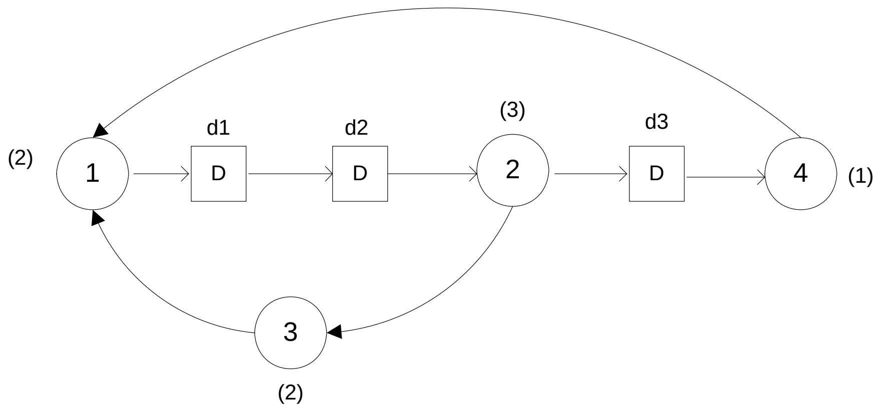
Solutions to Assignment #2
Chapter 2

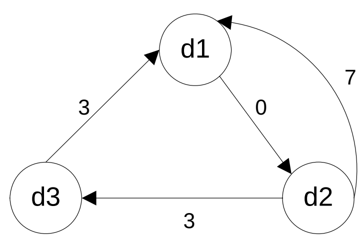
1. (a) The given DFG is



Computing of iteration bound by Longest Path Matrix (LPM) requires the following steps

I. Building the delay graph G_d from the DFG

$$d1 \rightarrow d2 : 0$$
$$d2 \rightarrow d1 : d2 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow d1 = 7$$
$$d2 \rightarrow d3 : d2 \rightarrow d3 = 3$$
$$d3 \rightarrow d1 : d3 \rightarrow 4 \rightarrow 1 \rightarrow d1 = 3$$



G_d

II. Build the $L^{(1)}$ matrix

$$L^{(1)} = \begin{bmatrix} -1 & 0 & -1 \\ 7 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix}$$

III. Construct the higher order matrices $L^{(m)}$ recursively

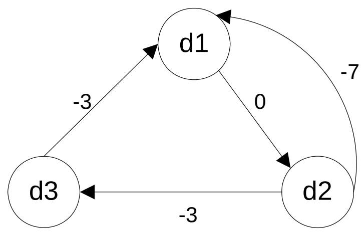
$$L^{(2)} = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 7 & -1 \\ -1 & 3 & -1 \end{bmatrix} \qquad L^{(3)} = \begin{bmatrix} 6 & 7 & -1 \\ 14 & 6 & 10 \\ 10 & -1 & 6 \end{bmatrix}$$

IV. Calculate the iteration bound T_∞

$$T_\infty = \max\left(\frac{7}{2},\frac{7}{2},\frac{6}{3},\frac{6}{3},\frac{6}{3}\right)$$
$$= 3.5$$

1. (b) Computing of iteration bound by Minimum Cycle Mean (MCM) requires the following steps

   I. Create $G_d^{'}$ from G_d



G'd

   II. Consider d1 as the source node (s) then

$$f^{(0)} = \begin{bmatrix} 0 \\ \infty \\ \infty \end{bmatrix} \quad f^{(1)} = \begin{bmatrix} \infty \\ 0 \\ \infty \end{bmatrix} \quad f^{(2)} = \begin{bmatrix} -7 \\ \infty \\ -3 \end{bmatrix} \quad f^{(3)} = \begin{bmatrix} -6 \\ -7 \\ \infty \end{bmatrix}$$

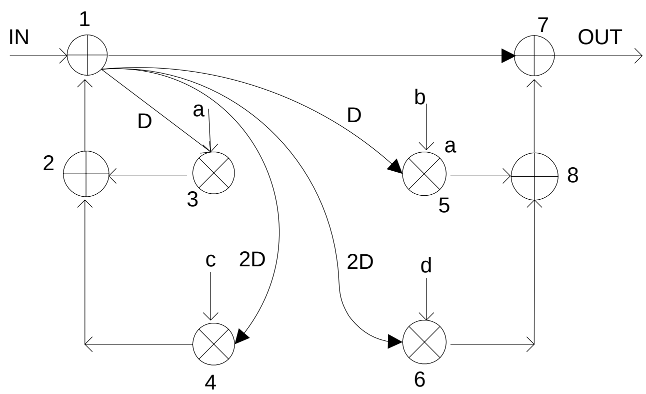
   III. Calculate $\max\limits_{i\in\{1,2,3\}\ \&\ m\in\{0,1,2\}}\left(\frac{f^{(3)}(i)-f^{(m)}(i)}{3-m}\right)$

	m=0	*m=1*	*m=2*	$\left(\frac{f^{(3)}(i)-f^{(m)}(i)}{3-m}\right)$
i=1	-2	- ∞	1	1
i=2	- ∞	$-\frac{7}{2}$	- ∞	$-\frac{7}{2}$
i=3	∞	∞	∞	∞

