



# Language Models

Computational Linguistics: Jordan Boyd-Graber  
University of Maryland  
INTRODUCTION

Slides adapted from Philip Koehn

## Roadmap

After this class, you'll be able to:

- Understand probability distributions through the metaphor of the Chinese Restaurant Process
- Be able to calculate Kneser-Ney smoothing
- Understand the role of contexts in language models

## Intuition

- Some words are “sticky”
- “San Francisco” is very common (high ungram)
- But Francisco only appears after one word

## Intuition

- Some words are “sticky”
- “San Francisco” is very common (high ungram)
- But Francisco only appears after one word
- Our goal: to tell a statistical story of bay area restaurants to account for this phenomenon
- How to model this phenomena

## Interpolation

- Higher and lower order  $n$ -gram models have different strengths and weaknesses
  - high-order  $n$ -grams are sensitive to more context, but have sparse counts
  - low-order  $n$ -grams consider only very limited context, but have robust counts
- Combine them

$$\begin{aligned} p_I(w_3 | w_1, w_2) = & \lambda_1 p_1(w_3) \\ & + \lambda_2 p_2(w_3 | w_2) \\ & + \lambda_3 p_3(w_3 | w_1, w_2) \end{aligned}$$

## Back-Off

- Trust the highest order language model that contains n-gram

$$p_n^{BO}(w_i | w_{i-n+1}, \dots, w_{i-1}) = \begin{cases} \alpha_n(w_i | w_{i-n+1}, \dots, w_{i-1}) & \text{if } \text{count}_n(w_{i-n+1}, \dots, w_i) > 0 \\ d_n(w_{i-n+1}, \dots, w_{i-1}) p_{n-1}^{BO}(w_i | w_{i-n+2}, \dots, w_{i-1}) & \text{else} \end{cases}$$

- Requires
  - adjusted prediction model  $\alpha_n(w_i | w_{i-n+1}, \dots, w_{i-1})$
  - discounting function  $d_n(w_1, \dots, w_{n-1})$

## Let's remember what a language model is

- It is a distribution over the next word in a sentence
- Given the previous  $n - 1$  words

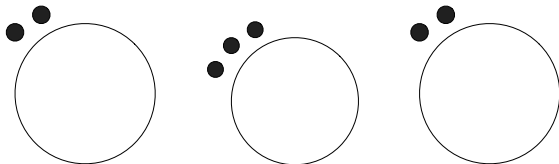
## Let's remember what a language model is

- It is a distribution over the next word in a sentence
- Given the previous  $n - 1$  words
- The challenge: backoff and sparsity



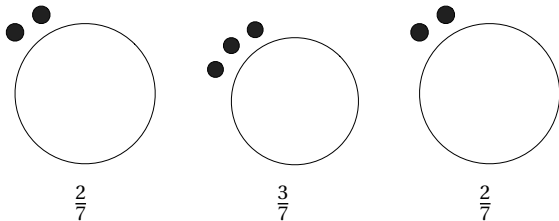
## The Chinese Restaurant as a Distribution

To generate a word, you first sit down at a table. You sit down at a table proportional to the number of people sitting at the table.



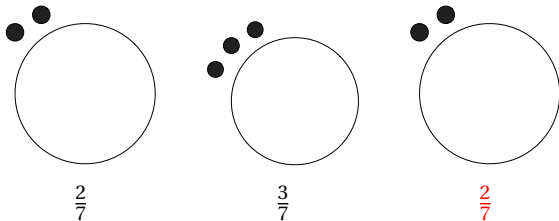
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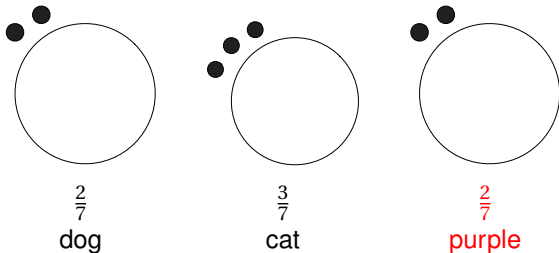
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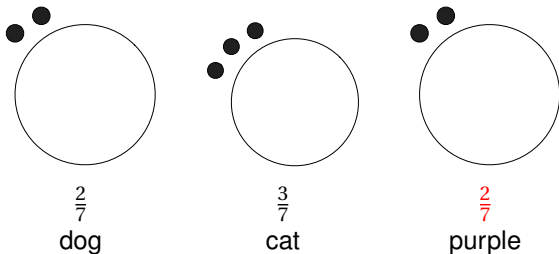
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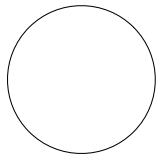
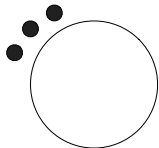
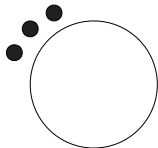
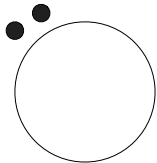
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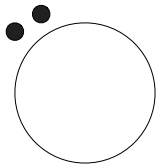
But this is just Maximum Likelihood

