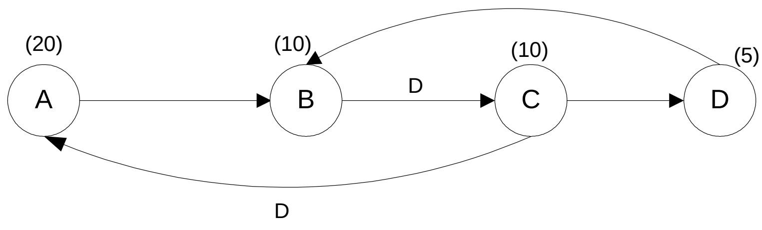
Solutions to Assignment #3
Chapter 4

2. The given DFG is



(a)The maximum sample rate is limited by the critical path from $A \rightarrow B$ (30 u.t.)

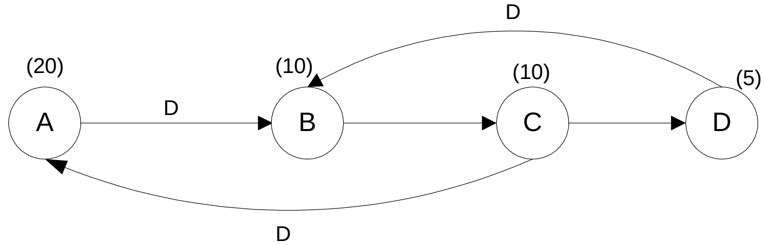
$$SampleRate_{\max} = \frac{1}{T_{critical}} = \frac{1}{30}$$

(b) The fundamental limit on the sample period is determined by the iteration bound as follows:

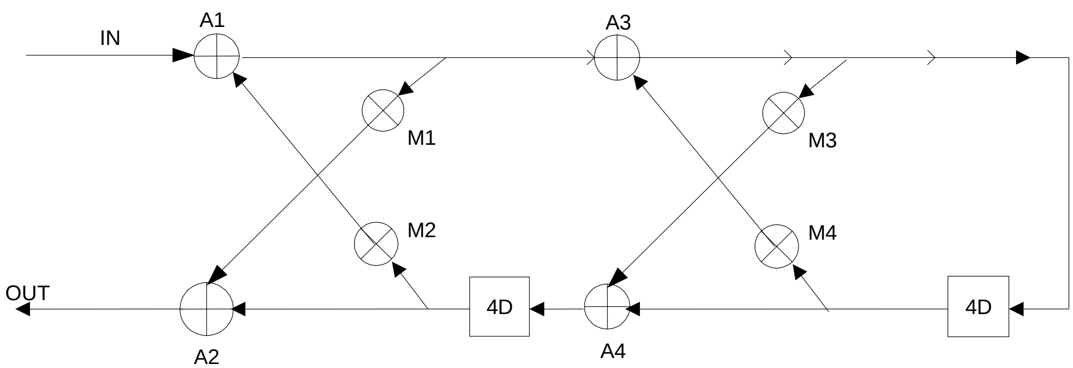
$$A \rightarrow B \rightarrow C \rightarrow A \quad with\ \ two\ \ delays$$
$$B \rightarrow C \rightarrow D \rightarrow B \quad with\ \ one\ \ delay$$

$$SamplePeriod_{\text{limited}} = T_{IBound} = \max\left[\frac{40}{2}, \frac{25}{1}\right] = 25 \text{ u.t.}$$

(c) The manually retimed circuit after applying cutset retiming along edges
$\{A \rightarrow B, C \rightarrow A\}$ and $\{D \rightarrow B, B \rightarrow C, C \rightarrow A\}$
is given by



3. The 4-level pipelined all pass 8th –order IIR digital filter DFG is given by



(a) The iteration bound for the circuit is determined with the cycles

$$A3 \rightarrow M4 \rightarrow A3 \quad with \quad four \quad delays$$

$$A1 \rightarrow A3 \rightarrow M3 \rightarrow A4 \rightarrow M2 \rightarrow A1 \quad with \quad four \ delays$$

