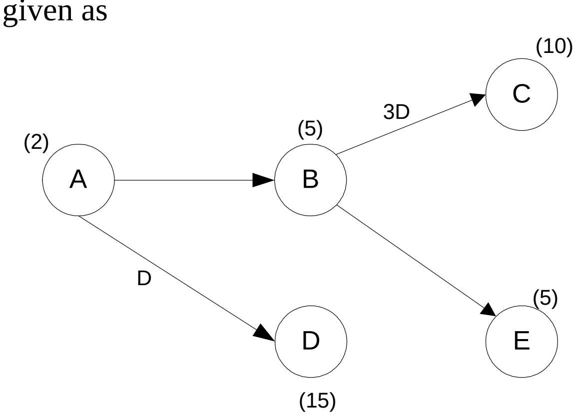
Solutions to Assignment #4
Chapter 5

5. The critical path lengths of an unfolded DFG only increase when a new arc is generated with no delays in the DFG; otherwise the path lengths remain the same. An arc with D delays in the original DFG produces J-D arcs with no delays in the J-unfolded DFG when J>D.

If J>D, then the J unfolded graph will contain an additional arc without delays, and the path length for the J-unfolded DFG increases through those nodes, compared to the (J-1) unfolded DFG.

Therefore, the critical path of the J-unfolded graph is greater than or equal to that of the (J-1) unfolded graph.

7. The DFG is given as



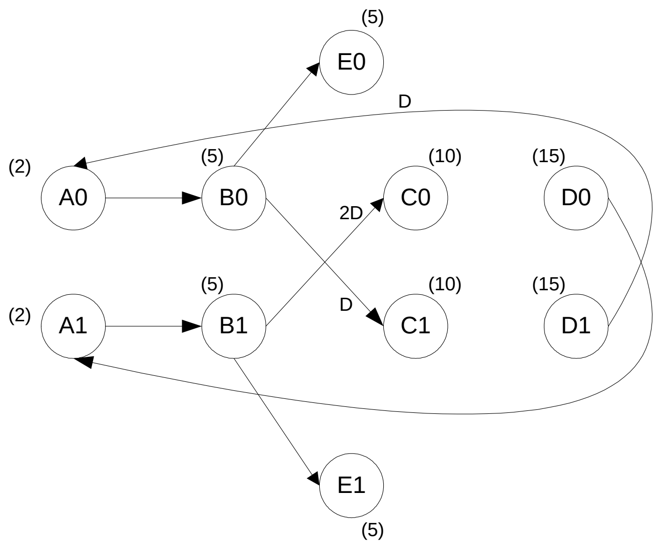
The computation for the critical path $A \rightarrow B \rightarrow E$ is given as 12 u.t. = T_{crit}^{0} .

Following the algorithm given in Problem 6, we have:

$$T_{crit}^{0} = 2+5+5=12;$$

$$J^{1} = \left\lceil \frac{12}{8} \right\rceil = 2$$

Applying unfolding of degree 2 we get



$T_{crit}^{1} = 2+15=17 > J^{1}\ x\ 8;$

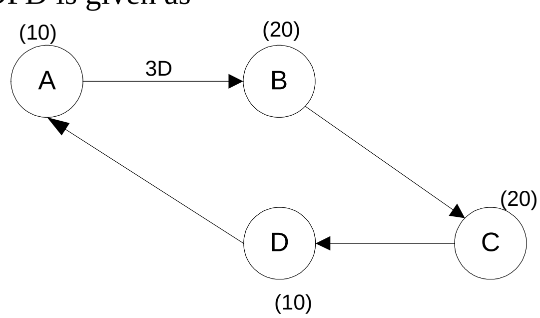
$J^{2} = \left\lceil \frac{17}{8} \right\rceil = 3$

