Support Vector Machines

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SLIDES ADAPTED FROM TOM MITCHELL, ERIC XING, AND LAUREN HANNAH

Roadmap

- Classification: machines labeling data for us
- Previously: logistic regression
- This time: SVMs
  - (another) example of linear classifier
  - State-of-the-art classification
  - Good theoretical properties

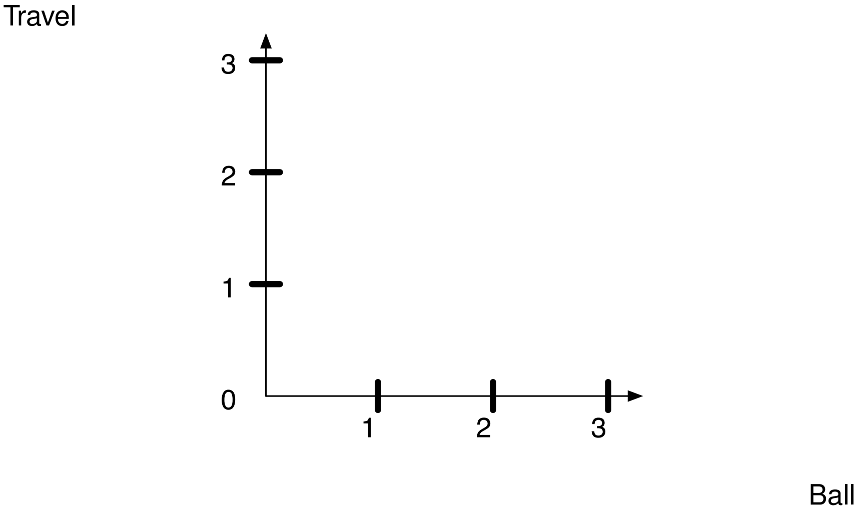
Thinking Geometrically

- Suppose you have two classes: vacations and sports
- Suppose you have four documents

Sports	Vacations
Doc_1: {ball, ball, ball, travel}	Doc_3: {travel, ball, travel}
Doc_2: {ball, ball}	Doc_4: {travel}

- What does this look like in vector space?

Put the documents in vector space



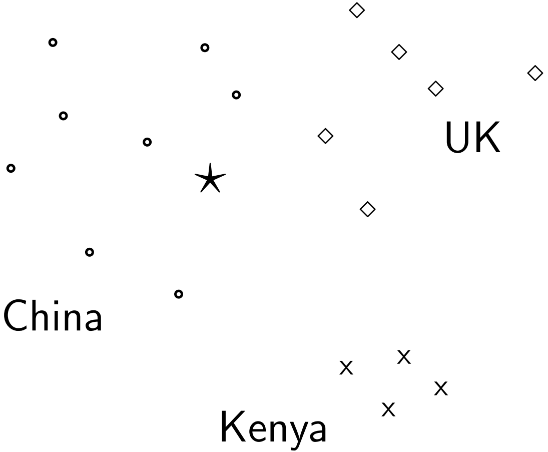
Vector space representation of documents

- Each document is a vector, one component for each term.
- Terms are axes.
- High dimensionality: 10,000s of dimensions and more
- How can we do classification in this space?

Vector space classification

- As before, the training set is a set of documents, each labeled with its class.
- In vector space classification, this set corresponds to a labeled set of points or vectors in the vector space.
- Premise 1: Documents in the same class form a **contiguous region**.
- Premise 2: Documents from different classes **don't overlap**.
- We define lines, surfaces, hypersurfaces to divide regions.

