Clustering

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SLIDES ADAPTED FROM DAVE BLEI AND LAUREN HANNAH

Clustering

Questions:

- how do we fit clusters?
- how many clusters should we use?
- how should we evaluate model fit?

K-Means

How do we fit the clusters?

- simplest method: K-means
- requires: real-valued data
- idea:
    - pick K initial cluster means
    - associate all points closest to mean k with cluster k
    - use points in cluster k to update mean for that cluster
    - re-associate points closest to new mean for k with cluster k
    - use new points in cluster k to update mean for that cluster
    - ...
    - stop when no change between updates

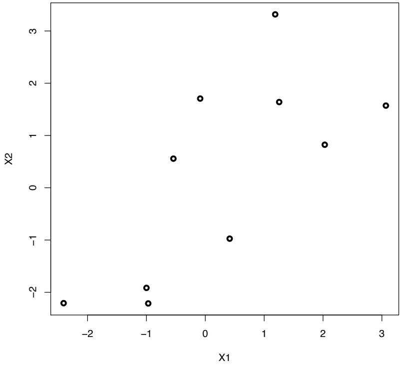
K-Means

Animation at:
`http://shabal.in/visuals/kmeans/1.html`

K-Means: Example

Data:

x_1	x_2
0.4	-1.0
-1.0	-2.2
-2.4	-2.2
-1.0	-1.9
-0.5	0.6
-0.1	1.7
1.2	3.3
3.1	1.6
1.3	1.6
2.0	0.8



Why topic models?

- Suppose you have a huge number of documents
- Want to know what's going on
- Can't read them all (e.g. every New York Times article from the 90's)
- Topic models offer a way to get a corpus-level view of major themes

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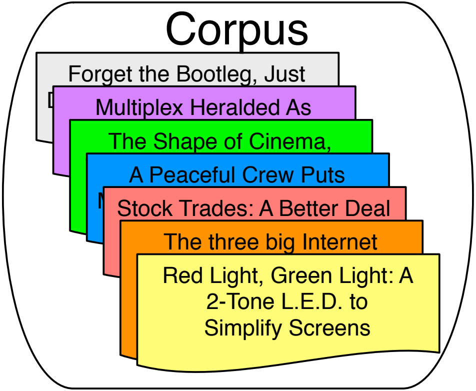
- Topic models offer a way to get a corpus-level view of major themes
- Unsupervised

Roadmap

- What are topic models
- How to know if you have good topic model
- How to go from raw data to topics

Conceptual Approach

From an **input corpus** and number of topics K $\longrightarrow$ words to topics



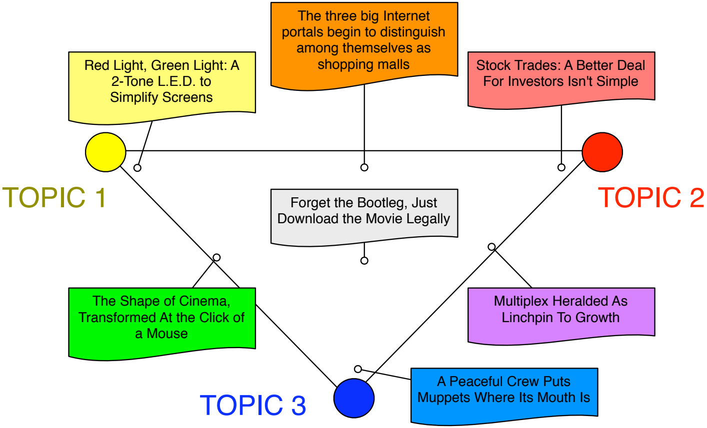
Conceptual Approach

From an input corpus and number of topics $K \longrightarrow$ **words to topics**

TOPIC 1	TOPIC 2	TOPIC 3
computer, technology, system, service, site, phone, internet, machine	sell, sale, store, product, business, advertising, market, consumer	play, film, movie, theater, production, star, director, stage

Conceptual Approach

- For each document, what topics are expressed by that document?



Topics from Science

human	evolution	disease	computer
genome	evolutionary	host	models
dna	species	bacteria	information
genetic	organisms	diseases	data
genes	life	resistance	computers
sequence	origin	bacterial	system
gene	biology	new	network
molecular	groups	strains	systems
sequencing	phylogenetic	control	model
map	living	infectious	parallel
information	diversity	malaria	methods
genetics	group	parasite	networks
mapping	new	parasites	software
project	two	united	new
sequences	common	tuberculosis	simulations

Why should you care?

- Neat way to explore / understand corpus collections
  - E-discovery
  - Social media
  - Scientific data
- NLP Applications
  - Word Sense Disambiguation
  - Discourse Segmentation
  - Machine Translation
- Psychology: word meaning, polysemy
- Inference is (relatively) simple

Matrix Factorization Approach

$$\underbrace{\begin{bmatrix} M \times K \end{bmatrix}}_{\text{Topic Assignment}} \times \underbrace{\begin{bmatrix} K \times V \end{bmatrix}}_{\text{Topics}} \approx \underbrace{\begin{bmatrix} M \times V \end{bmatrix}}_{\text{Dataset}}$$

K Number of topics

M Number of documents

V Size of vocabulary

Matrix Factorization Approach

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Topic Assignment — Topics — Dataset

- K Number of topics
- M Number of documents
- V Size of vocabulary

- If you use singular value decomposition (SVD), this technique is called latent semantic analysis.
- Popular in information retrieval.

Alternative: Generative Model

- How your data came to be
- Sequence of Probabilistic Steps
- Posterior Inference

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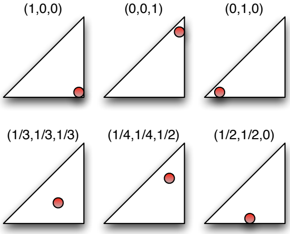
- Blei, Ng, Jordan. Latent **Dirichlet** Allocation. JMLR, 2003.

Multinomial Distribution

- Distribution over discrete outcomes
- Represented by non-negative vector that sums to one
- Picture representation



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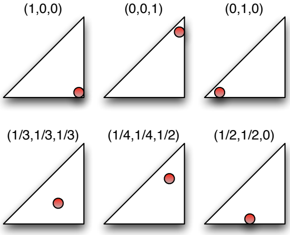

- Come from a Dirichlet distribution

Dirichlet Distribution

$$P(\boldsymbol{p} \mid \alpha \boldsymbol{m}) = \frac{\Gamma(\sum_k \alpha m_k)}{\prod_k \Gamma(\alpha m_k)} \prod_k p_k^{\alpha m_k - 1}$$

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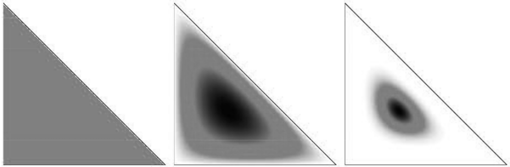

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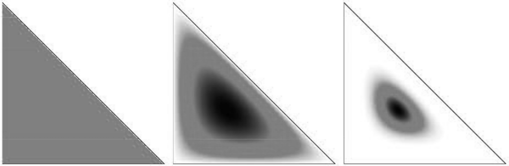

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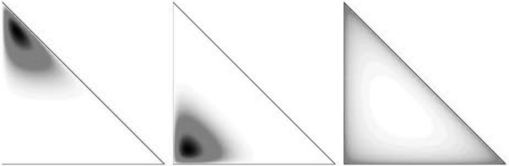

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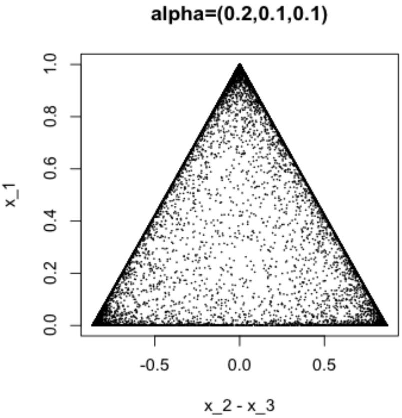
Dirichlet Distribution



Dirichlet Distribution

- If $\vec{\phi} \sim \text{Dir}(()\alpha)$, $\vec{w} \sim \text{Mult}(()\phi)$, and $n_k = |\{w_i : w_i = k\}|$ then

$$p(\phi|\alpha, \vec{w}) \propto p(\vec{w}|\phi)p(\phi|\alpha) \quad (1)$$

$$\propto \prod_k \phi^{n_k} \prod_k \phi^{\alpha_k - 1} \quad (2)$$

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- Conjugacy: this **posterior** has the same form as the **prior**

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Generative Model

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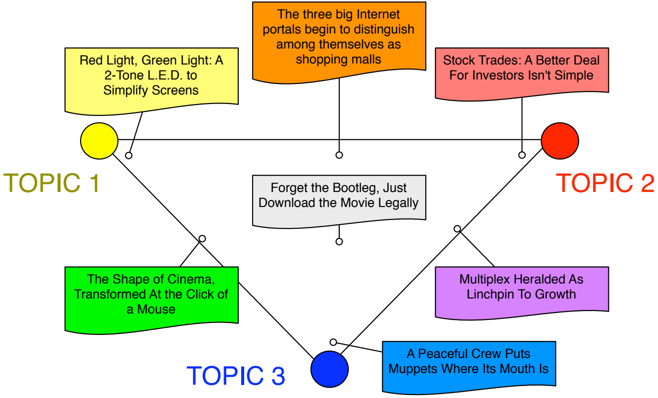

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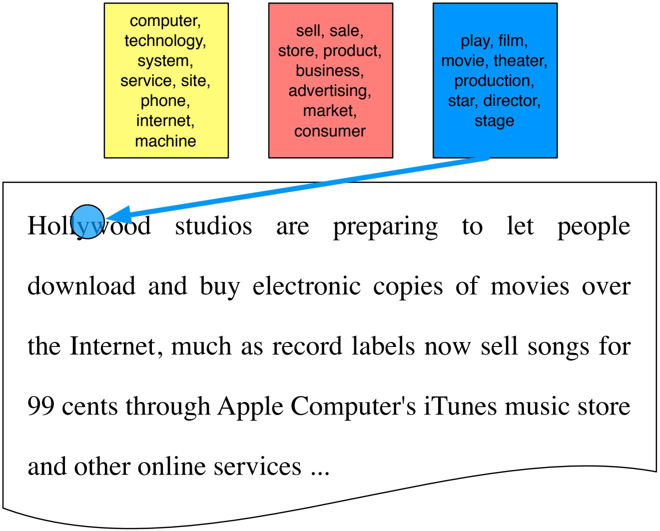

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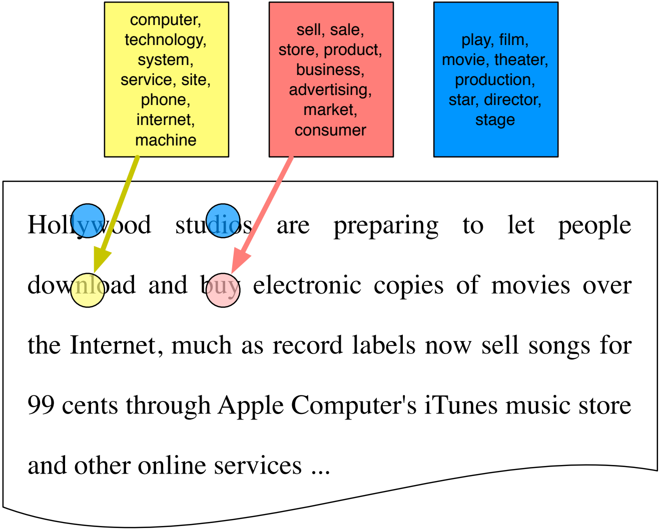