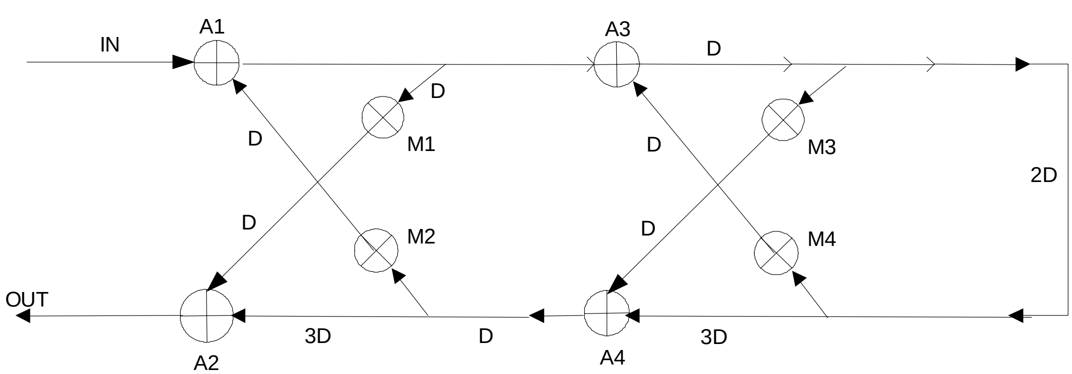
$$T_{IBound} = \max\left[\frac{3}{4}, \frac{7}{4}\right] = \frac{7}{4} \ u.t.$$

(b) The critical path time is from $M2 \rightarrow A1 \rightarrow A3 \rightarrow M3 \rightarrow A4$ with a

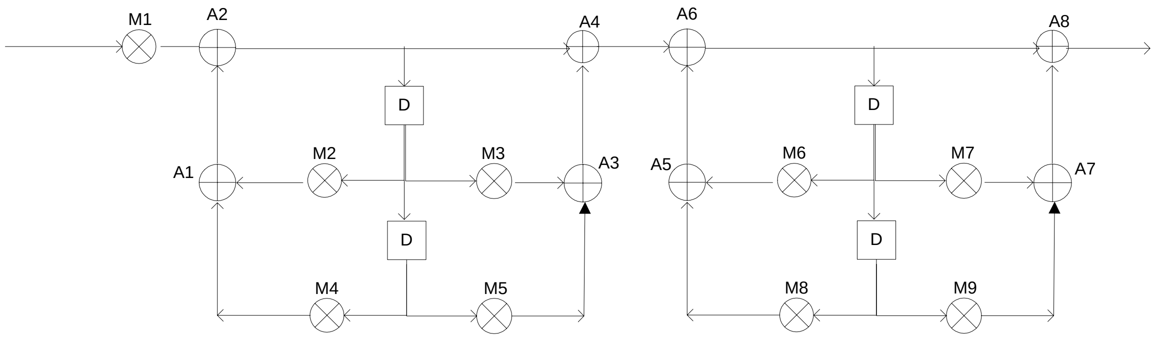
$$T_{critical} = 7 \ u.t.$$

Assuming each multiply requires 2 u.t. and each add requires 1u.t.

(c) Applying successive node retiming, we get the retimed circuit as



5. The 4th order IIR digital filter implemented as a cascade of 2 2nd-order sections is given by the circuit



(a) The two critical paths are given by

$$M4 \rightarrow A1 \rightarrow A2 \rightarrow A4 \rightarrow A6 \rightarrow A8 = 7\ u.t.$$

$$M2 \rightarrow A1 \rightarrow A2 \rightarrow A4 \rightarrow A6 \rightarrow A8 = 7\ u.t.$$

$$T_{critical} = 7\ u.t.$$

Assuming each multiply requires 2 u.t. and each add requires 1u.t.

The iteration bound for the circuit is determined by the cycles

$$A2 \rightarrow M2 \rightarrow A1 \rightarrow A2 \quad with\ one\ delay$$

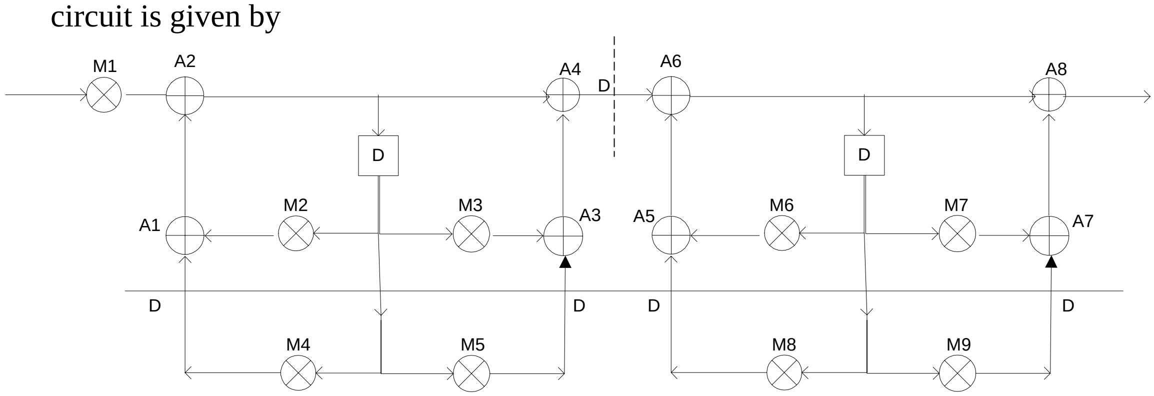
$$A2 \rightarrow M4 \rightarrow A1 \rightarrow A2 \quad with\ two\ delays$$