   IV. Calculate the iteration bound T_∞

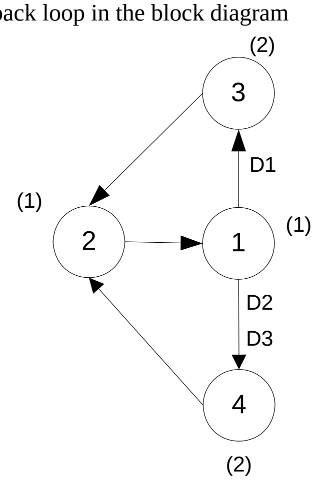
$$T_\infty = -\min_{i\in\{1,2,3\}}\left(1, \frac{-7}{2}, \infty\right)$$
$$= 3.5$$

3. (a) The given block diagram is



Computing of iteration bound by Longest Path Matrix (LPM) requires the following steps

I. Create the DFG for the feedback loop in the block diagram

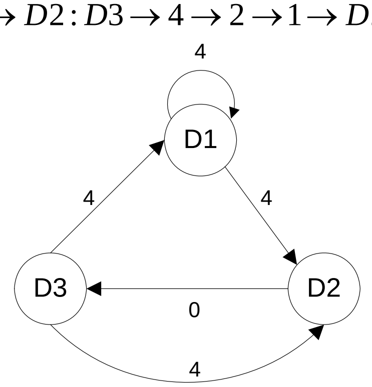


DFG OF FEEBACK LOOP WITH
3 LEFT DELAY ELEMENTS

A/Q addition and multiplication operations require (1)u.t. and (2) u.t. respectively

II. Building the delay graph G_d from the DFG

$$D1 \to D2 : D1 \to 3 \to 2 \to 1 \to D2 = 4$$
$$D1 \to D1 : D1 \to 3 \to 2 \to 1 \to D1 = 4$$
$$D2 \to D3 : 0$$
$$D3 \to D1 : D3 \to 4 \to 2 \to 1 \to D1 = 4$$
$$D3 \to D2 : D3 \to 4 \to 2 \to 1 \to D2 = 4$$



G_d

III. Build the $\mathrm{L}^{(1)}$ matrix

$$\mathrm{L}^{(1)} = \begin{bmatrix} 4 & 4 & -1 \\ -1 & -1 & 0 \\ 4 & 4 & -1 \end{bmatrix}$$

IV. Construct the higher order matrices $\mathrm{L}^{(m)}$ recursively

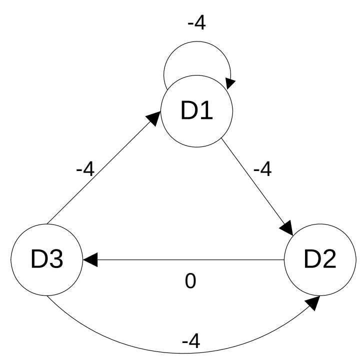
$$\mathrm{L}^{(2)} = \begin{bmatrix} 8 & 8 & 4 \\ 4 & 4 & -1 \\ 8 & 8 & 4 \end{bmatrix} \qquad \mathrm{L}^{(3)} = \begin{bmatrix} 12 & 12 & 8 \\ 8 & 8 & 4 \\ 12 & 12 & 8 \end{bmatrix}$$

V. Calculate the iteration bound T_∞

$$\mathrm{T}_\infty = \max\left(\frac{4}{1}, \frac{8}{2}, \frac{4}{2}, \frac{4}{2}, \frac{12}{3}, \frac{8}{3}, \frac{8}{3}\right)$$
$$= 4$$

3. (b) Computing of iteration bound by Minimum Cycle Mean (MCM) requires the following steps

I. Create $G_d^{'}$ from G_d



G'd

II. Consider D2 as the source node (“s”) then

$$f^{(0)} = \begin{bmatrix} \infty \\ 0 \\ \infty \end{bmatrix} \quad f^{(1)} = \begin{bmatrix} \infty \\ \infty \\ 0 \end{bmatrix} \quad f^{(2)} = \begin{bmatrix} -4 \\ -4 \\ \infty \end{bmatrix} \quad f^{(3)} = \begin{bmatrix} -8 \\ -8 \\ -4 \end{bmatrix}$$