Applying unfolding of degree 3 we get

$T_{crit}^{2} = 2+15=17 < J^{2}\ x\ 8;$

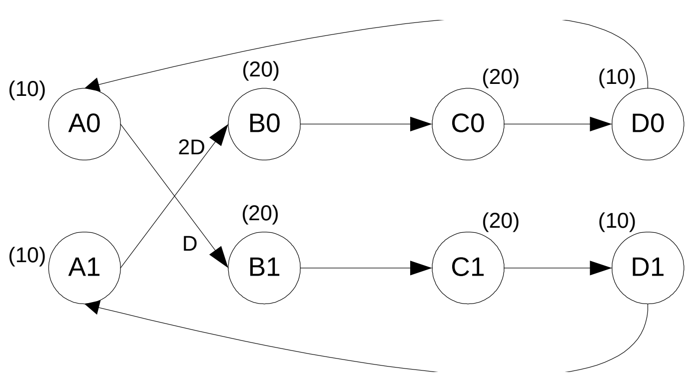
So the minimum unfolding factor to achieve iteration period of 8 u.t. is J=3.

8. The original DFD is given as

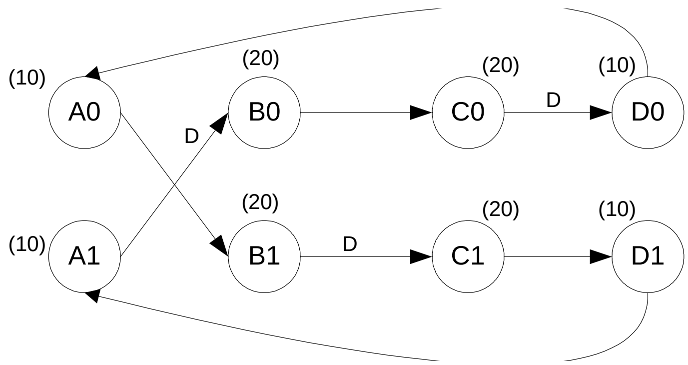


The computation for the critical path $B \to C \to D \to A$ is given as 20+20+10+10= 60 u.t.

(a) The 2-unfolded DFG is shown as



The retimed DFG after applying retiming on the the 2-unfolded DFG is given by



This unfolded DFG is retimed such that the computation time on the critical path is equal to 40 *u.t.*, and hence the sampling period is 20 *u.t.*

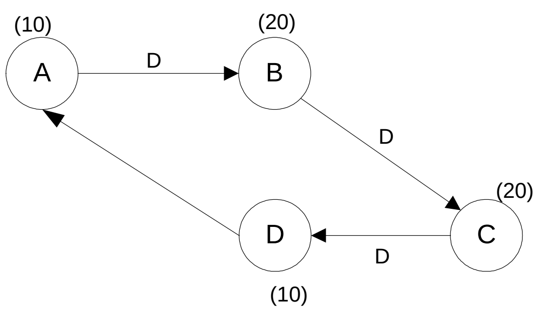
The retiming variable for each node is given as follows:

$$r'(A_0)=0 \;\; r'(B_0)=-1 \;\; r'(C_0)=-1 \;\; r'(D_0)=0$$
$$r'(A_1)=0 \;\; r'(B_1)=-1 \;\; r'(C_1)=0 \;\;\; r'(D_1)=0$$

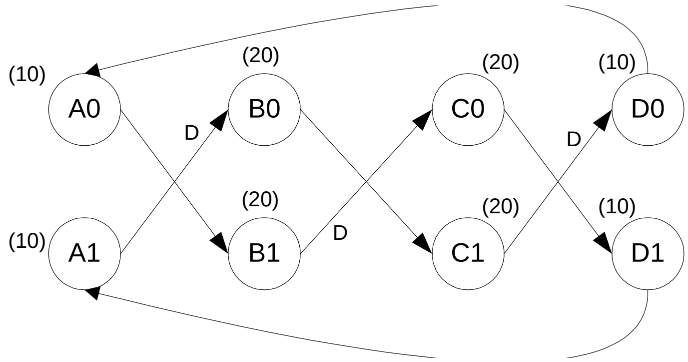
(b) The retiming function r for each node in the original DFG can be computed as

$$r(A)=r'(A_0)+r'(A_1)=0$$
$$r(B)=r'(B_0)+r'(B_1)=-2$$
$$r(C)=r'(C_0)+r'(C_1)=-1$$
$$r(D)=r'(D_0)+r'(D_1)=0$$

