Compressed Sensing

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Outline

1. Introduction
   - Compressed Sensing

2. Problem Formulation
   - Sparse Signal
   - Problem Statement

3. Proposed Solution
   - Near Optimal Information
   - Near Optimal Reconstruction

4. Applications
   - Example

5. Conclusion
Motivation

- Why go to so much effort to acquire all the data when most of the what we get will be thrown away?
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- Can't we just directly measure the part that won't end up being thrown away?
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- Can we just directly measure the part that won't end up being thrown away?
- Wouldn't it be possible to acquire the data in already compressed form so that one does not need to throw away anything?
A signal $x$ is $K$ sparse if its support is $i : x_i \neq 0$ is of cardinality less or equal to $K$. 
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Figure: 1 Sparse [richb]
Sparse Signal

Figure: 2 Sparse [richb]
Sparse Signal

Figure: 2 Sparse

Figure: 3 Sparse
Sparse Signal

- Have few non-zero coefficients.
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**Transform Compression:** The object has transform coefficients $\theta_i = \langle x, \Psi_i \rangle$ and are assumed sparse in the sense that, for some $0 < p < 2$ and for some $R > 0$:

$$||\theta||_p = \left( \sum_i |\theta_i|^p \right)^{1/p} \leq R$$
Compressive data acquisition

- When data is sparse/compressible, can directly acquire a condensed representation with no/little information loss through dimensionality reduction

\[ y = \Phi x \]
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- \( n = O(\sqrt{m \log(m)}) \) measurements instead of \( m \) measurements.
Problem Statement

\[ x \in \mathbb{R}^m \]
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- Class $X$ of such signals
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- Design an information operator $I_n : X \rightarrow \mathbb{R}^n$ that samples $n$ pieces of information about $x$
- Design an Algorithm $A : \mathbb{R}^n \rightarrow \mathbb{R}^m$ that offers an approximate reconstruction of $x$. 
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- Design an Algorithm \( A : \mathbb{R}^n \rightarrow \mathbb{R}^m \) that offers an approximate reconstruction of \( x \).
- Here the information operator takes the form

\[
I_n(x) = (\langle \xi_1, x \rangle, \ldots, \langle \xi_n, x \rangle)
\]

where the \( \xi_i \) are sampling kernels
Problem Statement

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- Interested in $\ell^2$ error of reconstruction

$$E_n(X) = \inf_{A_n, l_n^A} \sup_{x \in X} ||x - A_n(l_n(x))||_2$$
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$$E_n(X) = \inf_{A_n, I_n} \sup_{x \in X} ||x - A_n(I_n(x))||_2$$

- Why not adaptive:
  For $0 < p < 1$

$$E_n(X_{p,m}(R)) \leq 2^{1/p} E_n(X_{p,m}^{Adapt}(R))$$

Adaptive information is of minimal help.
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- $\Phi_J$ submatrix of $\Phi$ with columns $J$. 
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- Assume $\Psi$ to be identity for simplicity
- $J \subset \{1, 2, \ldots, m\}$
- $\Phi_J$ submatrix of $\Phi$ with columns $J$
- $V_J$ denote range of $\Phi$ in $\mathbb{R}^n$
Near Optimal Information

CS conditions

- Structural conditions on an $n \times m$ matrix which imply that its nullspace is optimal.
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- CS2: On each subspace $V_J$ we have the inequality
  $$\|v\|_1 \geq \eta_2 \cdot \sqrt{n} \cdot \|v\|_2 \forall v \in V_J$$
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- **CS2**: On each subspace \( V_J \) we have the inequality
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  uniformly in \( |J| < \rho n / \log(m) \)
  - This says that linear combinations of small groups of columns give vectors that look much like random noise, at least as far as the comparison of \( \ell^1 \) and \( \ell^2 \) norms is concerned.
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  - This says that linear combinations of small groups of columns give vectors that look much like random noise, at least as far as the comparison of \( \ell^1 \) and \( \ell^2 \) norms is concerned.
  - Every \( V_J \) slices through the \( \ell^1_m \) in such a way that the resulting convex section is actually close to spherical.
CS conditions

