Language Models

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Introduction

Slides adapted from Philip Koehn

Roadmap

After this lecture, you'll be able to:

- Understand probability distributions through the metaphor of the Chinese Restaurant Process
- Be able to calculate Kneser-Ney smoothing
- Understand the role of contexts in language models

Intuition

- Some words are “sticky”
- “San Francisco” is very common (high ungram)
- But Francisco only appears after one word

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- Some words are “sticky”
- “San Francisco” is very common (high ungram)
- But Francisco only appears after one word
- Our goal: to tell a statistical story of bay area restaurants to account for this phenomenon
- How to model this phenomena

Interpolation

- Higher and lower order n-gram models have different strengths and weaknesses
  - high-order n-grams are sensitive to more context, but have sparse counts
  - low-order n-grams consider only very limited context, but have robust counts
- Combine them

$$
\begin{aligned}
p_I(w_3|w_1,w_2) = \quad & \lambda_1\, p_1(w_3) \\
& + \lambda_2\, p_2(w_3|w_2) \\
& + \lambda_3\, p_3(w_3|w_1,w_2)
\end{aligned}
$$

Back-Off

- Trust the highest order language model that contains n-gram

$$p_n^{BO}(w_i|w_{i-n+1},...,w_{i-1}) =$$
$$= \begin{cases} \alpha_n(w_i|w_{i-n+1},...,w_{i-1}) \\ \qquad\qquad\qquad \text{if count}_n(w_{i-n+1},...,w_i) > 0 \\ d_n(w_{i-n+1},...,w_{i-1})\, p_{n-1}^{BO}(w_i|w_{i-n+2},...,w_{i-1}) \\ \qquad\qquad\qquad \text{else} \end{cases}$$

- Requires
  - adjusted prediction model $\alpha_n(w_i|w_{i-n+1},...,w_{i-1})$
  - discounting function $d_n(w_1,...,w_{n-1})$

Let's remember what a language model is

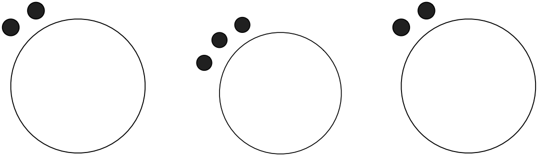
- It is a distribution over the next word in a sentence
- Given the previous $n-1$ words

Let's remember what a language model is

- It is a distribution over the next word in a sentence
- Given the previous $n-1$ words
- The challenge: backoff and sparsity

The Chinese Restaurant as a Distribution

To generate a word, you first sit down at a table. You sit down at a table proportional to the number of people sitting at the table.



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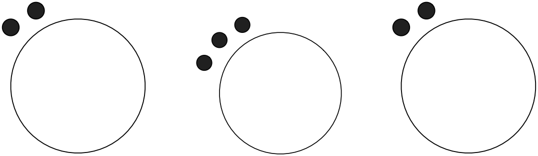

$\frac{2}{7}$ $\frac{3}{7}$ $\frac{2}{7}$

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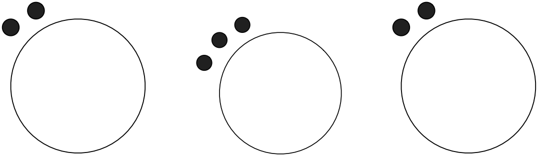

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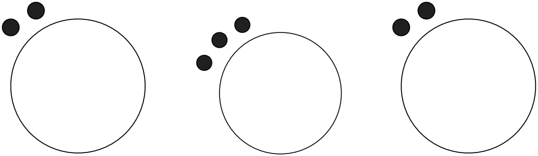

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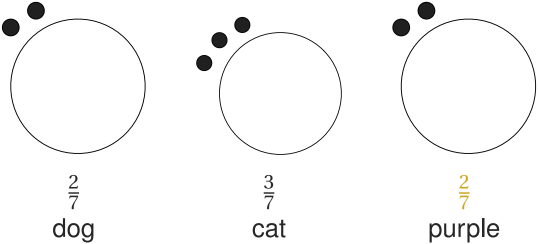
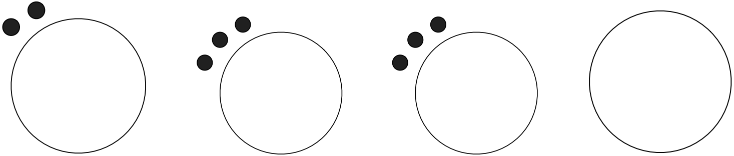


But this is just Maximum Likelihood

Why are we talking about Chinese Restaurants?