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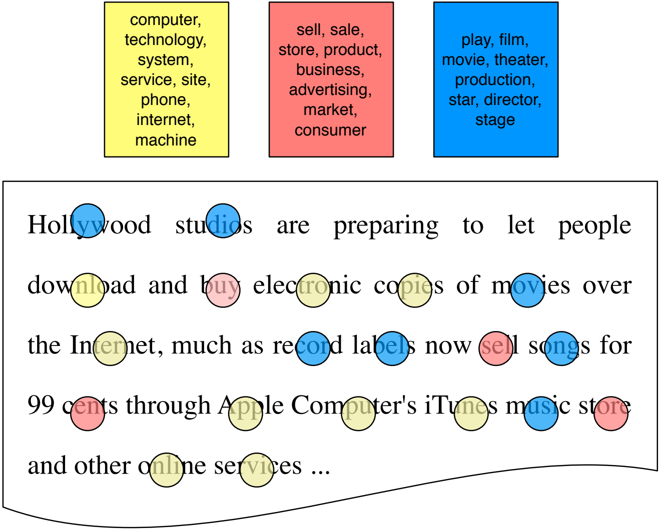
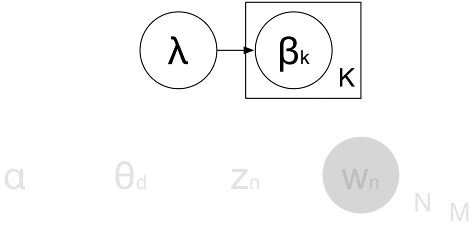


Generative Model Approach



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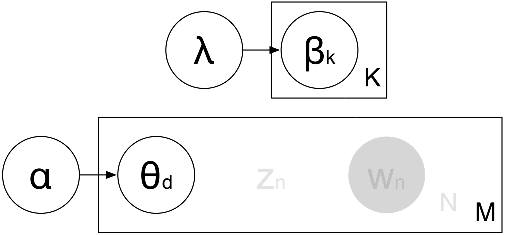

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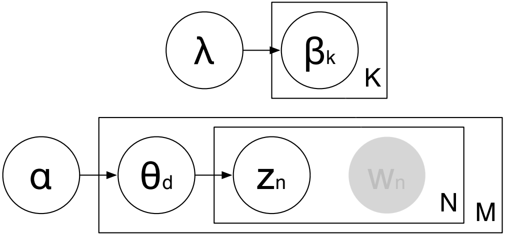

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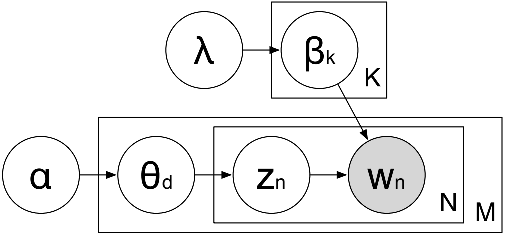

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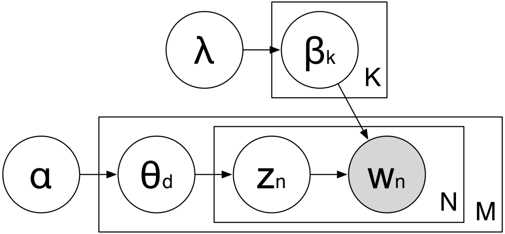

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Topic Models: What's Important

- Topic models
  - Topics to word types
  - Documents to topics
  - Topics to word types—multinomial distribution
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- Focus in this talk: statistical methods
  - Model: story of how your data came to be
  - Latent variables: missing pieces of your story
  - Statistical inference: filling in those missing pieces
- We use latent Dirichlet allocation (LDA), a fully Bayesian version of pLSI, probabilistic version of LSA

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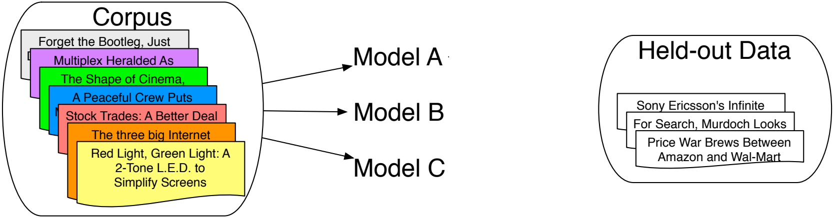
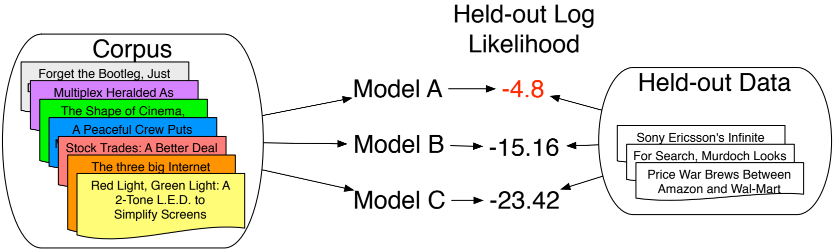


$$P(\boldsymbol{w} \mid \boldsymbol{w}', \boldsymbol{z}', \alpha \boldsymbol{m}, \beta \boldsymbol{u}) = \sum_{\boldsymbol{z}} P(\boldsymbol{w}, \boldsymbol{z} \mid \boldsymbol{w}', \boldsymbol{z}', \alpha \boldsymbol{m}, \beta \boldsymbol{u})$$

How you compute it is important too (Wallach et al. 2009)

Evaluation



Measures predictive power, not what the topics are

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Word Intrusion

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Word Intrusion

1. Take the highest probability words from a topic

Original Topic

dog, cat, horse, pig, cow

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3. We ask users to find the word that doesn't belong

Hypothesis

If the topics are interpretable, users will consistently choose true intruder

Word Intrusion

1 / 10

crash	accident	board	agency	tibetan	safety

2 / 10

commercial	network	television	advertising	viewer	layoff

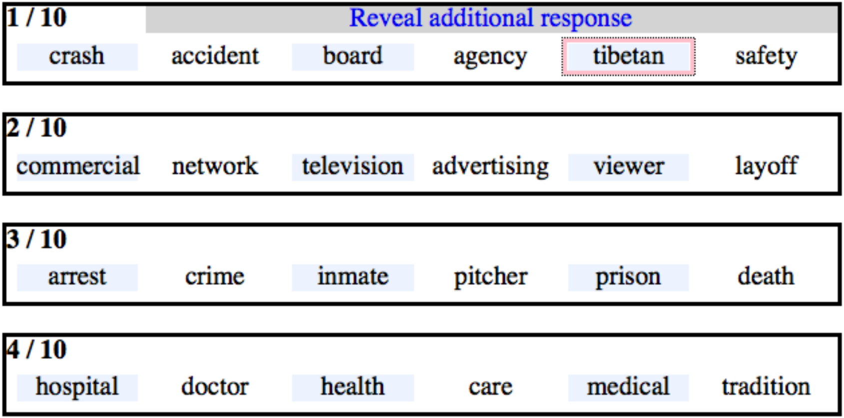
3 / 10

arrest	crime	inmate	pitcher	prison	death

4 / 10

hospital	doctor	health	care	medical	tradition

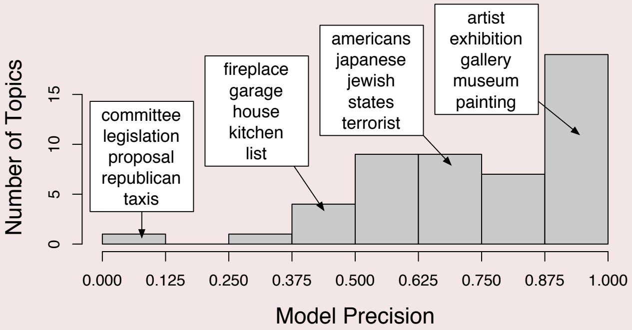
Word Intrusion



- Order of words was shuffled
- Which intruder was selected varied
- Model precision: percentage of users who clicked on intruder

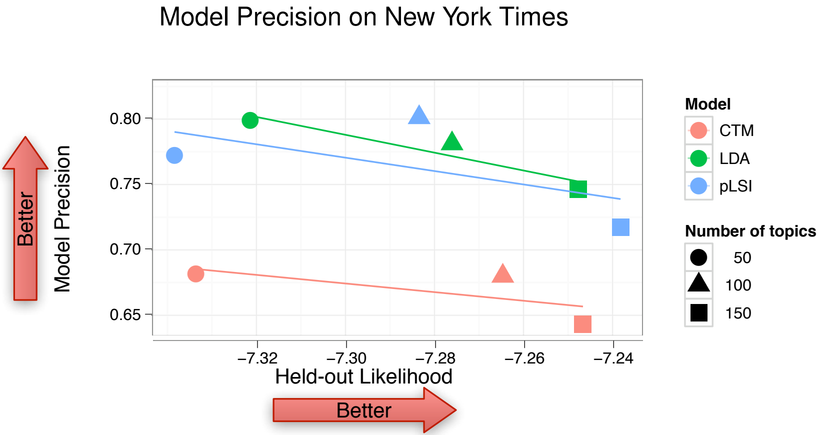
Word Intrusion: Which Topics are Interpretable?

New York Times, 50 LDA Topics



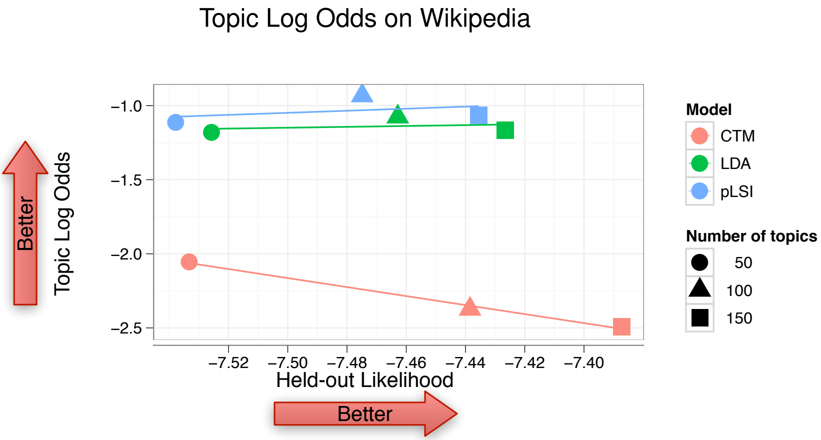
Model Precision: percentage of correct intruders found

Interpretability and Likelihood



within a model, higher likelihood $\neq$ higher interpretability

Interpretability and Likelihood



across models, higher likelihood $\neq$ higher interpretability

Evaluation Takeaway

- Measure what you care about
- If you care about prediction, likelihood is good
- If you care about a particular task, measure that

Inference

- We are interested in posterior distribution

$$p(Z|X,\Theta) \tag{4}$$

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- Here, latent variables are topic assignments z and topics θ. X is the words (divided into documents), and Θ are hyperparameters to Dirichlet distributions: α for topic proportion, λ for topics.