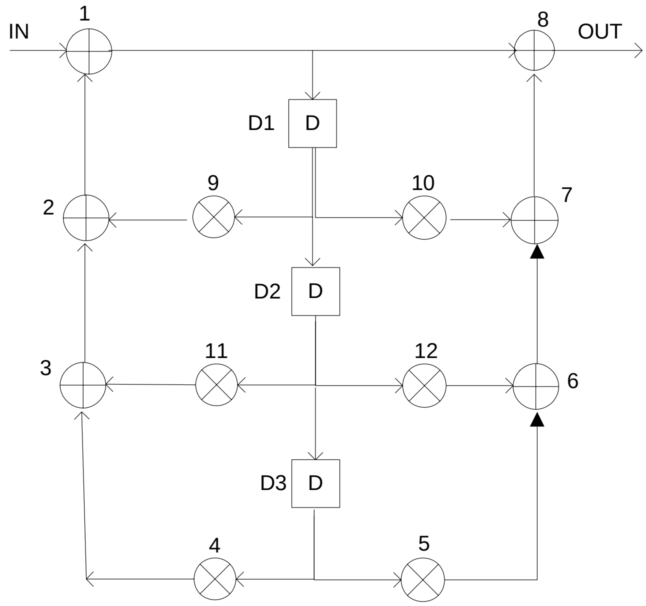
III. Calculate $\max\limits_{i \in \{1,2,3\} \,\&\, m \in \{0,1,2\}} \left(\dfrac{f^{(3)}(i) - f^{(m)}(i)}{3-m} \right)$

	$m=0$	$m=1$	$m=2$	$\left(\dfrac{f^{(3)}(i)-f^{(m)}(i)}{3-m}\right)$
$i=1$	$-\infty$	$-\infty$	-4	-4
$i=2$	$-8/3$	$-\infty$	-4	$-8/3$
$i=3$	$-\infty$	-2	$-\infty$	-2

IV. Calculate the iteration bound T_∞

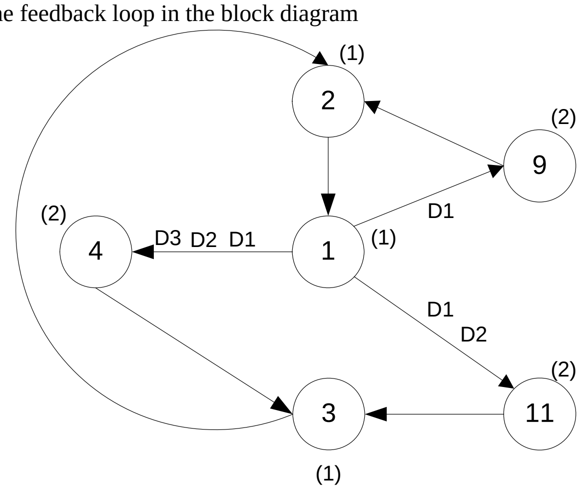
$$T_\infty = -\min_{i\in\{1,2,3\}}\left(-4,\frac{-8}{3},-2\right)$$
$$= 4$$

4. (a) The given block diagram is



Computing of iteration bound by Longest Path Matrix (LPM) requires the following steps

I. Create the DFG for the feedback loop in the block diagram

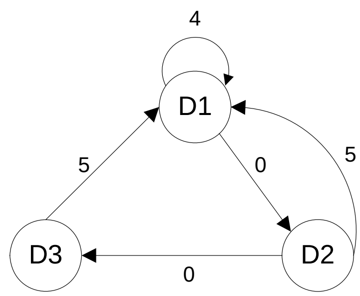


DFG OF FEEBACK LOOP WITH
3 DELAY ELEMENTS

The nodes are number 1-11. A/Q addition and multiplication operations require (1)u.t. and (2) u.t. respectively

II. Creating the delay graph G_d from the DFG

$$
\begin{aligned}
&D1 \to D2 : 0 \\
&D1 \to D1 : D1 \to 9 \to 2 \to 1 \to D1 = 4 \\
&D2 \to D1 : D2 \to 11 \to 3 \to 2 \to 1 \to D1 = 5 \\
&D2 \to D3 : 0 \\
&D3 \to D1 : D3 \to 4 \to 3 \to 2 \to 1 \to D1 = 5
\end{aligned}
$$



G_d

III. Build the $L^{(1)}$ matrix

$$
L^{(1)} = \begin{bmatrix} 4 & 0 & -1 \\ 5 & -1 & 0 \\ 5 & -1 & -1 \end{bmatrix}
$$