Always one more table . . .



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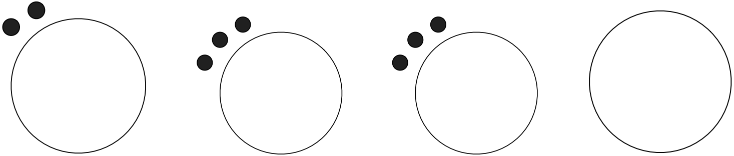
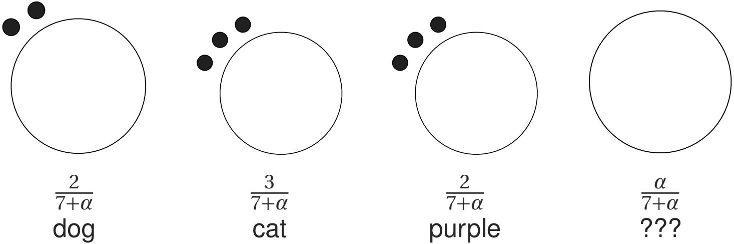


$\frac{2}{7+\alpha}$ $\frac{3}{7+\alpha}$ $\frac{2}{7+\alpha}$ $\frac{\alpha}{7+\alpha}$

Always one more table . . .



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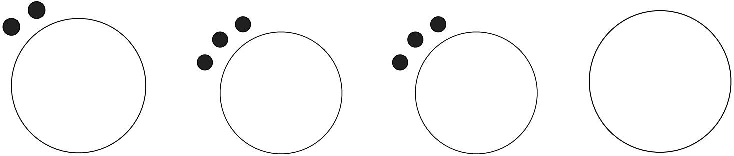
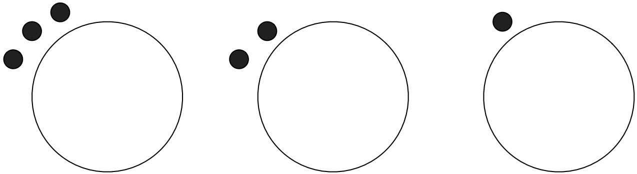


$\frac{2}{7+\alpha}$	$\frac{3}{7+\alpha}$	$\frac{2}{7+\alpha}$	$\frac{\alpha}{7+\alpha}$
dog	cat	purple	???

What to do with a new table?



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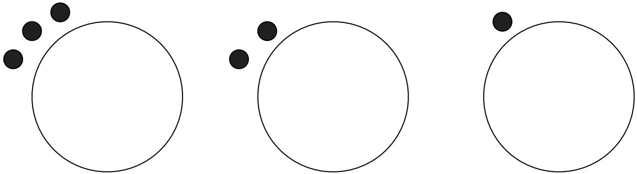

What can be a base distribution?

- Uniform (Dirichlet smoothing)

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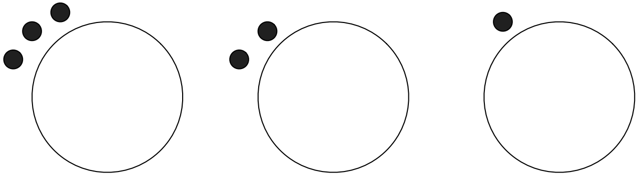
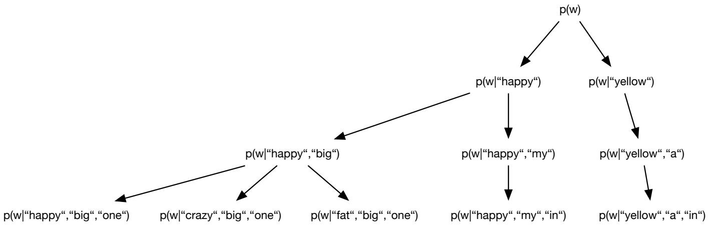


What can be a base distribution?

