Solution: increase size ... this makes images blurry
- Decrease hole size to make image sharp
  - Image is blurry again due to diffraction effects

2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.
Cameras with Lenses

- Increasing pinhole size to increase amount of light captured causes blur.
- Instead use lenses to capture more of the photons leaving object,
- Similar to eye.
Human Eye

- Lens
- Pupil to control amount of light
- Retinal screen
- Fovea, and surround.
- Note that image is inverted!
- Stuff in front of screen but we don’t see it!

http://www.cas.vanderbilt.edu/bsci111b/eye/human-eye.jpg
Interaction of light with matter

- Absorption
- Scattering
- Refraction
- Reflection
- Other effects:
  - Diffraction: deviation of straight propagation in the presence of obstacles
  - Fluorescence: absorption of light of a given wavelength by a fluorescent molecule causes reemission at another wavelength
Figure 1-7

Reflection and refraction at the interface between two homogeneous media with indexes of refraction $n_1$ and $n_2$. 
A thin lens. Rays passing through the point $O$ are not refracted. Rays parallel to the optical axis are focused on the focal point $F'$. 
Those interested read Section 1.2.1 in the book on paraxial geometric optics
Similar triangles \(<P'F'S'>, <ROF'>\) and \(<PSF><QOF>\) \(\rightarrow\)

\[(z' - f)(-z - f) = f^2\]

\[\frac{1}{z'} + \frac{1}{-z} = \frac{1}{f}\]
Assumptions for thin lens equation

- Lens surfaces are spherical
- Incoming light rays make a small angle with the optical axis
- The lens thickness is small compared to the radii of curvature
- The refractive index is the same for the media on both sides of the lens
Spherical aberration (from 3rd order optics)
Other aberrations

- Astigmatism: unevenness of the cornea
- Distortion: different areas of lens have different focal length
- Coma: point not on optical axis is depicted as asymmetrical comet-shaped blob
- Chromatic aberration
Summary

• Camera loses information about depth.
  – A model of the camera tells us what information is lost.

• This will be important when we want to recover this information. Examples:
  – Motion: with multiple images.
  – Recognition: using a model.
  – Shape: how is boundary of smooth object related to its image?
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Modeling How Surfaces Reflect Light

• First, language for describing light
  – Striking a surface;
  – Leaving a surface.

• Next, how do we model the relationship between the two.
  – This depends on the material;
  – Eg., cloth or mirror.
Irradiance, $E$

- Light power per unit area (watts per square meter) incident on a surface.
- If surface tilts away from light, same amount of light strikes bigger surface (less irradiance).
Radiance, $L$

- Amount of light radiated from a surface into a given solid angle per unit area (watts per square meter per per steradian).
- Note: the area is the foreshortened area, as seen from the direction that the light is being emitted.
BRDF

Horn, 1986

**Figure 10-7.** The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction $(\theta_e, \phi_e)$ to the irradiance resulting from illumination from the direction $(\theta_i, \phi_i)$.

\[
BRDF = f(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{L(\theta_e, \phi_e)}{E(\theta_i, \phi_i)}
\]
BRDF Not Always Appropriate

http://graphics.stanford.edu/papers/bssrdf/  
(Jensen, Marschner, Levoy, Hanrahan)
Special Cases: Lambertian

\[ f(\theta_i, \phi_i, \theta_e, \phi_e) = k \frac{1}{\pi} \]

- Albedo is fraction of light reflected.
- Diffuse objects (cloth, matte paint).
- Brightness doesn’t depend on viewpoint.
- Does depend on angle between light and surface.

\[ L(\theta_e, \phi_e) \propto \cos(\theta) \]
Lambertian Examples

Scene

(Oren and Nayar)

Lambertian sphere as the light moves.

(Steve Seitz)
Specular surfaces

• Another important class of surfaces is specular, or mirror-like.
  – radiation arriving along a direction leaves along the specular direction
  – reflect about normal
  – some fraction is absorbed, some reflected
  – on real surfaces, energy usually goes into a lobe of directions

Specular surfaces

• Brightness depends on viewing direction.

Phong’s model

- Vision algorithms rarely depend on the exact shape of the specular lobe.
- Typically:
  - very, very small --- mirror
  - small -- blurry mirror
  - bigger -- see only light sources as “specularities”
  - very big -- faint specularities
- Phong’s model
  - reflected energy falls off with $\cos^n(\delta \theta)$

(Forsyth & Ponce)
Lambertian + specular

- Two parameters: how shiny, what kind of shiny.
- Advantages
  - easy to manipulate
  - very often quite close true
- Disadvantages
  - some surfaces are not
    - e.g. underside of CD’s, feathers of many birds, blue spots on many marine crustaceans and fish, most rough surfaces, oil films (skin!), wet surfaces
  - Generally, very little advantage in modeling behavior of light at a surface in more detail -- it is quite difficult to understand behavior of L+S surfaces (but in graphics???)
Lambertian + Specular + Ambient

- Ambient to be explained.

Modeling Light Sources

• Light strikes a surface from every direction in front of the object.
• Light in a scene can be complex:
Can vary with direction.

(from Debevec)
Also with position

(from Langer and Zucker)
And Along a Straight Line

(from Narasimhnan and Nayar)

Useful to use simplified models.
Simplest model: distant point source

- All light in scene comes from same direction.
- With same intensity
- Consequences:
  - Shadows are black.
  - Light represented as direction & intensity
\[ \vec{l} = l \cdot \vec{l} \]
\[
\begin{cases}
\vec{l} & \text{is direction of light} \\
l & \text{is intensity of light}
\end{cases}
\]

\[ i = \max(0, \lambda(\vec{l} \cdot \hat{n})) \]

- \( i \) is radiance
- \( \lambda \) is albedo
- \( \hat{n} \) is surface normal
Ambient Component

- Assume each surface normal receives equal light from all directions.

\[ i = \alpha \lambda \]

- Diffuse lighting, no cast shadows.
- Ambient + point source turns out to be good approximation to next model.
Distant Light

Sky

• Light is function of direction.
• Same at every scene point.
• Point, elongated, diffuse.
Conclusions

• Projection loses info; we can understand this with geometry.
• Light reaching camera depends on surfaces and lighting; we can understand this with physics.
• Reflection also loses information.
• Our models are always simplified.
• Just because you can see doesn’t mean the relation between the world and images is intuitive.

“(The world) saw shadows black until Monet discovered they were coloured,…”

Maugham, Of Human Bondage