$$p(\vec{z},\vec{\beta},\vec{\theta}|\vec{w},\alpha,\lambda) \tag{5}$$

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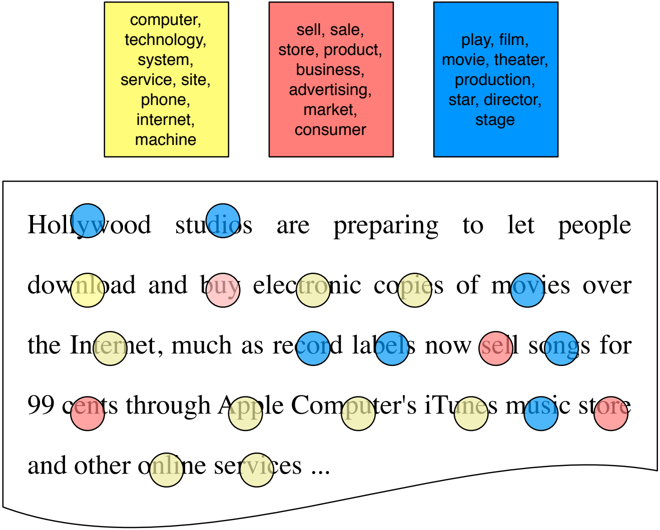
$$p(\vec{z},\vec{\beta},\vec{\theta}|\vec{w},\alpha,\lambda) \tag{5}$$

$$p(\vec{w},\vec{z},\vec{\theta},\vec{\beta}|\alpha,\lambda) = \prod_k p(\beta_k|\lambda)\prod_d p(\theta_d|\alpha)\prod_n p(z_{d,n}|\theta_d)p(w_{d,n}|\beta_{z_{d,n}})$$

Gibbs Sampling

- A form of Markov Chain Monte Carlo
- Chain is a sequence of random variable states
- Given a state $\{z_1, \dots z_N\}$ given certain technical conditions, drawing $z_k \sim p(z_1, \dots z_{k-1}, z_{k+1}, \dots z_N | X, \Theta)$ for all k (repeatedly) results in a Markov Chain whose stationary distribution is the posterior.
- For notational convenience, call $\vec{z}$ with $z_{d,n}$ removed $\vec{z}_{-d,n}$

Inference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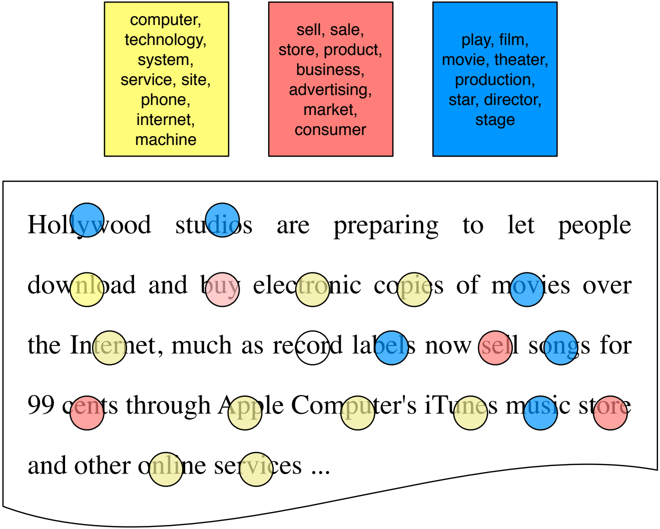
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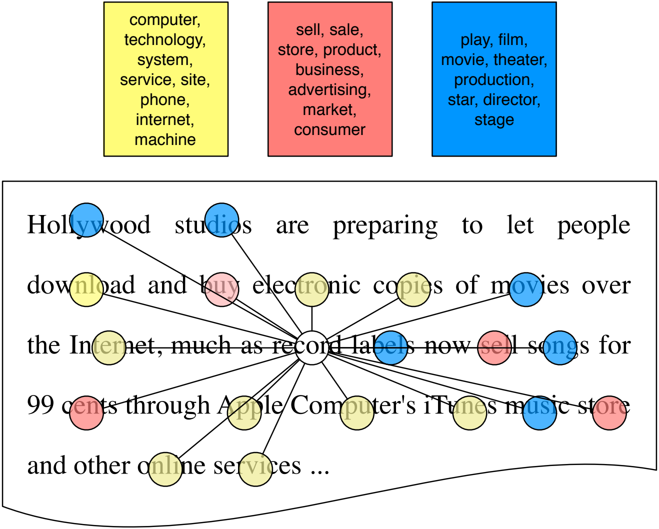
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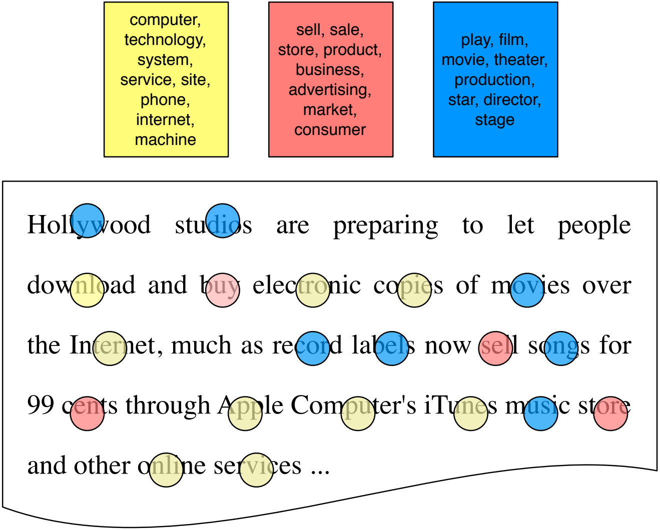
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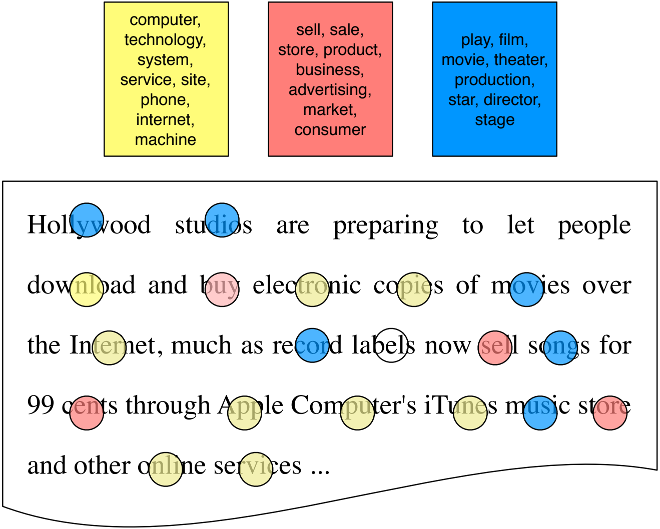
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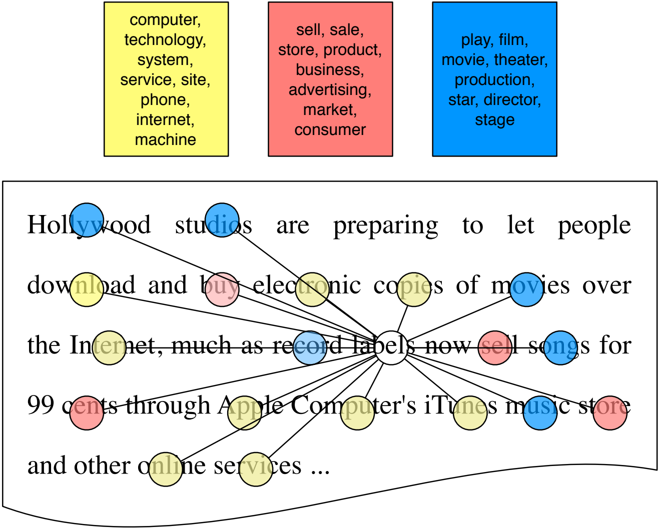
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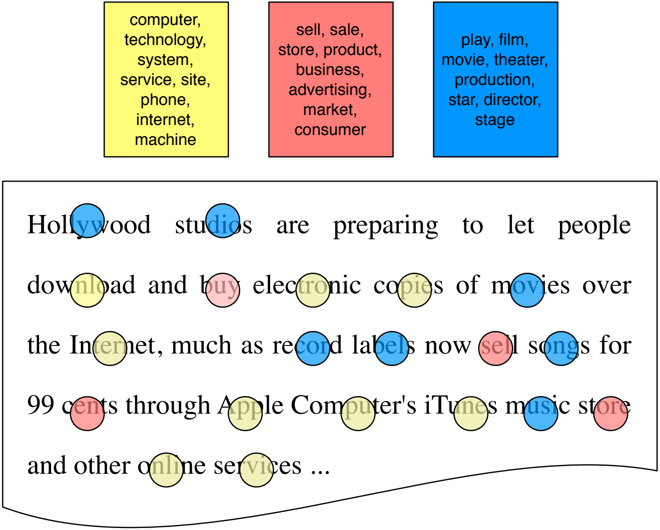
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Gibbs Sampling

- For LDA, we will sample the topic assignments
- Thus, we want:

$$p(z_{d,n} = k | \vec{z}_{-d,n}, \vec{w}, \alpha, \lambda) = \frac{p(z_{d,n} = k, \vec{z}_{-d,n} | \vec{w}, \alpha, \lambda)}{p(\vec{z}_{-d,n} | \vec{w}, \alpha, \lambda)}$$

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- The topics and per-document topic proportions are integrated out / marginalized
- Let $n_{d,i}$ be the number of words taking topic i in document d. Let $v_{k,w}$ be the number of times word w is used in topic k.

$$= \frac{\int_{\theta_d} \left(\prod_{i \neq k} \theta_d^{\alpha_i + n_{d,i} - 1} \right) \theta_d^{\alpha_k + n_{d,i}} d\theta_d \int_{\beta_k} \left(\prod_{i \neq w_{d,n}} \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) \beta_{k,w_{d,n}}^{\lambda_i + v_{k,i}} d\beta_k}{\int_{\theta_d} \left(\prod_i \theta_d^{\alpha_i + n_{d,i} - 1} \right) d\theta_d \int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k}$$

Gibbs Sampling

- For LDA, we will sample the topic assignments
- The topics and per-document topic proportions are integrated out / marginalized / Rao-Blackwellized
- Thus, we want:

$$p(z_{d,n} = k | \vec{z}_{-d,n}, \vec{w}, \alpha, \lambda) = \frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Gibbs Sampling

- Integral is normalizer of Dirichlet distribution

$$\int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k = \frac{\prod_i^V \Gamma\left(\beta_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \beta_i + v_{k,i}\right)}$$

Gibbs Sampling

- Integral is normalizer of Dirichlet distribution

$$\int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k = \frac{\prod_i^V \Gamma\left(\beta_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \beta_i + v_{k,i}\right)}$$

- So we can simplify

$$\frac{\int_{\theta_d} \left(\prod_{i \neq k} \theta_d^{\alpha_i + n_{d,i} - 1} \right) \theta_d^{\alpha_k + n_{d,i}} d\theta_d \int_{\beta_k} \left(\prod_{i \neq w_{d,n}} \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) \beta_{k,w_{d,n}}^{\lambda_i + v_{k,i}} d\beta_k}{\int_{\theta_d} \left(\prod_i \theta_d^{\alpha_i + n_{d,i} - 1} \right) d\theta_d \int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k} =$$

$$\frac{\frac{\Gamma\left(\alpha_k + n_{d,k} + 1\right)}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i} + 1\right)} \prod_{i \neq k}^K \Gamma\left(\alpha_k + n_{d,k}\right)}{\frac{\prod_i^K \Gamma\left(\alpha_i + n_{d,i}\right)}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i}\right)}} \frac{\frac{\Gamma\left(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i} + 1\right)} \prod_{i \neq w_{d,n}}^V \Gamma\left(\lambda_k + v_{k,w_{d,n}}\right)}{\frac{\prod_i^V \Gamma\left(\lambda_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i}\right)}}$$