IV. Construct the higher order matrices $L^{(m)}$ recursively

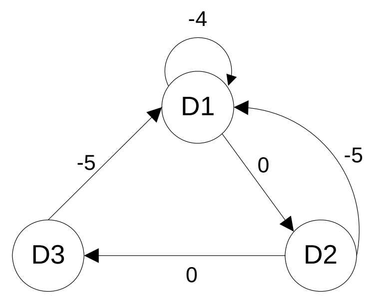
$$
L^{(2)} = \begin{bmatrix} 8 & 4 & 0 \\ 9 & 5 & -1 \\ 9 & 5 & -1 \end{bmatrix} \qquad L^{(3)} = \begin{bmatrix} 12 & 8 & 4 \\ 13 & 9 & 5 \\ 13 & 9 & 5 \end{bmatrix}
$$

V. Calculate the iteration bound T_∞

$$
\begin{aligned}
T_\infty &= \max\left(\frac{4}{1}, \frac{8}{2}, \frac{5}{2}, \frac{12}{3}, \frac{9}{3}, \frac{5}{3}\right) \\
&= 4
\end{aligned}
$$

4. (b) Computing of iteration bound by Minimum Cycle Mean (MCM) requires the following steps

I. Create G'_d from G_d



G'$_d$

II. Consider D1 as the source node ("s") then

$$f^{(0)} = \begin{bmatrix} 0 \\ \infty \\ \infty \end{bmatrix} \quad f^{(1)} = \begin{bmatrix} -4 \\ 0 \\ \infty \end{bmatrix} \quad f^{(2)} = \begin{bmatrix} -8 \\ -4 \\ 0 \end{bmatrix} \quad f^{(3)} = \begin{bmatrix} -12 \\ -8 \\ -4 \end{bmatrix}$$

III. Calculate $\max\limits_{i\in\{1,2,3\}\ \&\ m\in\{0,1,2\}} \left(\dfrac{f^{(3)}(i) - f^{(m)}(i)}{3-m} \right)$

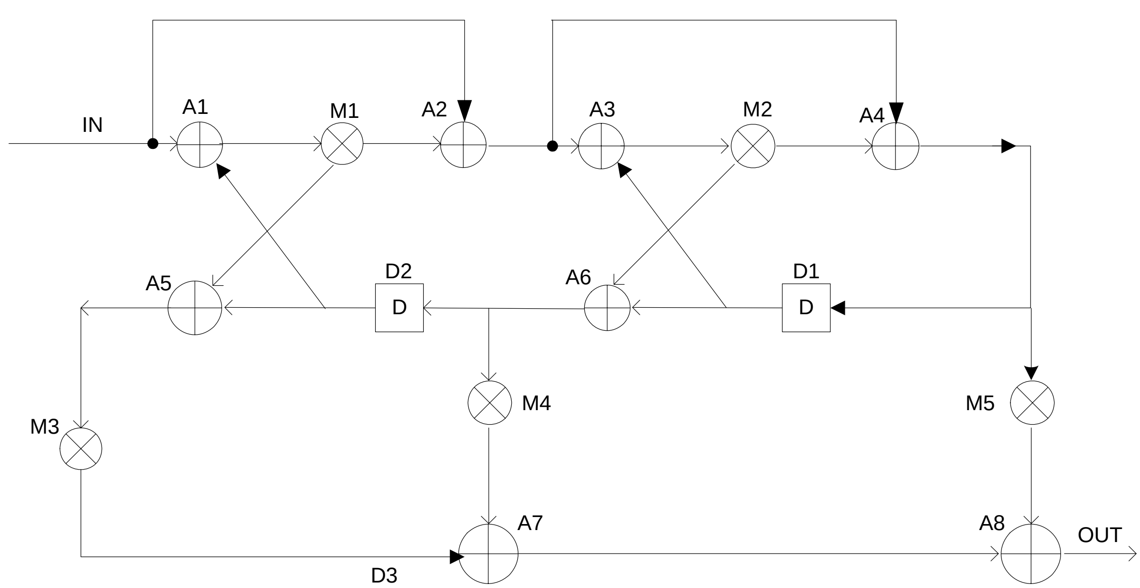
	m=0	*m=1*	*m=2*	$\left(\dfrac{f^{(3)}(i) - f^{(m)}(i)}{3-m} \right)$
i=1	-4	-4	-4	-4
i=2	- ∞	-4	-4	-4
i=3	- ∞	- ∞	-4	-4