With this retiming function, the original DFG can be retimed as follows:



Its 2-unfolded version is the following:



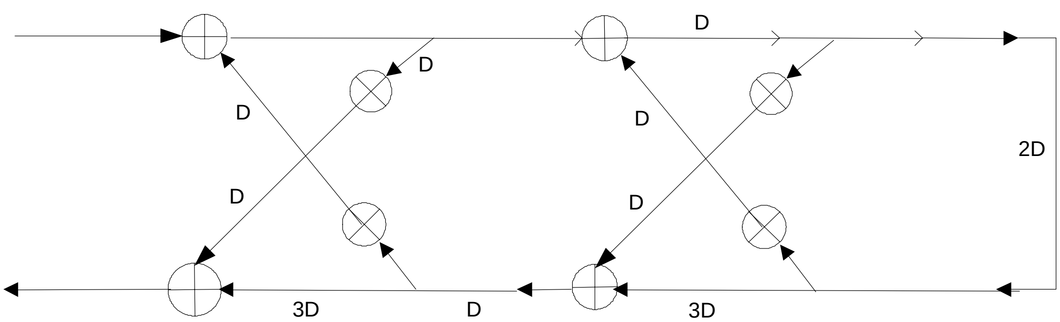
$T_{crit} = B_0 \to C_1 = 40\ u.t.$

The W and D matrices for the retimed version of the original DFG is given as

$$W(U,V) = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad D(U,V) = \begin{bmatrix} 10 & 30 & 40 & 60 \\ 60 & 20 & 40 & 50 \\ 40 & 60 & 20 & 30 \\ 20 & 40 & 60 & 10 \end{bmatrix}$$

As can be verified, for all pairs of nodes U, V in the retimed version of the original DFG,
If D (U, V) $\geq$ 40, then W (U, V) + r (V)-r (U) $\geq$ 2 holds.

11 (a). The retimed DFG is shown as



The computation time for critical path for the retimed graph is 4 u.t. The iteration bound of the original graph is given as $T\infty = \max\left[\frac{6}{4}, \frac{14}{4}\right] = \frac{14}{4}\ u.t.$

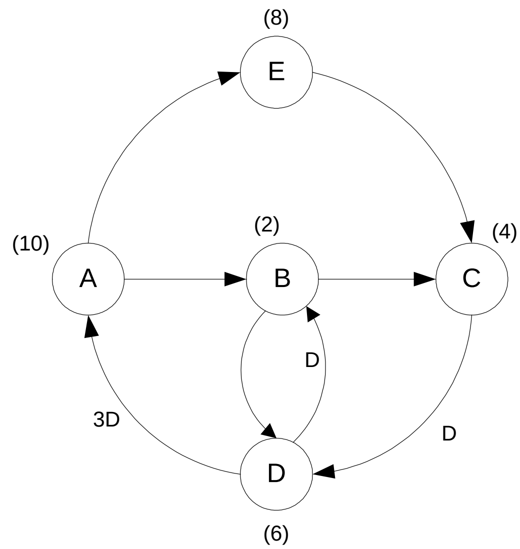
(b) The minimum unfolding factor J such that the J-unfolded DFG can be retimed to achieve a critical path computation time equal to J X T_∞, or a sample period equal to T_∞,is J=4. This can be verified by solving the retiming problems for the original DFG:

- For all edges $U \xrightarrow{e} V$ in the DFG, r(U)-r(V)$\leq$ w(e);