- Family of quotient norms on $\mathbb{R}^n$.

\[ Q_{J_c}(v) = \min \| \theta \|_{\ell^1(J_c)} \text{ subject to } \Phi_{J_c} \theta = v \]

These describe the minimal $\ell^1$ norm representation of $v$ achievable using only specified subsets of columns of $\Phi$. 
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- CS3: On each subspace $V_J$

$$Q_J^c(v) \geq \eta_3 / \sqrt{\log(m/n)} ||v||_1, v \in V_J$$

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- This says that for every vector in some $V_J$, the associated quotient norm $Q_{Jc}$ is never dramatically better than the simple $\ell_1$ norm on $(R)^n$. 

Near Optimal Information
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- Matrices satisfying these conditions are ubiquitous for large $n$ and $m$. 

Near Optimal Information

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- Matrices satisfying these conditions are ubiquitous for large $n$ and $m$.

- Possible to choose $\eta_i$ and $\rho$ independent of $n$ and of $m$.
Ubiquity

Theorem: Let \((n, m_n)\) be a sequence of problem sizes with \(n \to \infty\), \(n < m_n\), and \(m \sim A.n^\gamma\), \(A > 0\) and \(\gamma \geq 1\). There exist \(\eta_i > 0\) and \(\rho > 0\) depending only on \(A\) and \(\gamma\) so that, for each \(\delta > 0\) the proportion of all \(n \times m\) matrices \(\Phi\) satisfying CS1-CS3 with parameters \((\eta_i)\) and \(\rho\) eventually exceeds \(1 - \delta\).
Near Optimal Information

Φ matrix

• Random Projection Φ
Near Optimal Information

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Near Optimal Information

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**ϕ** matrix

- Random Projection $Φ$
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- But preserves structure and information in sparse signals with high probability.
**Near Optimal Information**

**Φ matrix**

- Random Projection Φ
- Not full rank $n \times m$ where $n < m$
- Loses information, in general
- But preserves structure and information in sparse signals with high probability.
- Generated by randomly sampling the columns, with different columns iid random uniform on $S^{n-1}$
Reconstruction Algorithm

- Least norm solution

\[
\min \|\theta(x)\|_p \text{ subject to } y_n = l_n(x) (P_p)
\]
Reconstruction Algorithm

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- Drawbacks:
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- Drawbacks:
  - \( p \) should be known
  - \( p < 1 \) is a nonconvex optimization problem.
  - Solving \( \ell^0 \) norm requires combinatorial optimization.
Near Optimal Reconstruction

Basis Pursuit

- Let $A_{n \times 2m} = [\Phi - \Phi]$
Near Optimal Reconstruction

Basis Pursuit

- Let $A_{n \times 2m} = [\Phi - \Phi]$
- Linear Program

$$\min_z 1^T z \text{ subject to } Az = y_n, \ x \geq 0.$$
Near Optimal Reconstruction

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- has solution $z^* = [u^* v^*]$
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- Linear Program
  
  $\min_z 1^T z \text{ subject to } Az = y_n, \ x \geq 0.$

- has solution $z^* = [u^* \ v^*]$
- $\theta^* = u^* - v^*$
Near Optimal Reconstruction

Basis Pursuit

- Fix $\epsilon > 0$
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- $x$ such that $||\theta||_1 \leq cm^\alpha$, $\alpha < 1/2$. 
Near Optimal Reconstruction

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- Fix $\epsilon > 0$
- $x$ such that $||\theta||_1 \leq cm^\alpha$, $\alpha < 1/2$.
- Make $n \sim C_\epsilon \cdot m^{2\alpha} \log(m)$ measurements.
Near Optimal Reconstruction