- Uniform (Dirichlet smoothing)
- Specific contexts $\longrightarrow$ less-specific contexts (backoff)

A hierarchy of Chinese Restaurants



Seating Assignments

`<s> a a a b a c </s>`

Seating Assignments

`<s> a a a b a c </s>`

Unigram Restaurant

<s> Restaurant

a Restaurant

b Restaurant

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

<s> Restaurant

$\boxed{*}^{1}$

a Restaurant

b Restaurant

c Restaurant

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Unigram Restaurant

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b Restaurant

a Restaurant

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Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

a [1]

<s> Restaurant

a [1]

a Restaurant

b Restaurant

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

a 1

<s> Restaurant

a 1

a Restaurant

* 1

b Restaurant

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

$\boxed{\texttt{a}}^{1}$

<s> Restaurant

$\boxed{\texttt{a}}^{1}$

a Restaurant

$\boxed{*}^{1}$

b Restaurant

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

a 2

<s> Restaurant

a 1

a Restaurant

a 1

b Restaurant

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

a [2]

<s> Restaurant

a [1]

a Restaurant

a [1]

b Restaurant

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

a [2]

<s> Restaurant

a [1]

a Restaurant

a [2]

b Restaurant

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

a [2]

<s> Restaurant

a [1]

a Restaurant

a [2] * [1]

b Restaurant

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

$\boxed{\texttt{a}}^{2}$ $\boxed{*}^{1}$

<s> Restaurant

$\boxed{\texttt{a}}^{1}$

a Restaurant

$\boxed{\texttt{a}}^{2}$ $\boxed{*}^{1}$

b Restaurant

c Restaurant

Seating Assignments

<s> a a a b a c </s>

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a [2] b [1]

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a [2] b [1]

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a [1]

a Restaurant

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c Restaurant

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Unigram Restaurant

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a Restaurant

$\boxed{a}^{2}$ $\boxed{b}^{1}$

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c Restaurant

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b Restaurant

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a Restaurant

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c Restaurant

Seating Assignments

<s> a a a b a c <//s>

Unigram Restaurant

a[2] b[1]

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a[1]

a Restaurant

a[2] b[1]

b Restaurant

*[1]

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

$\boxed{a}^3$ $\boxed{b}^1$

<s> Restaurant

$\boxed{a}^1$

b Restaurant

$\boxed{a}^1$

a Restaurant

$\boxed{a}^2$ $\boxed{b}^1$

c Restaurant

Seating Assignments

<s> a a a b a c </s>

Unigram Restaurant

a 3 b 1

<s> Restaurant

a 1

b Restaurant

a 1

a Restaurant

a 2 b 1

c Restaurant

Seating Assignments

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Unigram Restaurant

$\boxed{\texttt{a}}^{3}$ $\boxed{\texttt{b}}^{1}$

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b Restaurant

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Seating Assignments

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Unigram Restaurant

a[3] b[1] *[1]

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</s>^{1}

Real examples

- San Francisco

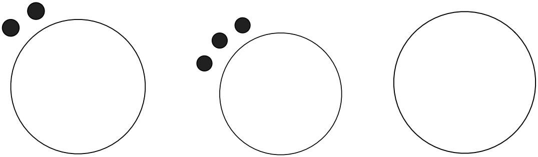
Real examples

- San Francisco
- Star Spangled Banner

Real examples

- San Francisco
- Star Spangled Banner
- Bottom Line: Counts go to the context that explains it best

The rich get richer



$\frac{2}{5+\theta}$ $\frac{3}{5+\theta}$ $\frac{\theta}{5+\theta}$

Computing the Probability of an Observation

$$p(w = x|\vec{s}, \theta, u) = \underbrace{\frac{c_{u,x}}{\theta + c_{u,\cdot}}}_{\text{existing table}} + \underbrace{\frac{\theta}{\theta + c_{u,\cdot}} p(w = x|\vec{s}, \theta, \pi(u))}_{\text{new table}} \tag{1}$$

- Word type x
- Seating assignments $\vec{s}$
- Concentration θ
- Context u
- Number seated at table serving x in restaurant u, $c_{u,x}$
- Number seated at all tables in restaurant u, $c_{u,\cdot}$
- The backoff context $\pi(u)$

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Example: $p(w = \mathrm{b}|\vec{s}, \theta = 1.0, u = \mathrm{a})$