- If D(U,V) $\geq$ c, then r (U) - r (V) $\leq$W(U,V) - J ,

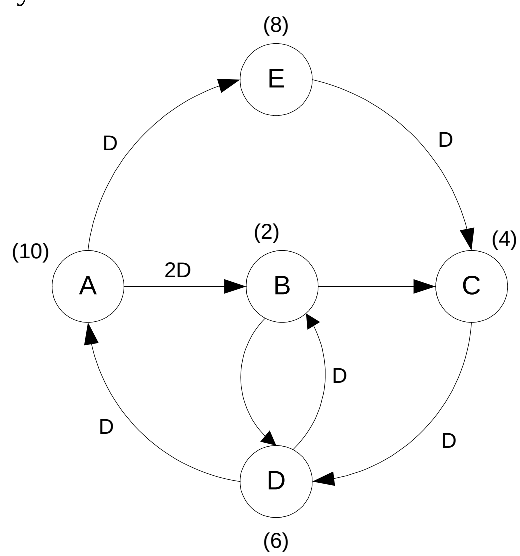
with increasing J until coming to a solution (note that $c = J \times \frac{7}{2}$.

16. The original DFD is given by



(a) The iteration bound of the original DFG is $T_\infty = \max(\frac{8}{1}, \frac{12}{2}, \frac{18}{3}, \frac{22}{4}, \frac{28}{4})$ =8 u.t..
However, its critical path is $A \to E \to C$ and has a computation time equal to 22 u.t. Hence the actual iteration period for the original DFG is 22 u.t.

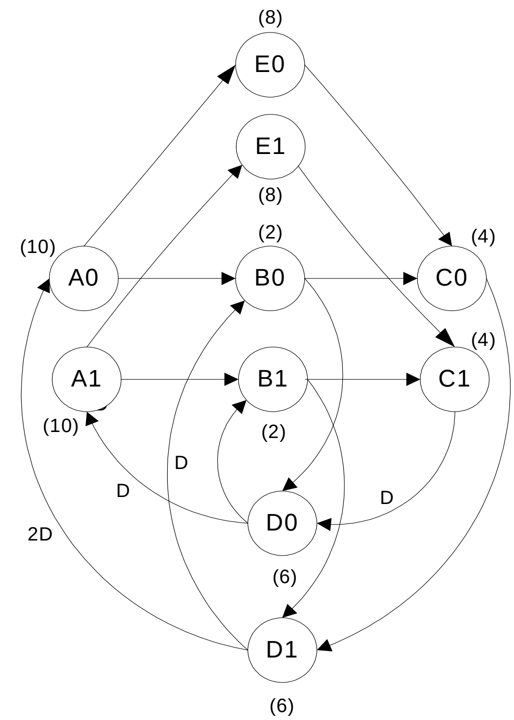
(b)Applying node retiming on nodes A and E, the original DFG can be retimed to a retimed graph given by



The minimum iteration period of the retimed graph is given by 10 u.t. This clock period cannot be further reduced since it is equal to the computation time of node A.

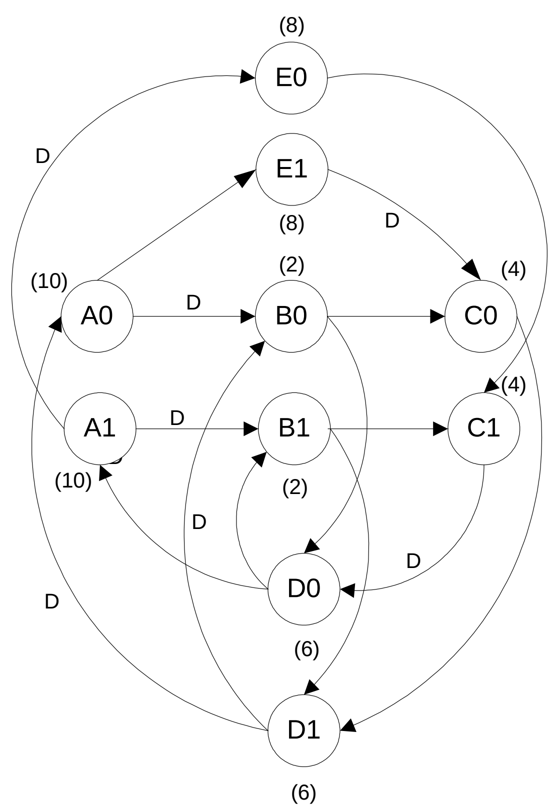
(c) Unfolding the original DFG by a factor of 2 gives a DFG with critical path