Gamma Function Identity

$$z = \frac{\Gamma(z+1)}{\Gamma(z)} \tag{6}$$

$$\frac{\frac{\Gamma(\alpha_k + n_{d,k} + 1)}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i} + 1\right)} \prod_{i \neq k}^K \Gamma(\alpha_k + n_{d,k})}{\frac{\prod_i^K \Gamma(\alpha_i + n_{d,i})}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i}\right)}} \frac{\frac{\Gamma(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i} + 1\right)} \prod_{i \neq w_{d,n}}^V \Gamma(\lambda_k + v_{k,w_{d,n}})}{\frac{\prod_i^V \Gamma(\lambda_i + v_{k,i})}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i}\right)}}$$

$$= \frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Gibbs Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i} \tag{7}$$

- Number of times document d uses topic k
- Number of times topic k uses word type $w_{d,n}$
- Dirichlet parameter for document to topic distribution
- Dirichlet parameter for topic to word distribution
- How much this document likes topic k
- How much this topic likes word $w_{d,n}$

Gibbs Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i} \tag{7}$$

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- Number of times document d uses topic k
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- Dirichlet parameter for document to topic distribution
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- How much this document likes topic k
- How much this topic likes word $w_{d,n}$

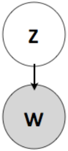
Sample Document

Etruscan	trade	price	temple	market

Sample Document

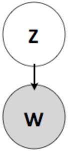
Etruscan	trade	price	temple	market

Randomly Assign Topics

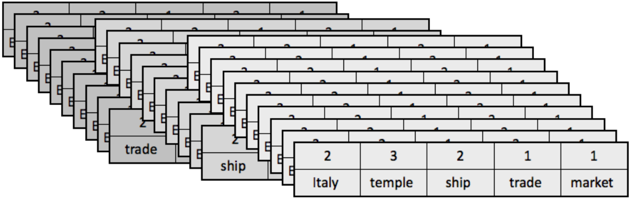


3	2	1	3	1
Etruscan	trade	price	temple	market

Randomly Assign Topics



3	2	1	3	1
Etruscan	trade	price	temple	market



Total Topic Counts

3	2	1	3	1
Etruscan	trade	price	temple	market

Total counts from **all** docs →

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	8	1
...			

Total Topic Counts

3	2	1	3	1
Etruscan	trade	price	temple	market

Total

	1	2	3
Etruscan	1	0	35
market	50	0	1

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Total Topic Counts

3	2	1	3	1
Etruscan	trade	price	temple	market

Total

	1	2	3
Etruscan	1	0	35
market	50	0	1

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

We want to sample this word . . .

3	**2**	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
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We want to sample this word . . .

3	**2**	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	**8**	1
...			

Decrement its count

3	?	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	**7**	1
...			

What is the conditional distribution for this topic?

3	?	1	3	1
Etruscan	trade	price	temple	market

Part 1: How much does this document like each topic?

3	?	1	3	1
Etruscan	trade	price	temple	market

Part 1: How much does this document like each topic?

3	?	1	3	1
Etruscan	trade	price	temple	market

Part 1: How much does this document like each topic?

3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1 Topic 2 Topic 3

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Part 1: How much does this document like each topic?

3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1 Topic 2 Topic 3

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Part 2: How much does each topic like the word?

3	?	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
trade	10	7	1

Part 2: How much does each topic like the word?

3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1 Topic 2 Topic 3

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Part 2: How much does each topic like the word?

3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1 Topic 2 Topic 3

Sampling Equation

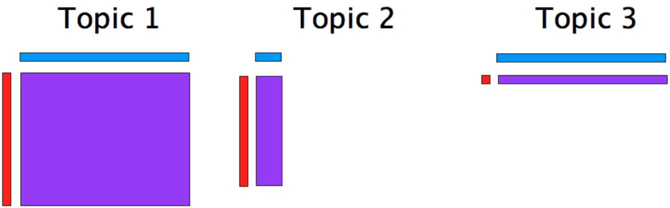
$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Geometric interpretation

3	?	1	3	1
Etruscan	trade	price	temple	market

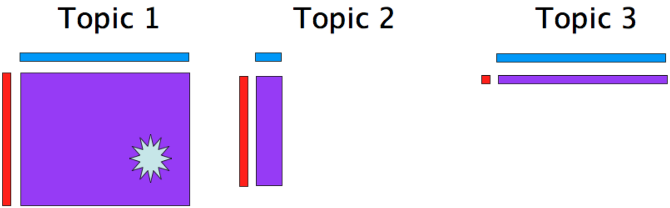
Geometric interpretation

3	?	1	3	1
Etruscan	trade	price	temple	market



Geometric interpretation

3	?	1	3	1
Etruscan	trade	price	temple	market



Update counts

3	?	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	**10**	7	1
...			

Update counts

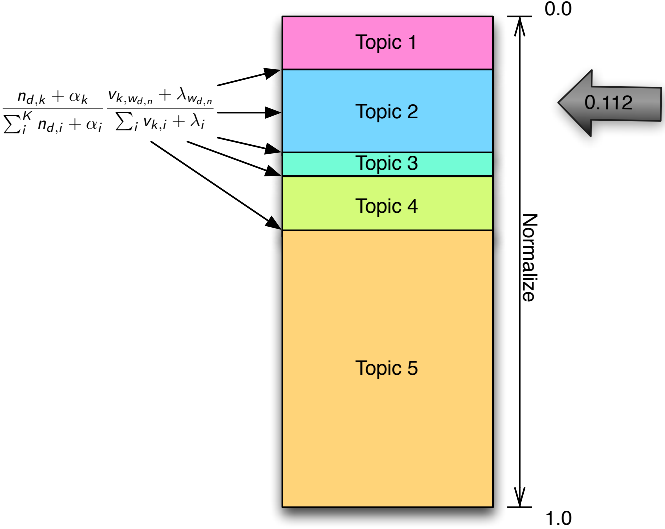
3	**1**	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	**11**	7	1
...			

Update counts

3	1	1	3	1
Etruscan	trade	price	temple	market

Details: how to sample from a distribution



Algorithm

1. For each iteration i:
   1.1 For each document d and word n currently assigned to z_{old}:
      1.1.1 Decrement $n_{d,z_{old}}$ and $v_{z_{old},w_{d,n}}$
      1.1.2 Sample $z_{new} = k$ with probability proportional to $\frac{n_{d,k}+\alpha_k}{\sum_i^K n_{d,i}+\alpha_i} \frac{v_{k,w_{d,n}}+\lambda_{w_{d,n}}}{\sum_i v_{k,i}+\lambda_i}$
      1.1.3 Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$

Implementation

Algorithm

1. For each iteration i:
   1.1 For each document d and word n currently assigned to z_{old}:
      1.1.1 Decrement $n_{d,z_{old}}$ and $v_{z_{old},w_{d,n}}$
      1.1.2 Sample $z_{new} = k$ with probability proportional to $\frac{n_{d,k}+\alpha_k}{\sum_i^K n_{d,i}+\alpha_i} \frac{v_{k,w_{d,n}}+\lambda_{w_{d,n}}}{\sum_i v_{k,i}+\lambda_i}$
      1.1.3 Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$

Desiderata

- Hyperparameters: Sample them too (slice sampling)
- Initialization: Random
- Sampling: Until likelihood converges
- Lag / burn-in: Difference of opinion on this
- Number of chains: Should do more than one

Available implementations

- Mallet (http://mallet.cs.umass.edu)
- LDAC (http://www.cs.princeton.edu/ blei/lda-c)
- Topicmod (http://code.google.com/p/topicmod)

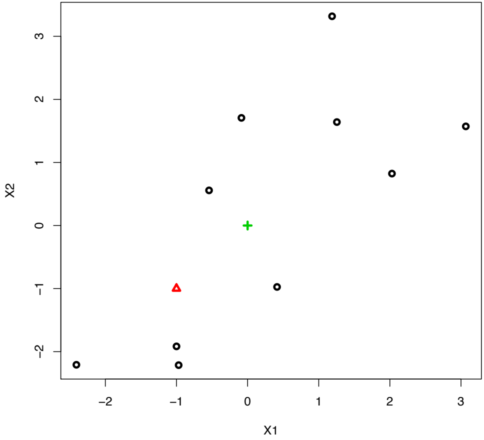
Wrapup

- Topic Models: Tools to uncover themes in large document collections
- Another example of Gibbs Sampling
- In class: Gibbs sampling example

K-Means: Example

Pick K centers (randomly):

$$(-1, -1) \text{ and } (0, 0)$$



K-Means: Example

Calculate distance between points and those centers:

x_1	x_2	$(-1,-1)$	$(0,0)$
0.4	-1.0	1.4	1.1
-1.0	-2.2	1.2	2.4
-2.4	-2.2	1.9	3.3
-1.0	-1.9	0.9	2.2
-0.5	0.6	1.6	0.8
-0.1	1.7	2.9	1.7
1.2	3.3	4.8	3.5
3.1	1.6	4.8	3.4
1.3	1.6	3.5	2.1
2.0	0.8	3.5	2.2

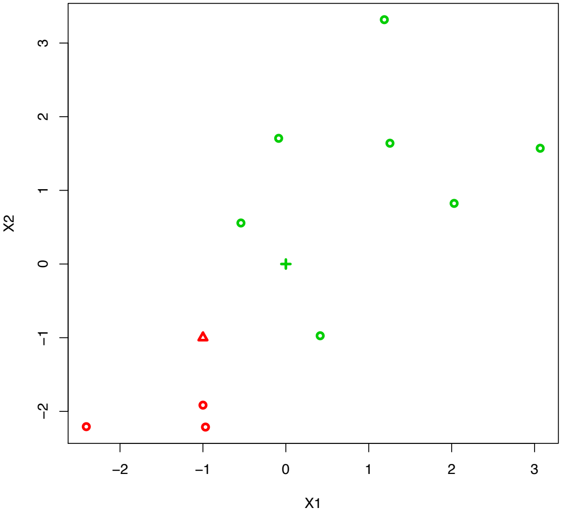
K-Means: Example

Choose mean with smaller distance:

x_1	x_2	$(-1,-1)$	$(0,0)$
0.4	-1.0	1.4	**1.1**
-1.0	-2.2	**1.2**	2.4
-2.4	-2.2	**1.9**	3.3
-1.0	-1.9	**0.9**	2.2
-0.5	0.6	1.6	**0.8**
-0.1	1.7	2.9	**1.7**
1.2	3.3	4.8	**3.5**
3.1	1.6	4.8	**3.4**
1.3	1.6	3.5	**2.1**
2.0	0.8	3.5	**2.2**

K-Means: Example

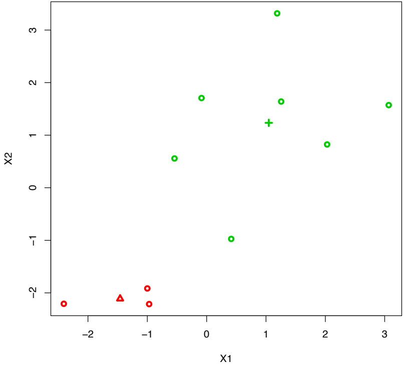
New clusters:



K-Means: Example

Refit means for each cluster:

- cluster 1: $(-1.0, -2.2)$, $(-2.4, -2.2)$, $(-1.0, -1.9)$
- new mean: $(-1.5, -2.1)$
- cluster 2: $(0.4, -1.0)$, $(-0.5, 0.6)$, $(-0.1, 1.7)$, $(1.2, 3.3)$, $(3.1, 1.6)$, $(1.3, 1.6)$, $(2.0, 0.8)$
- new mean: $(1.0, 1.2)$



K-Means: Example

Recalculate distances for each cluster:

x_1	x_2	$(-1.5, -2.1)$	$(1.0, 1.2)$
0.4	-1.0	2.2	2.3
-1.0	-2.2	0.5	4.0
-2.4	-2.2	1.0	4.9
-1.0	-1.9	0.5	3.8
-0.5	0.6	2.8	1.7
-0.1	1.7	4.1	1.2
1.2	3.3	6.0	2.1
3.1	1.6	5.8	2.0
1.3	1.6	4.6	0.5
2.0	0.8	4.6	1.1

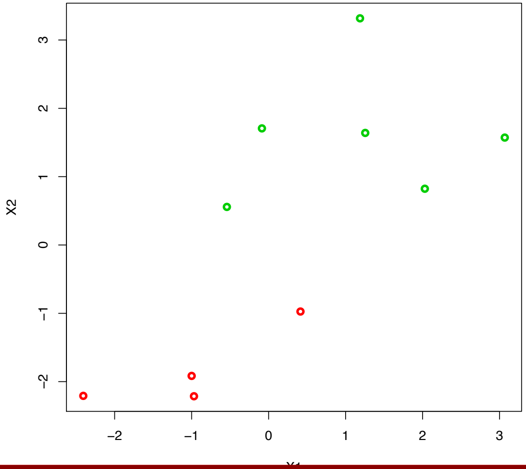
K-Means: Example

Choose mean with smaller distance:

x_1	x_2	$(-1.5,-2.1)$	$(1.0,1.2)$
0.4	-1.0	**2.2**	2.3
-1.0	-2.2	**0.5**	4.0
-2.4	-2.2	**1.0**	4.9
-1.0	-1.9	**0.5**	3.8
-0.5	0.6	2.8	**1.7**
-0.1	1.7	4.1	**1.2**
1.2	3.3	6.0	**2.1**
3.1	1.6	5.8	**2.0**
1.3	1.6	4.6	**0.5**
2.0	0.8	4.6	**1.1**

K-Means: Example

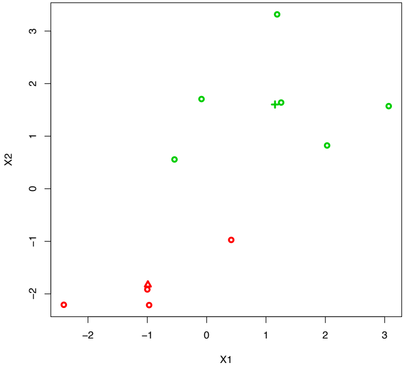
New clusters:



K-Means: Example

Refit means for each cluster:

- cluster 1: $(0.4, -1.0)$, $(-1.0, -2.2)$, $(-2.4, -2.2)$, $(-1.0, -1.9)$
- new mean: $(-1.0, -1.8)$
- cluster 2: $(-0.5, 0.6)$, $(-0.1, 1.7)$, $(1.2, 3.3)$, $(3.1, 1.6)$, $(1.3, 1.6)$, $(2.0, 0.8)$
- new mean: $(1.2, 1.6)$



K-Means: Example

Recalculate distances for each cluster:

x_1	x_2	$(-1.0, -1.8)$	$(1.2, 1.6)$
0.4	-1.0	1.6	2.7
-1.0	-2.2	0.4	4.4
-2.4	-2.2	1.5	5.2
-1.0	-1.9	0.1	4.1
-0.5	0.6	2.4	2.0
-0.1	1.7	3.6	1.2
1.2	3.3	5.6	1.7
3.1	1.6	5.3	1.9
1.3	1.6	4.1	0.1
2.0	0.8	4.0	1.2

K-Means: Example

Select smallest distance and compare these clusters with previous:

Table: New Clusters

x_1	x_2	$(-1.0,-1.8)$	$(1.2,1.6)$
0.4	-1.0	**1.6**	2.7
-1.0	-2.2	**0.4**	4.4
-2.4	-2.2	**1.5**	5.2
-1.0	-1.9	**0.1**	4.1
-0.5	0.6	2.4	**2.0**
-0.1	1.7	3.6	**1.2**
1.2	3.3	5.6	**1.7**
3.1	1.6	5.3	**1.9**
1.3	1.6	4.1	**0.1**
2.0	0.8	4.0	**1.2**

Table: Old Clusters

$(-1.5,-2.1)$	$(1.0,1.2)$
2.2	2.3
0.5	4.0
1.0	4.9
0.5	3.8
2.8	**1.7**
4.1	**1.2**
6.0	**2.1**
5.8	**2.0**
4.6	**0.5**
4.6	**1.1**

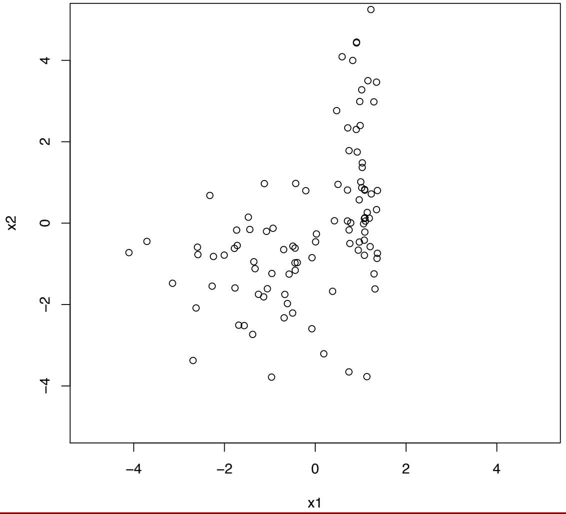
K-Means in Practice

K-means can be used for *image segmentation*

- partition image into multiple segments
- find boundaries of objects
- make art

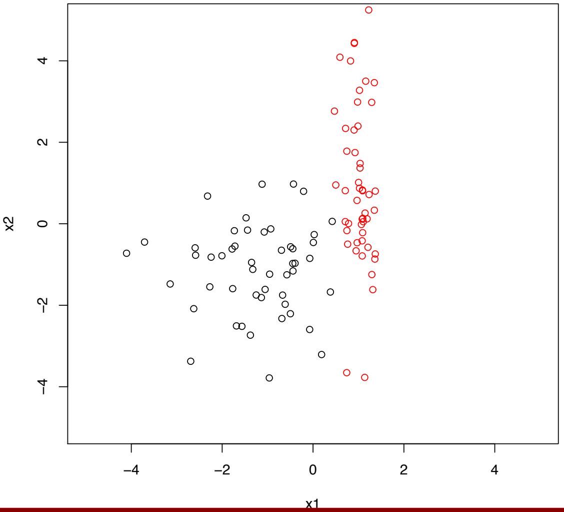
K-Means Clustering

What if our data look like this?



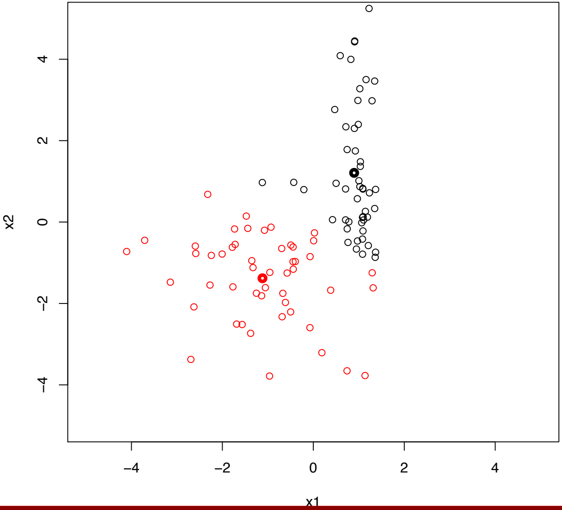
K-Means Clustering

True clustering:



K-Means Clustering

K-means clustering ($K = 2$):



Why topic models?

- Suppose you have a huge number of documents
- Want to know what's going on
- Can't read them all (e.g. every New York Times article from the 90's)
- Topic models offer a way to get a corpus-level view of major themes

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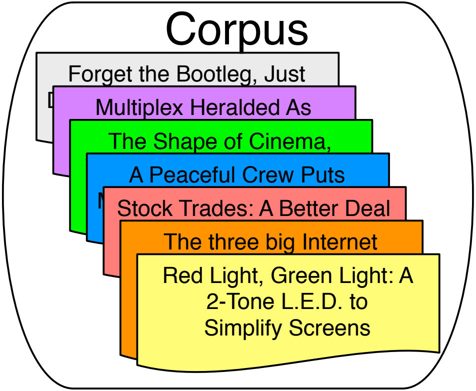
- Topic models offer a way to get a corpus-level view of major themes
- Unsupervised

Roadmap

- What are topic models
- How to know if you have good topic model
- How to go from raw data to topics

Conceptual Approach

From an **input corpus** and number of topics $K \longrightarrow$ words to topics



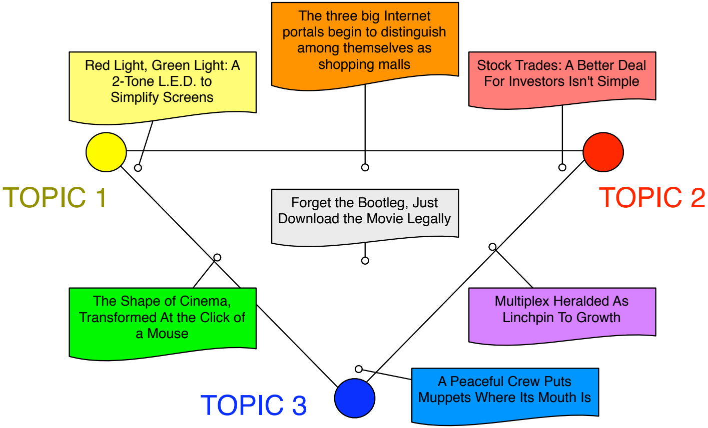
Conceptual Approach

From an input corpus and number of topics $K \longrightarrow$ **words to topics**

TOPIC 1	TOPIC 2	TOPIC 3
computer, technology, system, service, site, phone, internet, machine	sell, sale, store, product, business, advertising, market, consumer	play, film, movie, theater, production, star, director, stage

Conceptual Approach

- For each document, what topics are expressed by that document?



Topics from Science

human	evolution	disease	computer
genome	evolutionary	host	models
dna	species	bacteria	information
genetic	organisms	diseases	data
genes	life	resistance	computers
sequence	origin	bacterial	system
gene	biology	new	network
molecular	groups	strains	systems
sequencing	phylogenetic	control	model
map	living	infectious	parallel
information	diversity	malaria	methods
genetics	group	parasite	networks
mapping	new	parasites	software
project	two	united	new
sequences	common	tuberculosis	simulations

Why should you care?