Unigram Restaurant

a [3] b [1] c [1] </s> [1]

<s> Restaurant

a [1]

a Restaurant

a [2] b [1] c [1]

b Restaurant

a [1]

c Restaurant

</s> [1]

$$p(w = \mathrm{b}|\ldots) = \frac{c_{\mathrm{a},\mathrm{b}}}{\theta + c_{u,\cdot}} + \frac{\theta}{\theta + c_{u,\cdot}} p(w = x|\vec{s}, \theta, \pi(u)) \tag{2}$$

Example: $p(w = \mathrm{b}|\vec{s}, \theta = 1.0, u = \mathrm{a})$

Unigram Restaurant

a 3 b 1 c 1 </s> 1

<s> Restaurant

a 1

a Restaurant

a 2 b 1 c 1

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a 1

c Restaurant

</s> 1

$$p(w = \mathrm{b}|\ldots) = \frac{c_{\mathrm{a},\mathrm{b}}}{\theta + c_{u,\cdot}} + \frac{\theta}{\theta + c_{u,\cdot}} p(w = x|\vec{s}, \theta, \pi(u)) \qquad (2)$$

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$$p(w = \mathtt{b} | \ldots) = \frac{1}{\theta + c_{u,\cdot}} + \frac{\theta}{\theta + c_{u,\cdot}} p(w = x | \vec{s}, \theta, \pi(u)) \qquad (2)$$

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$\boxed{\texttt{a}}^{1}$

c Restaurant

$\boxed{\texttt{</s>}}^{1}$

$$p(w = \texttt{b} | \ldots) = \frac{1}{1.0 + 4} + \frac{1.0}{1.0 + 4} p(w = x | \vec{s}, \theta, \pi(u)) \tag{2}$$

Example: $p(w=\texttt{b}|\vec{s},\theta=1.0,u=\texttt{a})$

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Example: $p(w = \mathtt{b} \mid \vec{s}, \theta = 1.0, u = \mathtt{a})$

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c Restaurant

</s>^{1}

$$p(w = \mathtt{b} \mid \ldots) = \frac{1}{1.0 + 4} + \frac{1.0}{1.0 + 4} p(w = x \mid \vec{s}, \theta, \pi(\emptyset)) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

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c Restaurant

</s>^{1}

$$p(w = \mathrm{b}|\ldots) = \frac{1}{5} + \frac{1}{5}\left(\frac{c_{\emptyset,\mathrm{b}}}{c_{\emptyset,\cdot}+\theta} + \frac{\theta}{c_{\emptyset,\cdot}+\theta}\,\frac{1}{V}\right) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

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<s> Restaurant

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</s>^{1}

$$p(w = \mathtt{b} | \ldots) = \frac{1}{5} + \frac{1}{5}\left(\frac{c_{\emptyset,\mathtt{b}}}{c_{\emptyset,\cdot} + \theta} + \frac{\theta}{c_{\emptyset,\cdot} + \theta}\,\frac{1}{5} \right) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

Unigram Restaurant

$\boxed{\mathtt{a}}^{3}$ $\boxed{\mathtt{b}}^{1}$ $\boxed{\mathtt{c}}^{1}$ $\boxed{\mathtt{</s>}}^{1}$

<s> Restaurant

$\boxed{\mathtt{a}}^{1}$

a Restaurant

$\boxed{\mathtt{a}}^{2}$ $\boxed{\mathtt{b}}^{1}$ $\boxed{\mathtt{c}}^{1}$

b Restaurant

$\boxed{\mathtt{a}}^{1}$

c Restaurant

$\boxed{\mathtt{</s>}}^{1}$

$$p(w = \mathtt{b} | \ldots) = \frac{1}{5} + \frac{1}{5}\left(\frac{c_{\emptyset,\mathtt{b}}}{c_{\emptyset,\cdot} + 1.0} + \frac{1.0}{c_{\emptyset,\cdot} + 1.0}\frac{1}{5}\right) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

Unigram Restaurant

a [3] b [1] c [1] </s> [1]