Why are we talking about Chinese Restaurants?

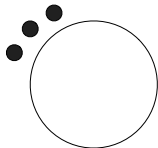
Always one more table ...



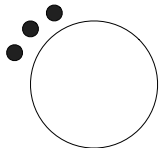
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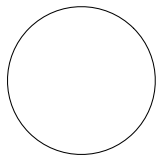
$$\frac{2}{7+\alpha}$$



$$\frac{3}{7+\alpha}$$

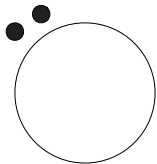


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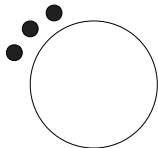


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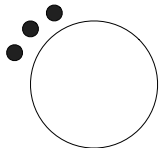
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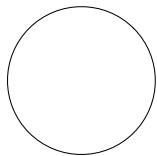
$\frac{2}{7+\alpha}$   
dog



$\frac{3}{7+\alpha}$   
cat



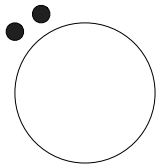
$\frac{2}{7+\alpha}$   
purple



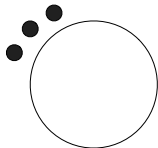
$\frac{\alpha}{7+\alpha}$   
???



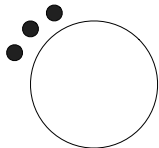
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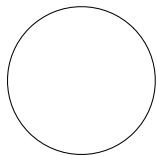
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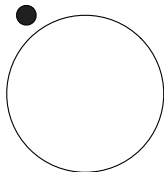
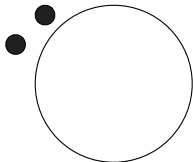
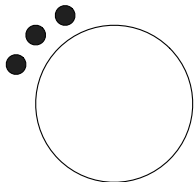


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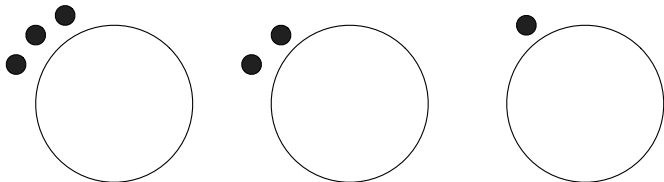


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## What to do with a new table?



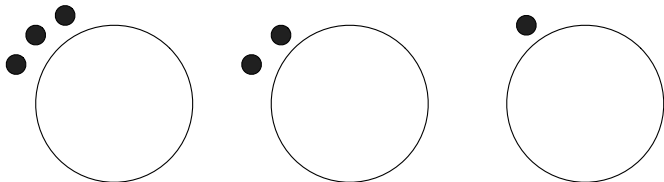
## What to do with a new table?



## What can be a base distribution?

- Uniform (Dirichlet smoothing)

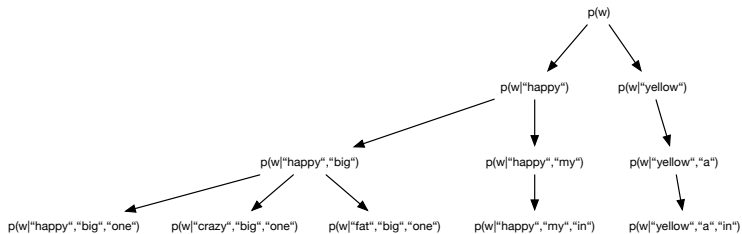
## What to do with a new table?