Classes in the vector space



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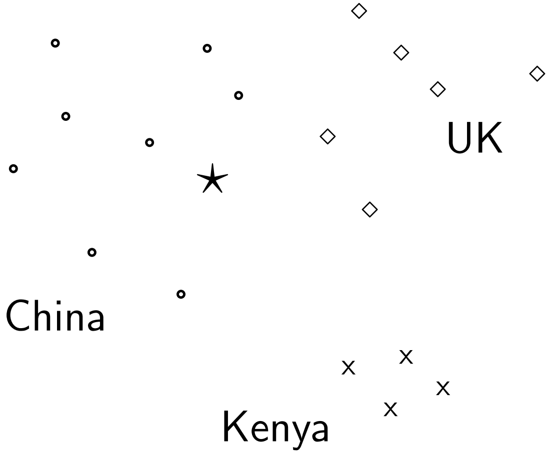
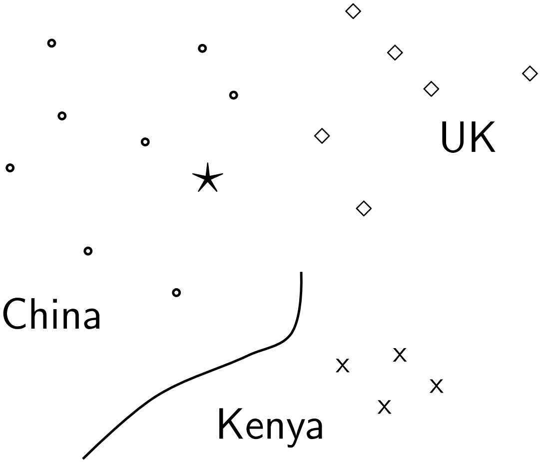


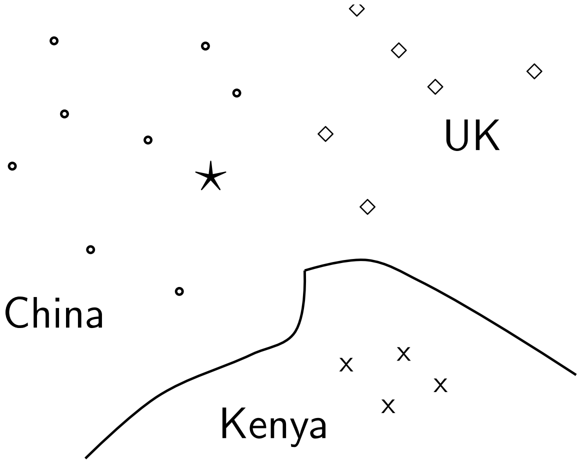
Should the document ⋆ be assigned to `China`, `UK` or `Kenya`?

Classes in the vector space



Find separators between the classes

Classes in the vector space



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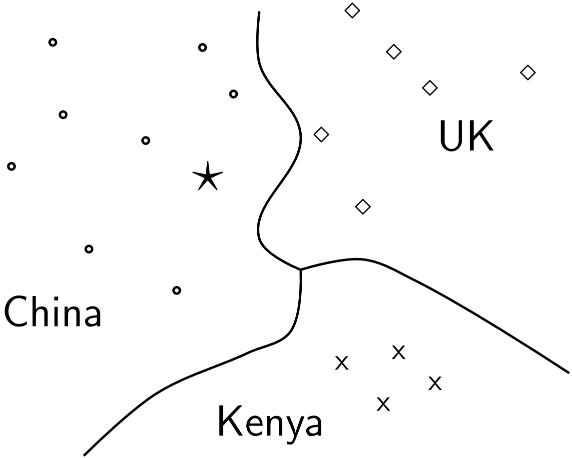
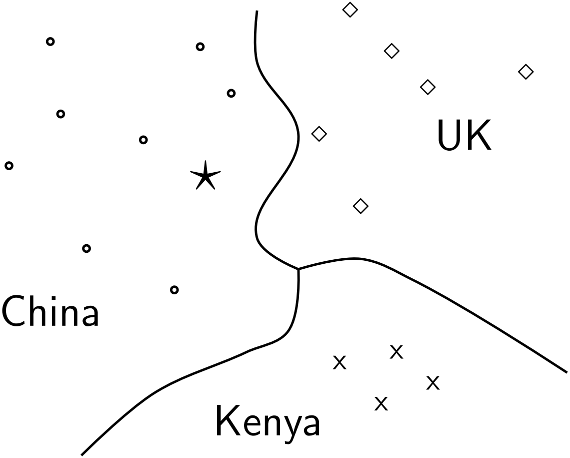


Based on these separators: ⋆ should be assigned to `China`

Classes in the vector space



How do we find separators that do a good job at classifying new documents like ⋆? – Main topic of today

Linear classifiers

- Definition:
  - A linear classifier computes a linear combination or weighted sum $\sum_i \beta_i x_i$ of the feature values.
  - Classification decision: $\sum_i \beta_i x_i > \beta_0$? ($\beta_0$ is our bias)
  - ...where β_0 (the threshold) is a parameter.
- We call this the **separator** or **decision boundary**.
- We find the separator based on training set.
- Methods for finding separator: logistic regression, linear SVM
- Assumption: The classes are **linearly separable**.