<s> Restaurant

a [1]

a Restaurant

a [2] b [1] c [1]

b Restaurant

a [1]

c Restaurant

</s> [1]

$$p(w = \mathtt{b} | \ldots) = \frac{1}{5} + \frac{1}{5}\left(\frac{1}{c_{\emptyset,\cdot} + 1.0} + \frac{1.0}{c_{\emptyset,\cdot} + 1.0}\frac{1}{5}\right) \tag{2}$$

Example: $p(w = \mathtt{b} | \vec{s}, \theta = 1.0, u = \mathtt{a})$

Unigram Restaurant

a^{3} b^{1} c^{1} </s>^{1}

<s> Restaurant

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b Restaurant

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c Restaurant

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$$p(w = \mathtt{b} | \ldots) = \frac{1}{5} + \frac{1}{5}\left(\frac{1}{6 + 1.0} + \frac{1.0}{6 + 1.0}\frac{1}{5}\right) \tag{2}$$

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<s> Restaurant

$\boxed{\mathtt{a}}^1$

a Restaurant

$\boxed{\mathtt{a}}^2$ $\boxed{\mathtt{b}}^1$ $\boxed{\mathtt{c}}^1$

b Restaurant

$\boxed{\mathtt{a}}^1$

c Restaurant

$\boxed{\mathtt{</s>}}^1$

$$p(w = \mathtt{b} | \dots) = \frac{1}{5} + \frac{1}{5}\left(\frac{1}{7} + \frac{1}{7}\frac{1}{5}\right) = 0.24 \tag{2}$$

Discounting

- Empirically, it helps favor the backoff if you have more tables
- Otherwise, it gets too close to maximum likelihood
- Idea is called discounting
- Steal a little bit of probability mass δ from every table and give it to the new table (backoff)

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Interpolated Kneser-Ney!

More advanced models

- Interpolated Kneser-Ney assumes **one table with a dish (word)** per restaurant
- Can get slightly better performance by assuming you can have duplicated tables: **Pitman-Yor** language model
- Requires Gibbs Sampling of the seating assignments
  - If you walk into a restaurant with your dish on a table, you sample whether to create a new dish or not
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- Bottom line: discrete representation of the context
- Neural language models use continuous representations to store context, which works better

Exercises

Exercise

- Start with restaurant we had before
- Assume you see `<s> b b a c </s>`; add those counts to tables
- Compute probability of `b` following `a` ($\theta = 1.0, \delta = 0.5$)
- Compute the probability of `a` following `b`
- Compute probability of `</s>` following `<s>`

A busy night at the restaurant

Unigram Restaurant

a ^{3} b ^{1} c ^{1} </s> ^{1}

<s> Restaurant

a ^{1}

a Restaurant

a ^{2} b ^{1} c ^{1}

b Restaurant

a ^{1}

c Restaurant

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A busy night at the restaurant

Unigram Restaurant

a ^{3} b ^{1} c ^{1} </s> ^{1}

<s> Restaurant

a ^{1} b ^{1}

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b Restaurant

a ^{1}

c Restaurant

</s> ^{1}

A busy night at the restaurant

Unigram Restaurant

$\mathtt{a}^{3}$ $\mathtt{b}^{2}$ $\mathtt{c}^{1}$ $\mathtt{</s>}^{1}$

<s> Restaurant

$\mathtt{a}^{1}$ $\mathtt{b}^{1}$

a Restaurant

$\mathtt{a}^{2}$ $\mathtt{b}^{1}$ $\mathtt{c}^{1}$

b Restaurant

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c Restaurant

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A busy night at the restaurant

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a^{3} b^{2} c^{1} </s>^{1}

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a^{1} b^{1}

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a^{2} b^{1} c^{1}

b Restaurant

a^{1} b^{1}

c Restaurant

</s>^{1}

A busy night at the restaurant

Unigram Restaurant

$\boxed{\texttt{a}}^{3}$ $\boxed{\texttt{b}}^{3}$ $\boxed{\texttt{c}}^{1}$ $\boxed{\texttt{</s>}}^{1}$