## What can be a base distribution?

- Uniform (Dirichlet smoothing)
- Specific contexts  $\rightarrow$  less-specific contexts (backoff)

## A hierarchy of Chinese Restaurants



## Seating Assignments

**Dataset:**

<s> a a a b a c </s>

## Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

<s> Restaurant

a Restaurant

b Restaurant

c Restaurant

## Seating Assignments

Dataset:

<s> a a a b a c </s>

Unigram Restaurant

<s> Restaurant

\*

a Restaurant

c Restaurant

b Restaurant



## Seating Assignments

Dataset:

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Unigram Restaurant

\*<sup>1</sup>

<s> Restaurant

\*<sup>1</sup>

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c Restaurant

b Restaurant

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b Restaurant

c Restaurant

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Dataset:

<s> a a a b a c </s>

Unigram Restaurant

a<sup>2</sup>

<s> Restaurant

a<sup>1</sup>

a Restaurant

a<sup>1</sup>

b Restaurant

c Restaurant

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<s> Restaurant

a<sup>1</sup>

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a<sup>1</sup>

b Restaurant

c Restaurant

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<s> a a a b a c </s>

Unigram Restaurant

a<sup>2</sup>

<s> Restaurant

a<sup>1</sup>

a Restaurant

a<sup>2</sup>

b Restaurant

c Restaurant

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<s> a a a b a c </s>

Unigram Restaurant

a<sup>2</sup>

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a<sup>2</sup> \*<sup>1</sup>

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a<sup>1</sup>

### b Restaurant

\*

### a Restaurant

a<sup>2</sup> b<sup>1</sup>

### c Restaurant

## Seating Assignments

Dataset:

<s> a a a b a c </s>

### Unigram Restaurant

a<sup>3</sup> b<sup>1</sup>

### <s> Restaurant

a<sup>1</sup>

### b Restaurant

a<sup>1</sup>

### a Restaurant

a<sup>2</sup> b<sup>1</sup>

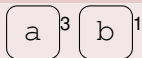
### c Restaurant

## Seating Assignments

Dataset:

<s> a a a b a c </s>

### Unigram Restaurant



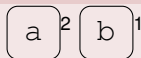
### <s> Restaurant



### b Restaurant



### a Restaurant



### c Restaurant

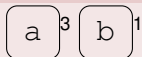


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Dataset:

<s> a a a b a c </s>

### Unigram Restaurant



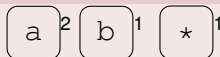
### <s> Restaurant



### b Restaurant



### a Restaurant



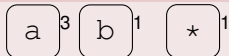
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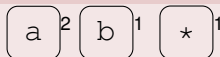
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### b Restaurant



### c Restaurant



## Seating Assignments

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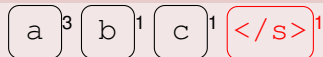


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### <s> Restaurant



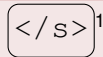
### a Restaurant



### b Restaurant



### c Restaurant



## Real examples

- San Francisco



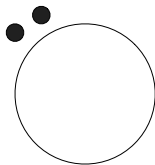
## Real examples

- San Francisco
- Star Spangled Banner

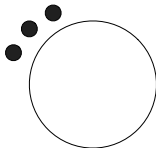
## Real examples

- San Francisco
- Star Spangled Banner
- Bottom Line: Counts go to the context that explains it best

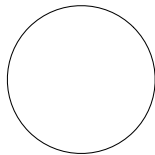
## The rich get richer



$$\frac{2}{5+\theta}$$



$$\frac{3}{5+\theta}$$



$$\frac{\theta}{5+\theta}$$

## Computing the Probability of an Observation

$$p(w = x | \vec{s}, \theta, u) = \underbrace{\frac{c_{u,x}}{\theta + c_{u,\cdot}}}_{\text{existing table}} + \underbrace{\frac{\theta}{\theta + c_{u,\cdot}} p(w = x | \vec{s}, \theta, \pi(u))}_{\text{new table}} \quad (1)$$

- Word type  $x$
- Seating assignments  $\vec{s}$
- Concentration  $\theta$
- Context  $u$
- Number seated at table serving  $x$  in restaurant  $u$ ,  $c_{u,x}$
- Number seated at all tables in restaurant  $u$ ,  $c_{u,\cdot}$
- The backoff context  $\pi(u)$