$$A6 \rightarrow M6 \rightarrow A5 \rightarrow A6 \quad with\ one\ delay$$

$$A6 \rightarrow M8 \rightarrow A5 \rightarrow A6 \quad with\ two\ delays$$

$$T_{IBound} = \max\left[\frac{4}{1}, \frac{4}{2}, \frac{4}{1}, \frac{4}{2}\right] = 4\ u.t.$$

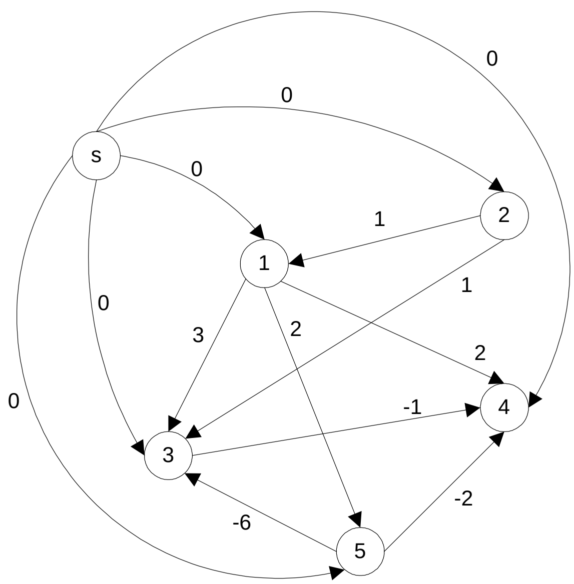
(b) The minimum achievable clock period obtained with pipelining and retiming is the iteration bound of the DFG, is equal to 4 u.t. The retimed and pipelined circuit is given by



9. The given set of inequalities is

$$r_1 - r_2 \le 1$$
$$r_3 - r_1 \le 3$$
$$r_4 - r_1 \le 2$$
$$r_4 - r_3 \le -1$$
$$r_3 - r_2 \le 1$$
$$r_5 - r_1 \le 2$$
$$r_3 - r_5 \le -6$$
$$r_4 - r_5 \le -2$$

The constraint graph for this set of inequalities is given by



(a)Using Bellman-Ford algorithm we get the value of $r^5(V)$ as the shortest path

$r^k(V)$	k=1	k=2	k=3	k=4	k=5
V=1	0	0	0	0	0
V=2	0	0	0	0	0
V=3	0	-6	-6	-6	-6
V=4	0	-2	-7	-7	-7
V=5	0	0	0	0	0
V=s	0	0	0	0	0

Thus $r_1 = r_2 = 0, r_3 = -6, r_4 = -7, r_5 = 0$

(b)Using Floyd-Warshall algorithm, the matrices are given by

$$R^{(1)} = \begin{bmatrix} \infty & \infty & 3 & 2 & 2 & \infty \\ 1 & \infty & 1 & \infty & \infty & \infty \\ \infty & \infty & \infty & -1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & -6 & -2 & \infty & \infty \\ 0 & 0 & 0 & 0 & 0 & \infty \end{bmatrix} \quad R^{(2)} = \begin{bmatrix} \infty & \infty & -4 & 0 & 2 & \infty \\ 1 & \infty & 1 & 0 & 3 & \infty \\ \infty & \infty & \infty & -1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & -6 & -7 & \infty & \infty \\ 0 & 0 & -6 & -2 & 0 & \infty \end{bmatrix}$$

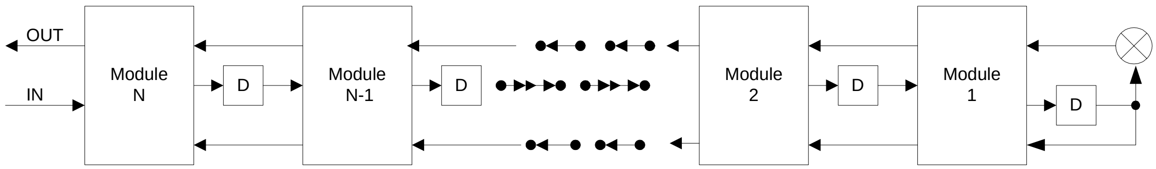
$$R^{(3)} = \begin{bmatrix} \infty & \infty & -4 & -5 & 2 & \infty \\ 1 & \infty & -3 & 0 & 3 & \infty \\ \infty & \infty & \infty & -1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & -6 & -7 & \infty & \infty \\ 0 & 0 & -6 & -7 & 0 & \infty \end{bmatrix} \quad R^{(4)} = \begin{bmatrix} \infty & \infty & -4 & -5 & 2 & \infty \\ 1 & \infty & -3 & -4 & 3 & \infty \\ \infty & \infty & \infty & -1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & -6 & -7 & \infty & \infty \\ 0 & 0 & -6 & -7 & 0 & \infty \end{bmatrix}$$