IV. Calculate the iteration bound T_∞

$$T_\infty = -\min_{i\in\{1,2,3\}} \left(-4,-4,-4\right)$$

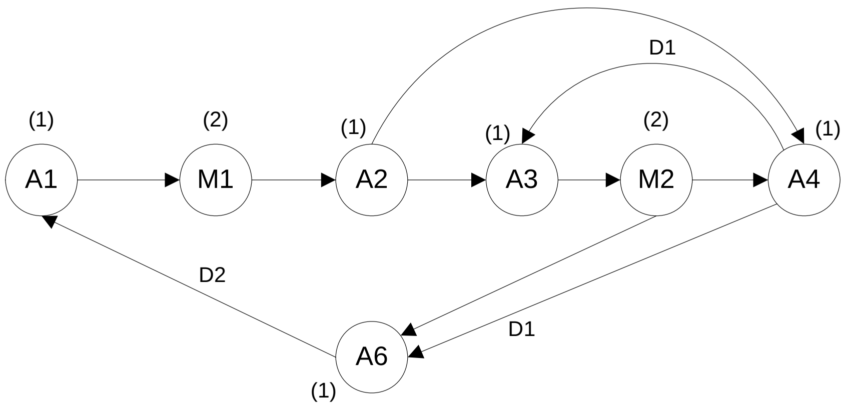
$$= 4$$

6. (a) The given block diagram is



Computing of iteration bound by Longest Path Matrix (LPM) requires the following steps

I. Create the DFG for the feedback loop in the block diagram

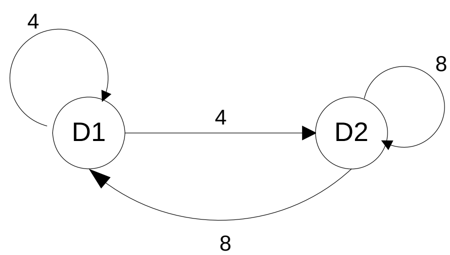


DFG OF FEEBACK LOOP IN THE UPPER PART
WITH 2 DELAY ELEMENTS

A/Q addition and multiplication operations require (1)u.t. and (2) u.t. respectively

II. Creating the delay graph G_d from the DFG

$$D1 \to D1 : D1 \to A3 \to M2 \to A4 \to D1 = 4$$
$$D1 \to D2 : D1 \to A3 \to M2 \to A6 \to D2 = 4$$
$$D2 \to D1 : D2 \to A1 \to M1 \to A2 \to A3 \to M2 \to A4 \to D1 = 8$$
$$D2 \to D2 : D2 \to A1 \to M1 \to A2 \to A3 \to M2 \to A6 \to D2 = 8$$



G_d

III. Build the $L^{(1)}$ matrix

$$L^{(1)} = \begin{bmatrix} 4 & 4 \\ 8 & 8 \end{bmatrix}$$

IV. Construct the higher order matrices $L^{(m)}$ recursively

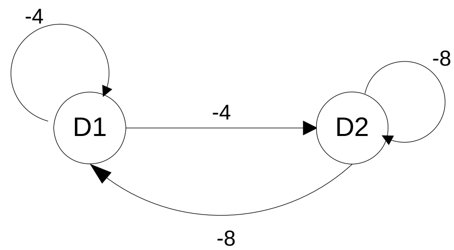
$$L^{(2)} = \begin{bmatrix} 12 & 12 \\ 16 & 16 \end{bmatrix}$$

V. Calculate the iteration bound T_∞

$$T_\infty = \max\left(\frac{4}{1}, \frac{8}{1}, \frac{12}{2}, \frac{16}{2}\right)$$
$$= 8$$

6. (b) Computing of iteration bound by Minimum Cycle Mean (MCM) requires the following steps

I. Create G'_d from G_d



G'd

II. Consider D2 as the source node ("s") then

$$f^{(0)} = \begin{bmatrix} \infty \\ 0 \end{bmatrix} \quad f^{(1)} = \begin{bmatrix} -8 \\ -8 \end{bmatrix} \quad f^{(2)} = \begin{bmatrix} -16 \\ -16 \end{bmatrix}$$

III. Calculate $\max_{i\in\{1,2\}\ \&\ m\in\{0,1\}}\left(\frac{f^{(2)}(i)-f^{(m)}(i)}{2-m}\right)$

	m=0	*m=1*	$\left(\frac{f^{(2)}(i)-f^{(m)}(i)}{2-m}\right)$
i=1	- ∞	-8	-8
i=2	-8	-8	-8

IV. Calculate the iteration bound T_∞

$$T_\infty = -\min_{i\in\{1,2\}}(-8,-8)$$
$$= 8$$