

1. Suppose we are trying to distinguish cats from dogs and the only measurement we have is weight. Let the prior probabilities for cats and dogs be $P(\text{CAT}) = 0.3$, $p(\text{DOG}) = 0.7$ and let the class conditional density functions for (integer-valued) weight given class be uniformly distributed as follows:

– $P(\text{Weight}|\text{CAT})$ is uniform on $[20, 39]$

– $P(\text{Weight}|\text{DOG})$ is uniform on $[30,59]$.

Explain how Bayes' rule can be used to classify an unknown animal having weight 35 pounds given the prior and conditional probabilities defined above.

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Solution: The Bayes formula for this problem is:

$$P(X | \text{weight}) = \frac{P(\text{weight} | X)P(X)}{\sum_{Y \in \{\text{CAT}, \text{DOG}\}} P(\text{weight} | Y)P(Y)}$$

where $X \in \{\text{CAT}, \text{DOG}\}$. Given $P(\text{CAT}) = 0.3$, $P(\text{DOG}) = 0.7$

$$p(\text{weight} | \text{CAT}) = \frac{1}{39 - 20 + 1} = \frac{1}{20}$$

$$p(\text{weight} | \text{DOG}) = \frac{1}{59 - 30 + 1} = \frac{1}{30},$$

we have

$$\begin{aligned} P(\text{CAT} | 35) &= \frac{P(35 | \text{CAT})P(\text{CAT})}{P(35 | \text{CAT})P(\text{CAT}) + P(35 | \text{DOG})P(\text{DOG})} \\ &= \frac{\frac{1}{20} \cdot 0.3}{\frac{1}{20} \cdot 0.3 + \frac{1}{30} \cdot 0.7} \approx 0.3913 \end{aligned}$$

$$\begin{aligned} P(\text{DOG} | 35) &= \frac{P(35 | \text{DOG})P(\text{DOG})}{P(35 | \text{CAT})P(\text{CAT}) + P(35 | \text{DOG})P(\text{DOG})} \\ &= \frac{\frac{1}{30} \cdot 0.7}{\frac{1}{20} \cdot 0.3 + \frac{1}{30} \cdot 0.7} \approx 0.6087 \end{aligned}$$

Since $P(\text{DOG} | 35) > P(\text{CAT} | 35)$, the animal is more likely to be a dog.