- Neat way to explore / understand corpus collections
  - E-discovery
  - Social media
  - Scientific data
- NLP Applications
  - Word Sense Disambiguation
  - Discourse Segmentation
  - Machine Translation
- Psychology: word meaning, polysemy
- Inference is (relatively) simple

Matrix Factorization Approach

$$\underbrace{\begin{bmatrix} M \times K \end{bmatrix}}_{\text{Topic Assignment}} \times \underbrace{\begin{bmatrix} K \times V \end{bmatrix}}_{\text{Topics}} \approx \underbrace{\begin{bmatrix} M \times V \end{bmatrix}}_{\text{Dataset}}$$

- K Number of topics
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Topic Assignment | Topics | Dataset

- K Number of topics
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- V Size of vocabulary

- If you use singular value decomposition (SVD), this technique is called latent semantic analysis.
- Popular in information retrieval.

Alternative: Generative Model

- How your data came to be
- Sequence of Probabilistic Steps
- Posterior Inference

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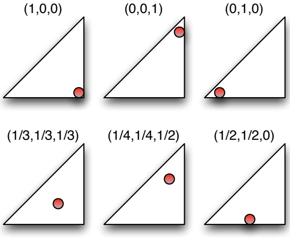
- Blei, Ng, Jordan. Latent **Dirichlet** Allocation. JMLR, 2003.

Multinomial Distribution

- Distribution over discrete outcomes
- Represented by non-negative vector that sums to one
- Picture representation



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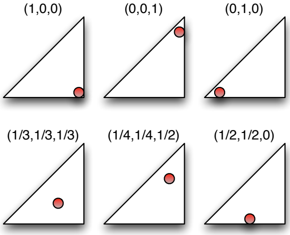

- Come from a Dirichlet distribution

Dirichlet Distribution

$$P(\boldsymbol{p} \mid \alpha \boldsymbol{m}) = \frac{\Gamma(\sum_k \alpha m_k)}{\prod_k \Gamma(\alpha m_k)} \prod_k p_k^{\alpha m_k - 1}$$

Dirichlet Distribution

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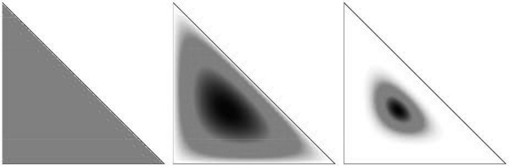

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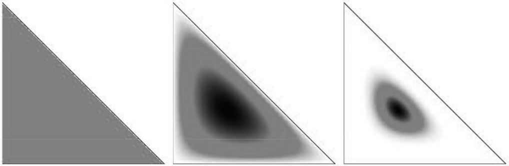

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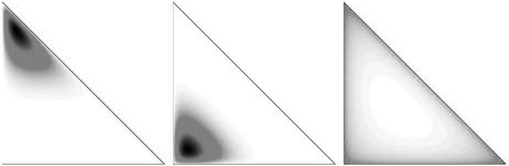

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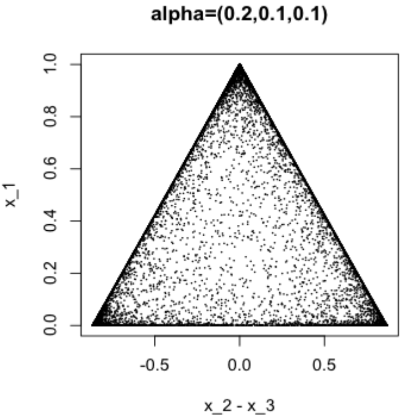
Dirichlet Distribution



Dirichlet Distribution

- If $\vec{\phi} \sim \text{Dir}(()\alpha)$, $\vec{w} \sim \text{Mult}(()\phi)$, and $n_k = |\{w_i : w_i = k\}|$ then

$$p(\phi|\alpha, \vec{w}) \propto p(\vec{w}|\phi)p(\phi|\alpha) \quad (1)$$

$$\propto \prod_k \phi^{n_k} \prod_k \phi^{\alpha_k - 1} \quad (2)$$

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- Conjugacy: this **posterior** has the same form as the **prior**

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Generative Model

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TOPIC 2

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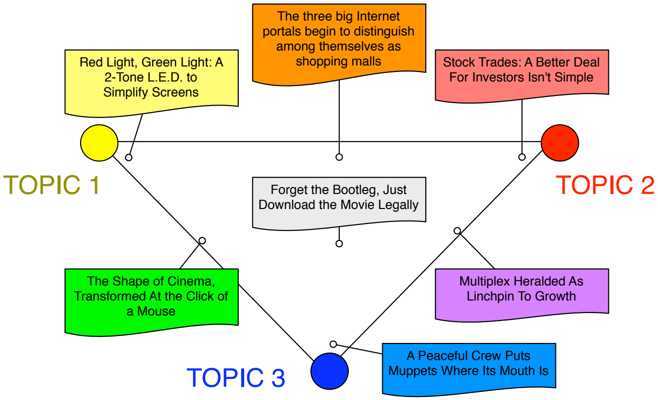
TOPIC 3

play, film, movie, theater, production, star, director, stage

Generative Model



Generative Model

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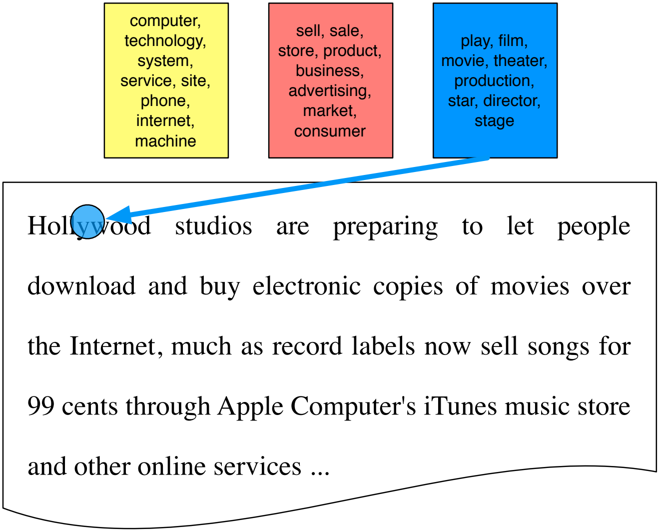
Hollywood studios are preparing to let people download and buy electronic copies of movies over the Internet, much as record labels now sell songs for 99 cents through Apple Computer's iTunes music store and other online services ...

Generative Model



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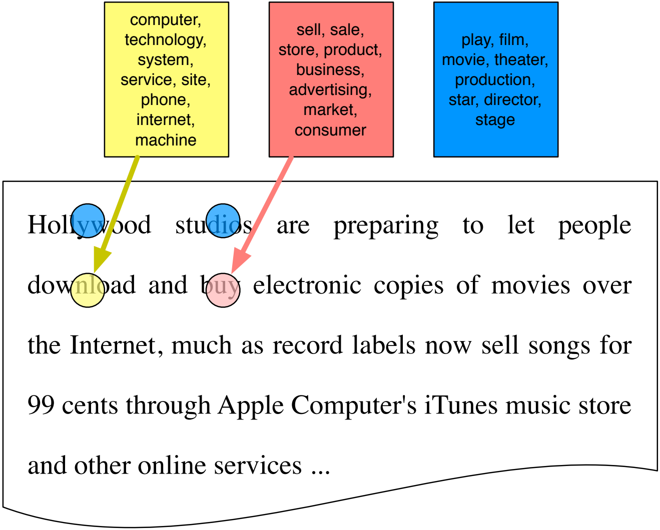

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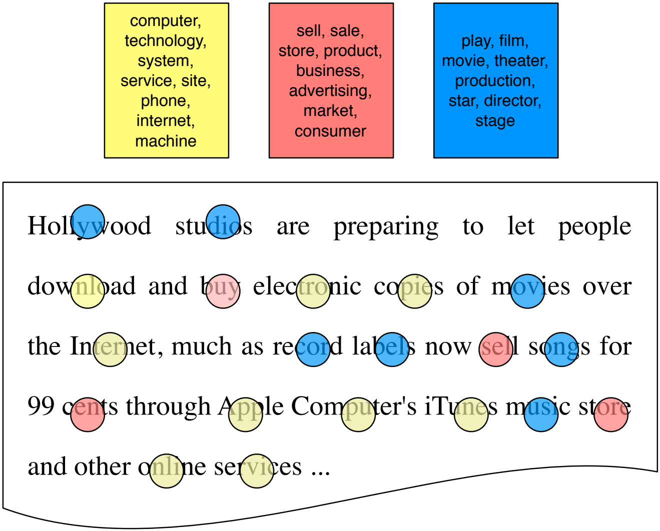
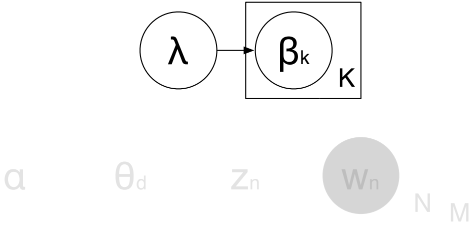


Generative Model Approach



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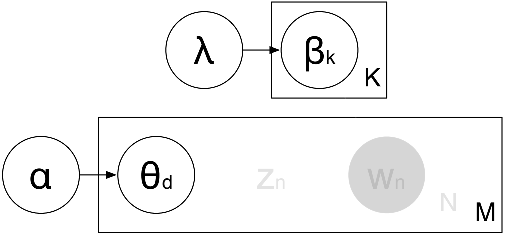

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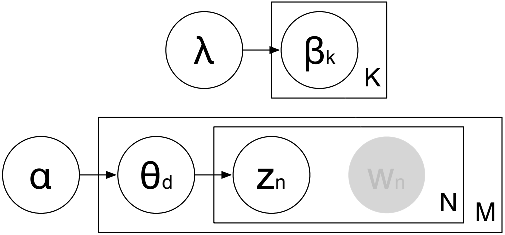

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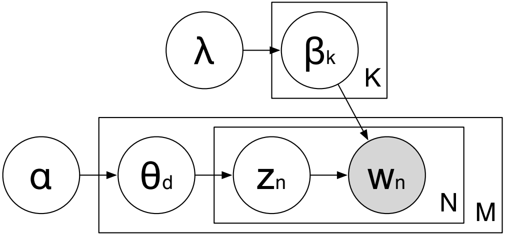

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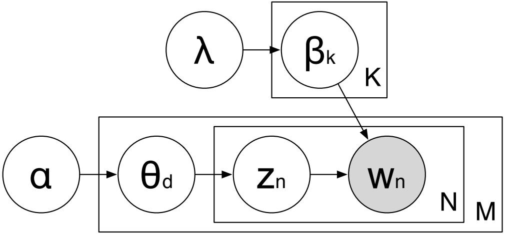

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Topic Models: What's Important

- Topic models
  - Topics to word types
  - Documents to topics
  - Topics to word types—multinomial distribution
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- Focus in this talk: statistical methods
  - Model: story of how your data came to be
  - Latent variables: missing pieces of your story
  - Statistical inference: filling in those missing pieces
- We use latent Dirichlet allocation (LDA), a fully Bayesian version of pLSI, probabilistic version of LSA

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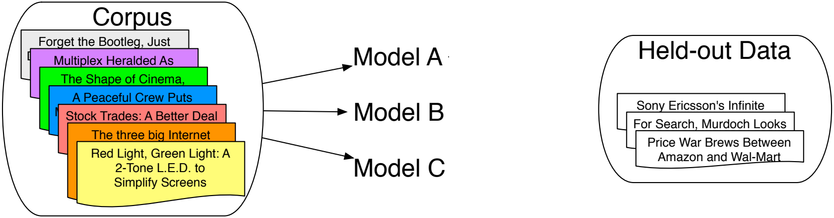