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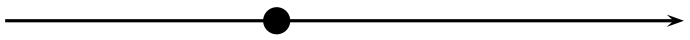
- Before, we just talked about equations. What's the geometric intuition?

A linear classifier in 1D



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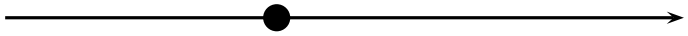

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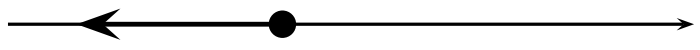

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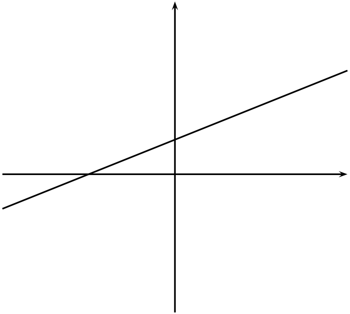

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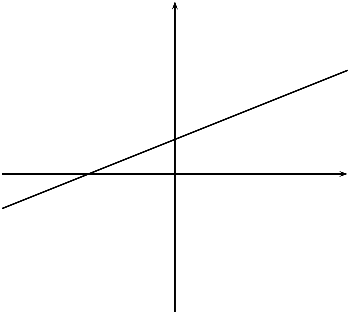

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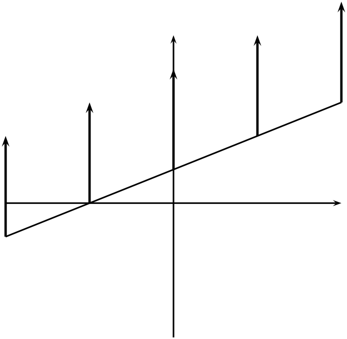

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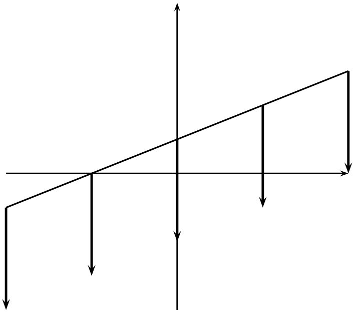

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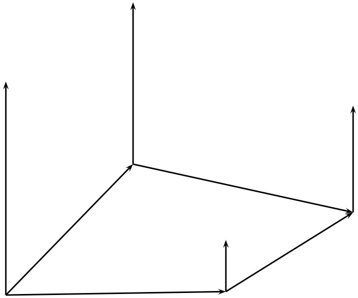

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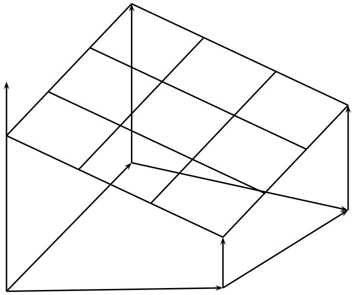

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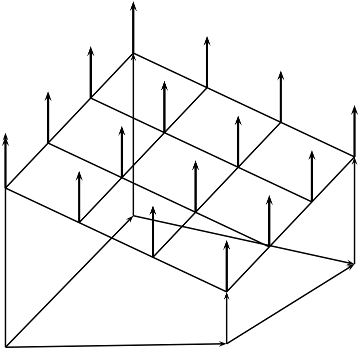

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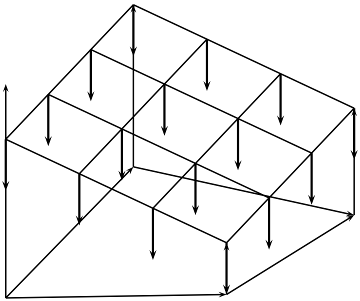

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