$A_0 \rightarrow E_0 \rightarrow C_0 \rightarrow D_1$



The resulting iteration period is equal to *28 u.t.*

Unfolding the retimed DFG by 2 gives a DFG with critical path $A_0 \rightarrow E_1$

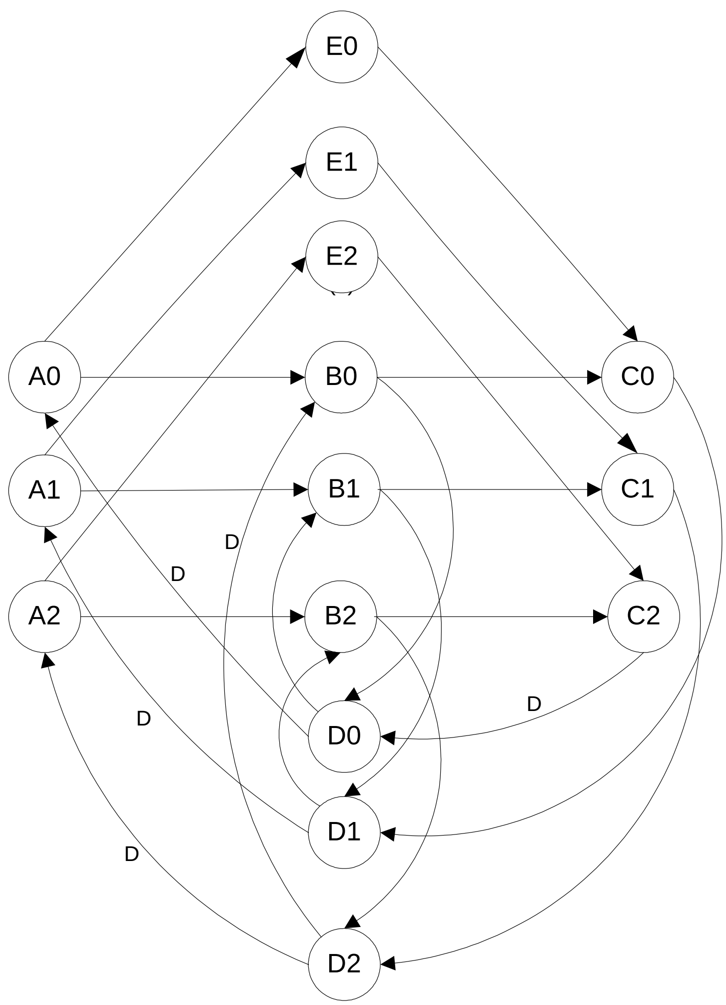


Hence the iteration period is equal to 18 u.t.

(d) Solving the following equation retiming problem for the original DFG

- For all edges $U \xrightarrow{e} V$ in the DFG, r(U)-r(V) $\leq$ w(e);
- If D(U,V) $\geq$ c, then r (U) - r (V) $\leq$W(U,V) - J ,

with increasing value of J until a solution is feasible and $c=8J$ .Then this J is the minimum unfolding factor. It turns out that the minimum unfolding factor is equal to 3, and the 3-unfolded DFG is shown below:



Without further retiming, this DFG has an iteration period equal to $J\, x T_{\infty}$ =24 *u.t.* where T_{∞} is the iteration bound of the original graph.