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How you compute it is important too (Wallach et al. 2009)

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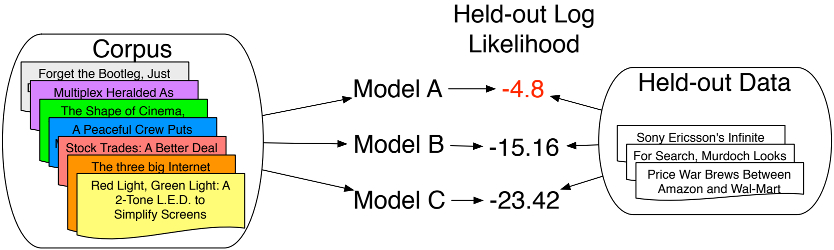

Measures predictive power, not what the topics are

$$P(\boldsymbol{w} \mid \boldsymbol{w}', \boldsymbol{z}', \alpha \boldsymbol{m}, \beta \boldsymbol{u}) = \sum_{\boldsymbol{z}} P(\boldsymbol{w}, \boldsymbol{z} \mid \boldsymbol{w}', \boldsymbol{z}', \alpha \boldsymbol{m}, \beta \boldsymbol{u})$$

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Word Intrusion

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Word Intrusion

1. Take the highest probability words from a topic

Original Topic

dog, cat, horse, pig, cow

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dog, cat, horse, pig, cow

2. Take a high-probability word from another topic and add it

Topic with Intruder

dog, cat, apple, horse, pig, cow

3. We ask users to find the word that doesn't belong

Hypothesis

If the topics are interpretable, users will consistently choose true intruder

Word Intrusion

1 / 10

crash	accident	board	agency	tibetan	safety

2 / 10

commercial	network	television	advertising	viewer	layoff

3 / 10

arrest	crime	inmate	pitcher	prison	death

4 / 10

hospital	doctor	health	care	medical	tradition

Word Intrusion

1 / 10	Reveal additional response				
crash	accident	board	agency	tibetan	safety
2 / 10					
commercial	network	television	advertising	viewer	layoff
3 / 10					
arrest	crime	inmate	pitcher	prison	death
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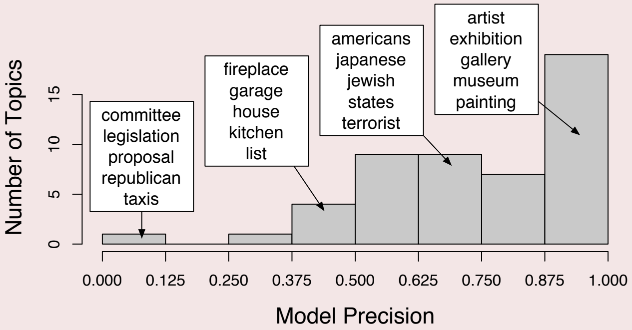
- Order of words was shuffled
- Which intruder was selected varied
- Model precision: percentage of users who clicked on intruder

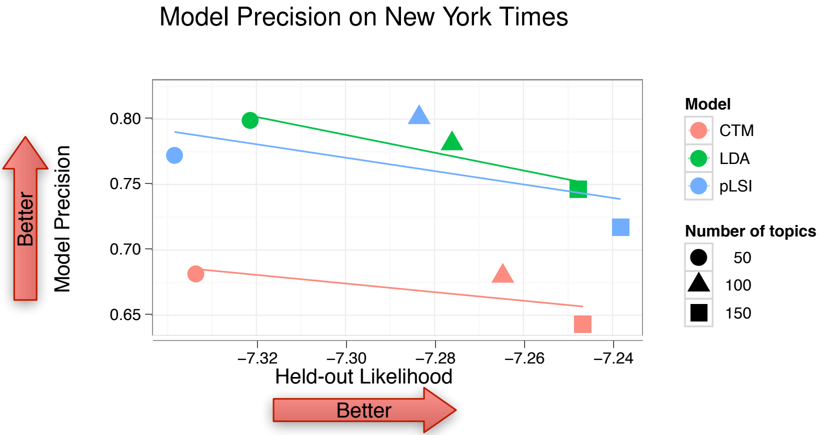
Word Intrusion: Which Topics are Interpretable?

New York Times, 50 LDA Topics



Model Precision: percentage of correct intruders found

Interpretability and Likelihood



within a model, higher likelihood $\neq$ higher interpretability

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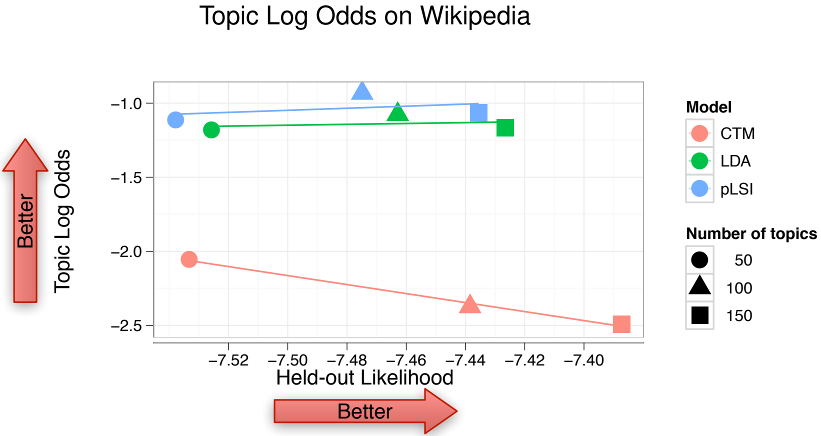

across models, higher likelihood $\neq$ higher interpretability

Evaluation Takeaway

- Measure what you care about
- If you care about prediction, likelihood is good
- If you care about a particular task, measure that

Sampling Token A

Assignments

$\text{Doc}_1 : z_A = 1, z_B = 2, z_C = 3, z_D = 1$
$\text{Doc}_2 : z_E = 2, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger dog iron pig
Topic 2 :pig hamburger iron cat
Topic 3 :dog iron cat

- $p(z_A = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$

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New assignment for (0, 0): 3

Sampling Token B

Assignments

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Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron cat
Topic 3 :dog dog iron cat

- $p(z_B = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$
- $p(z_B = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$

Sampling Token B

Assignments

Doc$_1$: $z_A = 3, z_B = 2, z_C = 3, z_D = 1$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron cat
Topic 3 :dog dog iron cat

- $p(z_B = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$
- $p(z_B = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_B = 3) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{4+5.000}\right) = 0.500 \times 0.222 = 0.111 = 0.111$

Sampling Token B

Assignments

Doc$_1$: $z_A = 3, z_B = 2, z_C = 3, z_D = 1$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron cat
Topic 3 :dog dog iron cat

- $p(z_B = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$
- $p(z_B = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_B = 3) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{4+5.000}\right) = 0.500 \times 0.222 = 0.111 = 0.111$

Sampling Token B

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 2, z_C = 3, z_D = 1$
$\text{Doc}_2 : z_E = 2, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron cat
Topic 3 :dog dog iron cat

- $p(z_B = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$
- $p(z_B = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_B = 3) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{4+5.000}\right) = 0.500 \times 0.222 = 0.111 = 0.111$

New assignment for (0, 1): 3

Sampling Token C

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 1$
$\text{Doc}_2 : z_E = 2, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron
Topic 3 :dog dog iron cat cat

Sampling Token C

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 1$

$\text{Doc}_2 : z_E = 2, z_F = 3, z_G = 1$

$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig

Topic 2 :pig hamburger iron

Topic 3 :dog dog iron cat cat

- $p(z_C = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$

Sampling Token C

Assignments	Topics
$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 1$	**Topic 1** :hamburger iron pig
$\text{Doc}_2 : z_E = 2, z_F = 3, z_G = 1$	**Topic 2** :pig hamburger iron
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$	**Topic 3** :dog dog iron cat cat

- $p(z_C = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$
- $p(z_C = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$

Sampling Token C

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 1$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron
Topic 3 :dog dog iron cat cat

- $p(z_C = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$
- $p(z_C = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_C = 3) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{4+5.000}\right) = 0.500 \times 0.222 = 0.111 = 0.111$

Sampling Token C

Assignments	Topics
$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 1$	**Topic 1** :hamburger iron pig
$\text{Doc}_2 : z_E = 2, z_F = 3, z_G = 1$	**Topic 2** :pig hamburger iron
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$	**Topic 3** :dog dog iron cat cat

- $p(z_C = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$
- $p(z_C = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_C = 3) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{4+5.000}\right) = 0.500 \times 0.222 = 0.111 = 0.111$

Sampling Token C

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 1$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron
Topic 3 :dog dog iron cat cat

- $p(z_C = 1) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.333 \times 0.125 = 0.042 = 0.042$
- $p(z_C = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_C = 3) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{4+5.000}\right) = 0.500 \times 0.222 = 0.111 = 0.111$

New assignment for (0, 2): 3

Sampling Token D

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 1$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron
Topic 3 :dog dog iron cat cat

Sampling Token D

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 1$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron
Topic 3 :dog dog iron cat cat

- $p(z_D = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.167 \times 0.143 = 0.024 = 0.024$

Sampling Token D

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 1$
$\text{Doc}_2 : z_E = 2, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron
Topic 3 :dog dog iron cat cat

- $p(z_D = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.167 \times 0.143 = 0.024 = 0.024$
- $p(z_D = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{3+5.000}\right) = 0.167 \times 0.250 = 0.042 = 0.042$

Sampling Token D

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 1$

Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$

Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig

Topic 2 :pig hamburger iron

Topic 3 :dog dog iron cat cat

- $p(z_D = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.167 \times 0.143 = 0.024 = 0.024$
- $p(z_D = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{3+5.000}\right) = 0.167 \times 0.250 = 0.042 = 0.042$
- $p(z_D = 3) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{5+5.000}\right) = 0.667 \times 0.100 = 0.067 = 0.067$

Sampling Token D

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 1$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron
Topic 3 :dog dog iron cat cat

- $p(z_D = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.167 \times 0.143 = 0.024 = 0.024$
- $p(z_D = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{3+5.000}\right) = 0.167 \times 0.250 = 0.042 = 0.042$
- $p(z_D = 3) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{5+5.000}\right) = 0.667 \times 0.100 = 0.067 = 0.067$

Sampling Token D

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 1$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron pig
Topic 2 :pig hamburger iron
Topic 3 :dog dog iron cat cat

- $p(z_D = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.167 \times 0.143 = 0.024 = 0.024$
- $p(z_D = 2) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{3+5.000}\right) = 0.167 \times 0.250 = 0.042 = 0.042$
- $p(z_D = 3) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{5+5.000}\right) = 0.667 \times 0.100 = 0.067 = 0.067$

New assignment for (0, 3): 3

Sampling Token E

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron
Topic 2 :pig hamburger iron
Topic 3 :pig dog dog iron cat cat

Sampling Token E

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron
Topic 2 :pig hamburger iron
Topic 3 :pig dog dog iron cat cat

- $p(z_E = 1) = \left(\frac{1+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{2+5.000}\right) = 0.400 \times 0.286 = 0.114 = 0.114$

Sampling Token E

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 2, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron
Topic 2 :pig hamburger iron
Topic 3 :pig dog dog iron cat cat

- $p(z_E = 1) = \left(\frac{1+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{2+5.000}\right) = 0.400 \times 0.286 = 0.114 = 0.114$
- $p(z_E = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$

Sampling Token E

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron
Topic 2 :pig hamburger iron
Topic 3 :pig dog dog iron cat cat