$$R^{(5)} = \begin{bmatrix} \infty & \infty & -4 & -5 & 2 & \infty \\ 1 & \infty & -3 & -4 & 3 & \infty \\ \infty & \infty & \infty & -1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & -6 & -7 & \infty & \infty \\ 0 & 0 & -6 & -7 & 0 & \infty \end{bmatrix} \quad R^{(6)} = \begin{bmatrix} \infty & \infty & -4 & -5 & 2 & \infty \\ 1 & \infty & -3 & -4 & 3 & \infty \\ \infty & \infty & \infty & -1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & -6 & -7 & \infty & \infty \\ 0 & 0 & -6 & -7 & 0 & \infty \end{bmatrix}$$

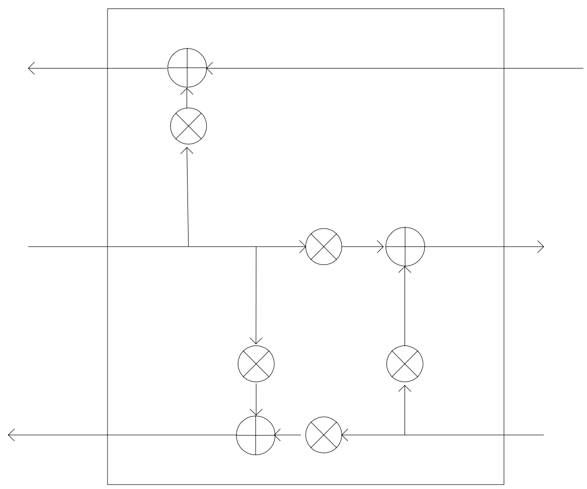
$$R^{(7)} = \begin{bmatrix} \infty & \infty & -4 & -5 & 2 & \infty \\ 1 & \infty & -3 & -4 & 3 & \infty \\ \infty & \infty & \infty & -1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & -6 & -7 & \infty & \infty \\ 0 & 0 & -6 & -7 & 0 & \infty \end{bmatrix}$$

Thus we get $r_1 = r_2 = 0, r_3 = -6, r_4 = -7, r_5 = 0$ from $R^{(7)}$

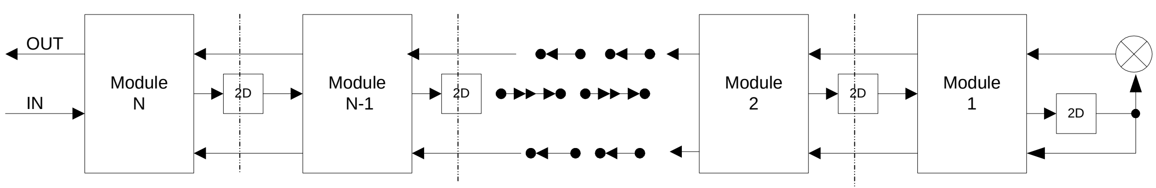
12. The N stage normalized lattice filter is given by

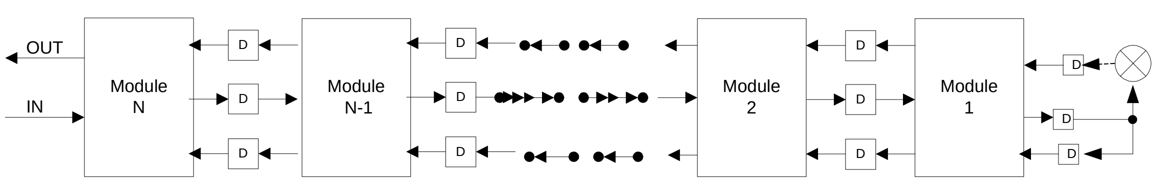


MODULE



(a)The critical path time =the minimum clock cycle = 3 X 25 =75 u.t.
(b) The 2-slow transformed and retimed filter structure is given by





The clock period of the retimed filter is 3 u.t. and the sample period is 2 X 3 =6 u.t.