$\texttt{<s>}$ Restaurant

$\boxed{\texttt{a}}^{1}$ $\boxed{\texttt{b}}^{1}$

$\texttt{a}$ Restaurant

$\boxed{\texttt{a}}^{2}$ $\boxed{\texttt{b}}^{1}$ $\boxed{\texttt{c}}^{1}$

$\texttt{b}$ Restaurant

$\boxed{\texttt{a}}^{1}$ $\boxed{\texttt{b}}^{1}$

$\texttt{c}$ Restaurant

$\boxed{\texttt{</s>}}^{1}$

A busy night at the restaurant

Unigram Restaurant

a^{3} b^{3} c^{1} $</s>^{1}$

<s> Restaurant

a^{1} b^{1}

a Restaurant

a^{2} b^{1} c^{1}

b Restaurant

a^{2} b^{1}

c Restaurant

$</s>^{1}$

A busy night at the restaurant

Unigram Restaurant

$\texttt{a}^{3}$ $\texttt{b}^{3}$ $\texttt{c}^{1}$ $\texttt{</s>}^{1}$

<s> Restaurant

$\texttt{a}^{1}$ $\texttt{b}^{1}$

a Restaurant

$\texttt{a}^{2}$ $\texttt{b}^{1}$ $\texttt{c}^{2}$

b Restaurant

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A busy night at the restaurant

Unigram Restaurant

a^3 b^3 c^1 </s>^{1}

<s> Restaurant

a^1 b^1

a Restaurant

a^2 b^1 c^2

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A busy night at the restaurant

Unigram Restaurant

$\mathtt{a}^{3}$ $\mathtt{b}^{3}$ $\mathtt{c}^{1}$ </s>^{1}

<s> Restaurant

$\mathtt{a}^{1}$ $\mathtt{b}^{1}$

a Restaurant

$\mathtt{a}^{2}$ $\mathtt{b}^{1}$ $\mathtt{c}^{2}$

b Restaurant

$\mathtt{a}^{2}$ $\mathtt{b}^{1}$

c Restaurant

</s>^{2}

As you see more data, bottom restaurants do more work.

$\mathtt{b}$ following $\mathtt{a}$

$$= \frac{1-\delta}{\theta+5} + \frac{\theta+3\delta}{\theta+5} p(\mathtt{b}) \tag{4}$$

$$= \frac{1-\delta}{\theta+5} + \frac{\theta+3\delta}{\theta+5} \left(\frac{3-\delta}{\theta+8} + \frac{\theta+4\delta}{\theta+8} \frac{1}{V} \right) \tag{5}$$

(6)

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$$\tag{6}$$

0.23

a following b

$$= \frac{2-\delta}{\theta+3} + \frac{\theta+2\delta}{\theta+3} p(\mathtt{a}) \tag{7}$$

$$= \frac{2-\delta}{\theta+3} + \frac{\theta+2\delta}{\theta+3} \left(\frac{3-\delta}{\theta+8} + \frac{\theta+4\delta}{\theta+8} \frac{1}{V} \right) \tag{8}$$

(9)

$\mathtt{a}$ following $\mathtt{b}$

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(9)

$\mathtt{a}$ following $\mathtt{b}$

$$
\begin{aligned}
&= \frac{2-\delta}{\theta+3} + \frac{\theta+2\delta}{\theta+3} p(\mathtt{a}) && (7)\\
&= \frac{2-\delta}{\theta+3} + \frac{\theta+2\delta}{\theta+3} \left(\frac{3-\delta}{\theta+8} + \frac{\theta+4\delta}{\theta+8} \frac{1}{V} \right) && (8)\\
& && (9)
\end{aligned}
$$

0.55

</s> following <s>

$$= \frac{\theta + 2\delta}{\theta + 2} p(</s>) \tag{10}$$

$$= \frac{\theta + 2\delta}{\theta + 2} \left(\frac{1 - \delta}{\theta + 8} + \frac{\theta + 4\delta}{\theta + 8} \frac{1}{V} \right) \tag{11}$$

(12)

</s> following <s>

$$= \frac{\theta + 2\delta}{\theta + 2} p(</\text{s}>) \tag{10}$$

$$= \frac{\theta + 2\delta}{\theta + 2} \left(\frac{1 - \delta}{\theta + 8} + \frac{\theta + 4\delta}{\theta + 8} \frac{1}{V} \right) \tag{11}$$

(12)

</s> following <s>

$$= \frac{\theta + 2\delta}{\theta + 2} p(\texttt{</s>}) \tag{10}$$

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$$\tag{12}$$

0.08