- $p(z_E = 1) = \left(\frac{1+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{2+5.000}\right) = 0.400 \times 0.286 = 0.114 = 0.114$
- $p(z_E = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$
- $p(z_E = 3) = \left(\frac{1+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{6+5.000}\right) = 0.400 \times 0.091 = 0.036 = 0.036$

Sampling Token E

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 2, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron
Topic 2 :pig hamburger iron
Topic 3 :pig dog dog iron cat cat

- $p(z_E = 1) = \left(\frac{1+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{2+5.000}\right) = 0.400 \times 0.286 = 0.114 = 0.114$
- $p(z_E = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$
- $p(z_E = 3) = \left(\frac{1+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{6+5.000}\right) = 0.400 \times 0.091 = 0.036 = 0.036$

Sampling Token E

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 2, z_F = 3, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger iron
Topic 2 :pig hamburger iron
Topic 3 :pig dog dog iron cat cat

- $p(z_E = 1) = \left(\frac{1+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{2+5.000}\right) = 0.400 \times 0.286 = 0.114 = 0.114$
- $p(z_E = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$
- $p(z_E = 3) = \left(\frac{1+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{6+5.000}\right) = 0.400 \times 0.091 = 0.036 = 0.036$

New assignment for (1, 0): 1

Sampling Token F

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger iron
Topic 2 :pig iron
Topic 3 :pig dog dog iron cat cat

Sampling Token F

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger iron
Topic 2 :pig iron
Topic 3 :pig dog dog iron cat cat

- $p(z_F = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.600 \times 0.125 = 0.075 = 0.075$

Sampling Token F

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger iron
Topic 2 :pig iron
Topic 3 :pig dog dog iron cat cat

- $p(z_F = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.600 \times 0.125 = 0.075 = 0.075$
- $p(z_F = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$

Sampling Token F

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$

Doc$_2$: $z_E = 1, z_F = 3, z_G = 1$

Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger iron

Topic 2 :pig iron

Topic 3 :pig dog dog iron cat cat

- $p(z_F = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.600 \times 0.125 = 0.075 = 0.075$
- $p(z_F = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$
- $p(z_F = 3) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{5+5.000}\right) = 0.200 \times 0.200 = 0.040 = 0.040$

Sampling Token F

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger iron
Topic 2 :pig iron
Topic 3 :pig dog dog iron cat cat

- $p(z_F = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.600 \times 0.125 = 0.075 = 0.075$
- $p(z_F = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$
- $p(z_F = 3) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{5+5.000}\right) = 0.200 \times 0.200 = 0.040 = 0.040$

Sampling Token F

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 3, z_G = 1$
$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger iron
Topic 2 :pig iron
Topic 3 :pig dog dog iron cat cat

- $p(z_F = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.600 \times 0.125 = 0.075 = 0.075$
- $p(z_F = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$
- $p(z_F = 3) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{5+5.000}\right) = 0.200 \times 0.200 = 0.040 = 0.040$

New assignment for (1, 1): 1

Sampling Token G

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron
Topic 2 :pig iron
Topic 3 :pig dog iron cat cat

Sampling Token G

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron
Topic 2 :pig iron
Topic 3 :pig dog iron cat cat

- $p(z_G = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{3+5.000}\right) = 0.600 \times 0.250 = 0.150 = 0.150$

Sampling Token G

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$

$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$

$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron

Topic 2 :pig iron

Topic 3 :pig dog iron cat cat

- $p(z_G = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{3+5.000}\right) = 0.600 \times 0.250 = 0.150 = 0.150$
- $p(z_G = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$

Sampling Token G

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$

$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$

$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron

Topic 2 :pig iron

Topic 3 :pig dog iron cat cat

- $p(z_G = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{3+5.000}\right) = 0.600 \times 0.250 = 0.150 = 0.150$
- $p(z_G = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$
- $p(z_G = 3) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{5+5.000}\right) = 0.200 \times 0.100 = 0.020 = 0.020$

Sampling Token G

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron
Topic 2 :pig iron
Topic 3 :pig dog iron cat cat

- $p(z_G = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{3+5.000}\right) = 0.600 \times 0.250 = 0.150 = 0.150$
- $p(z_G = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$
- $p(z_G = 3) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{5+5.000}\right) = 0.200 \times 0.100 = 0.020 = 0.020$

Sampling Token G

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron
Topic 2 :pig iron
Topic 3 :pig dog iron cat cat

- $p(z_G = 1) = \left(\frac{2+1.000}{2+3.000}\right) \times \left(\frac{1+1.000}{3+5.000}\right) = 0.600 \times 0.250 = 0.150 = 0.150$
- $p(z_G = 2) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{2+5.000}\right) = 0.200 \times 0.143 = 0.029 = 0.029$
- $p(z_G = 3) = \left(\frac{0+1.000}{2+3.000}\right) \times \left(\frac{0+1.000}{5+5.000}\right) = 0.200 \times 0.100 = 0.020 = 0.020$

New assignment for (1, 2): 1

Sampling Token H

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron
Topic 2 :pig iron
Topic 3 :pig dog iron cat cat

Sampling Token H

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron
Topic 2 :pig iron
Topic 3 :pig dog iron cat cat

- $p(z_H = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$

Sampling Token H

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$

$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$

$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron

Topic 2 :pig iron

Topic 3 :pig dog iron cat cat

- $p(z_H = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_H = 2) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{2+5.000}\right) = 0.500 \times 0.286 = 0.143 = 0.143$

Sampling Token H

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$

$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$

$\text{Doc}_3 : z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron

Topic 2 :pig iron

Topic 3 :pig dog iron cat cat

- $p(z_H = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_H = 2) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{2+5.000}\right) = 0.500 \times 0.286 = 0.143 = 0.143$
- $p(z_H = 3) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{5+5.000}\right) = 0.333 \times 0.200 = 0.067 = 0.067$

Sampling Token H

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron
Topic 2 :pig iron
Topic 3 :pig dog iron cat cat

- $p(z_H = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_H = 2) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{2+5.000}\right) = 0.500 \times 0.286 = 0.143 = 0.143$
- $p(z_H = 3) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{5+5.000}\right) = 0.333 \times 0.200 = 0.067 = 0.067$

Sampling Token H

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$

Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$

Doc$_3$: $z_H = 1, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog iron

Topic 2 :pig iron

Topic 3 :pig dog iron cat cat

- $p(z_H = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_H = 2) = \left(\frac{2+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{2+5.000}\right) = 0.500 \times 0.286 = 0.143 = 0.143$
- $p(z_H = 3) = \left(\frac{1+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{5+5.000}\right) = 0.333 \times 0.200 = 0.067 = 0.067$

New assignment for (2, 0): 2

Sampling Token I

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$

$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$

$\text{Doc}_3 : z_H = 2, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog

Topic 2 :pig iron iron

Topic 3 :pig dog iron cat cat

Sampling Token I

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron
Topic 3 :pig dog iron cat cat

- $p(z_I = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$

Sampling Token I

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron
Topic 3 :pig dog iron cat cat

- $p(z_I = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_I = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{2+1.000}{3+5.000}\right) = 0.667 \times 0.375 = 0.250 = 0.250$

Sampling Token I

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron
Topic 3 :pig dog iron cat cat

- $p(z_I = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_I = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{2+1.000}{3+5.000}\right) = 0.667 \times 0.375 = 0.250 = 0.250$
- $p(z_I = 3) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{4+5.000}\right) = 0.167 \times 0.111 = 0.019 = 0.019$

Sampling Token I

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$
$\text{Doc}_3 : z_H = 2, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron
Topic 3 :pig dog iron cat cat

- $p(z_I = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_I = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{2+1.000}{3+5.000}\right) = 0.667 \times 0.375 = 0.250 = 0.250$
- $p(z_I = 3) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{4+5.000}\right) = 0.167 \times 0.111 = 0.019 = 0.019$

Sampling Token I

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 3, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron
Topic 3 :pig dog iron cat cat

- $p(z_I = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_I = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{2+1.000}{3+5.000}\right) = 0.667 \times 0.375 = 0.250 = 0.250$
- $p(z_I = 3) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{4+5.000}\right) = 0.167 \times 0.111 = 0.019 = 0.019$

New assignment for (2, 1): 2

Sampling Token J

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

Sampling Token J

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

- $p(z_J = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$

Sampling Token J

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

- $p(z_J = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_J = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.667 \times 0.125 = 0.083 = 0.083$

Sampling Token J

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

- $p(z_J = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_J = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.667 \times 0.125 = 0.083 = 0.083$
- $p(z_J = 3) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{4+5.000}\right) = 0.167 \times 0.222 = 0.037 = 0.037$

Sampling Token J

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$

$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$

$\text{Doc}_3 : z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog

Topic 2 :pig iron iron iron

Topic 3 :pig dog cat cat

- $p(z_J = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_J = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.667 \times 0.125 = 0.083 = 0.083$
- $p(z_J = 3) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{4+5.000}\right) = 0.167 \times 0.222 = 0.037 = 0.037$

Sampling Token J

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

- $p(z_J = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_J = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.667 \times 0.125 = 0.083 = 0.083$
- $p(z_J = 3) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{1+1.000}{4+5.000}\right) = 0.167 \times 0.222 = 0.037 = 0.037$

New assignment for (2, 2): 2

Sampling Token K

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$
$\text{Doc}_3 : z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

Sampling Token K

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$
$\text{Doc}_3 : z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

- $p(z_K = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$

Sampling Token K

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

- $p(z_K = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_K = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{2+1.000}{3+5.000}\right) = 0.667 \times 0.375 = 0.250 = 0.250$

Sampling Token K

Assignments

Doc$_1$: $z_A = 3, z_B = 3, z_C = 3, z_D = 3$
Doc$_2$: $z_E = 1, z_F = 1, z_G = 1$
Doc$_3$: $z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

- $p(z_K = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_K = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{2+1.000}{3+5.000}\right) = 0.667 \times 0.375 = 0.250 = 0.250$
- $p(z_K = 3) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{4+5.000}\right) = 0.167 \times 0.111 = 0.019 = 0.019$

Sampling Token K

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$
$\text{Doc}_3 : z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

- $p(z_K = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_K = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{2+1.000}{3+5.000}\right) = 0.667 \times 0.375 = 0.250 = 0.250$
- $p(z_K = 3) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{4+5.000}\right) = 0.167 \times 0.111 = 0.019 = 0.019$

Sampling Token K

Assignments

$\text{Doc}_1 : z_A = 3, z_B = 3, z_C = 3, z_D = 3$
$\text{Doc}_2 : z_E = 1, z_F = 1, z_G = 1$
$\text{Doc}_3 : z_H = 2, z_I = 2, z_J = 2, z_K = 2$

Topics

Topic 1 :hamburger hamburger dog
Topic 2 :pig iron iron iron
Topic 3 :pig dog cat cat

- $p(z_K = 1) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{3+5.000}\right) = 0.167 \times 0.125 = 0.021 = 0.021$
- $p(z_K = 2) = \left(\frac{3+1.000}{3+3.000}\right) \times \left(\frac{2+1.000}{3+5.000}\right) = 0.667 \times 0.375 = 0.250 = 0.250$
- $p(z_K = 3) = \left(\frac{0+1.000}{3+3.000}\right) \times \left(\frac{0+1.000}{4+5.000}\right) = 0.167 \times 0.111 = 0.019 = 0.019$

New assignment for (2, 3): 2