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- Fix $\epsilon > 0$
- $x$ such that $||\theta||_1 \leq cm^\alpha$, $\alpha < 1/2$.
- Make $n \sim C_\epsilon . m^{2\alpha} \log(m)$ measurements.

$$||x - \hat{x}_{1,\theta}||_2 << \epsilon . ||x||_2$$
Reconstruction Algorithm

- When $P_0$ has a solution, $P_1$ will find it.
Near Optimal Reconstruction

Reconstruction Algorithm

- When $P_0$ has a solution, $P_1$ will find it.
- Theorem: Suppose that $\Phi$ satisfies CS1-CS3 with given positive constants $\rho, (\eta_i)$. There is a constant $\rho_0 > 0$ depending only on $\rho$ and $(\eta_i)$ and not on $n$ or $m$ so that, if $\theta$ has at most $\rho_0 n / \log(m)$ nonzeros, then $(P_0)$ and $(P_1)$ both have the same unique solution.
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i.e., Although the system of equations is massively undetermined, $\ell^1$ minimization and sparse solution coincide when the result is sufficiently sparse.
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Parial Fourier Ensemble

- Collection of $n \times m$ matrices made by sampling $n$ rows out of the $m \times m$ Fourier matrix,
Partial Fourier Ensemble

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- Concrete examples of $\Phi$ working within the CS framework
Partial Fourier Ensemble

- Collection of $n \times m$ matrices made by sampling $n$ rows out of the $m \times m$ Fourier matrix,
- Concrete examples of $\Phi$ working within the CS framework
- Allows $\ell^1$ minimization to reconstruct from such information for all $0 < p < 1$
Images of Bounded Variation

- $f(x), x \in [0, 1]^2$. 
Images of Bounded Variation

- $f(x), \ x \in [0, 1]^2$.
- Bounded in absolute value $\|f\|_{\infty} \leq B$. 
Images of Bounded Variation

- \( f(x), \ x \in [0, 1]^2 \).
- Bounded in absolute value \( \|f\|_\infty \leq B \).
- The wavelet coefficients follow \( \|\theta(j)\|_\infty \leq C.B.2^{-j} \).
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- Fix finest scale $j_1$
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- Take $j_0 = j_1/2$
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- Total of $c.j_1^2.4^{j_1/2}$ pieces of measured information.
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- Error of the same order of linear sampling with $4^j_1$ samples:

  $$\|f - \hat{f}\| \leq c2^{-j_1}$$

  with $c$ independent of $f$. 
Example

Piecewise $C^2$ Images with $C^2$ edges

- $C^2,2(B,L)$ of piecewise smooth $f(x)$, $x \in [0,1]^2$. 
Piecewise $C^2$ Images with $C^2$ edges

- $C^{2,2}(B, L)$ of piecewise smooth $f(x)$, $x \in [0, 1]^2$.
- Bounded in absolute value, first and second partial derivative by $B$. 
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- Curves total length $\leq L$. 


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- Take $j_0 = j_1/4$.
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- Total of $c.j_1^{5/2}.4^{j_1}/4$ pieces of measured information.
- Error of the same order of linear sampling with $4^{j_1}$ samples:

$$||f - \hat{f}|| \leq c2^{-j_1}$$

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  - Random sampling yields near optimal $I_n$ with overwhelmingly high probability.
Conclusion

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  • Conditions on $\Phi$ CS1-CS3
  • Information Operator $I_n(x) = \Phi \psi^T x$
  • Random sampling yields near optimal $I_n$ with overwhelmingly high probability.
  • Reconstructed $x$ by solving $\ell^1$ convex optimization.
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  - Random sampling yields near optimal $I_n$ with overwhelmingly high probability.
  - Reconstructed $x$ by solving $\ell^1$ convex optimization.
  - Exploits a priori signal sparsity information. [richb]
Thank You!