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**Example:**  $p(w = \mathbf{b} | \vec{s}, \theta = 1.0, u = \mathbf{a})$

### Unigram Restaurant

a<sup>3</sup> b<sup>1</sup> c<sup>1</sup> </s><sup>1</sup>

### <s> Restaurant

a<sup>1</sup>

### a Restaurant

a<sup>2</sup> b<sup>1</sup> c<sup>1</sup>

### b Restaurant

a<sup>1</sup>

### c Restaurant

</s><sup>1</sup>

$$p(w = \mathbf{b} | \dots) = \frac{c_{\mathbf{a}, \mathbf{b}}}{\theta + c_{u, \cdot}} + \frac{\theta}{\theta + c_{u, \cdot}} p(w = x | \vec{s}, \theta, \pi(u)) \quad (2)$$

**Example:**  $p(w = \mathbf{b} | \vec{s}, \theta = 1.0, u = \mathbf{a})$

### Unigram Restaurant

a<sup>3</sup> b<sup>1</sup> c<sup>1</sup> </s><sup>1</sup>

### <s> Restaurant

a<sup>1</sup>

### a Restaurant

a<sup>2</sup> b<sup>1</sup> c<sup>1</sup>

### b Restaurant

a<sup>1</sup>

### c Restaurant

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a<sup>1</sup>

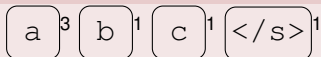
### c Restaurant

</s><sup>1</sup>

$$p(w = \mathbf{b} | \dots) = \frac{1}{\theta + c_{u,\cdot}} + \frac{\theta}{\theta + c_{u,\cdot}} p(w = x | \vec{s}, \theta, \pi(u)) \quad (2)$$

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### Unigram Restaurant



### <s> Restaurant



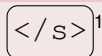
### a Restaurant



### b Restaurant



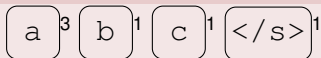
### c Restaurant



$$p(w = \mathbf{b} | \dots) = \frac{1}{1.0 + c_{u,\cdot}} + \frac{1.0}{1.0 + c_{u,\cdot}} p(w = x | \vec{s}, \theta, \pi(u)) \quad (2)$$

**Example:**  $p(w = \mathbf{b} | \vec{s}, \theta = 1.0, u = \mathbf{a})$

### Unigram Restaurant



### <s> Restaurant



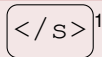
### a Restaurant



### b Restaurant



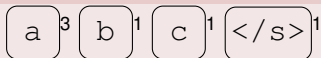
### c Restaurant



$$p(w = \mathbf{b} | \dots) = \frac{1}{1.0 + 4} + \frac{1.0}{1.0 + 4} p(w = x | \vec{s}, \theta, \pi(u)) \quad (2)$$

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### Unigram Restaurant



### $\langle /s \rangle$ Restaurant



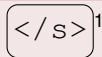
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### Unigram Restaurant

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a<sup>2</sup> b<sup>1</sup> c<sup>1</sup>

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$$p(w = \mathbf{b} | \dots) = \frac{1}{1.0 + 4} + \frac{1.0}{1.0 + 4} p(w = \mathbf{x} | \vec{s}, \theta, \pi(\emptyset)) \quad (2)$$

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$$p(w = \mathbf{b} | \dots) = \frac{1}{5} + \frac{1}{5} \left( \frac{c_{\emptyset, \mathbf{b}}}{c_{\emptyset, \cdot} + \theta} + \frac{\theta}{c_{\emptyset, \cdot} + \theta} \frac{1}{V} \right) \quad (2)$$

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### b Restaurant

a<sup>1</sup>

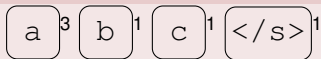
### c Restaurant

</s><sup>1</sup>

$$p(w = \mathbf{b} | \dots) = \frac{1}{5} + \frac{1}{5} \left( \frac{1}{c_{\emptyset, \cdot} + 1.0} + \frac{1.0}{c_{\emptyset, \cdot} + 1.0} \frac{1}{5} \right) \quad (2)$$

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### Unigram Restaurant



### <s> Restaurant



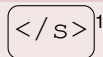
### a Restaurant



### b Restaurant



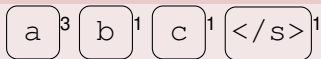
### c Restaurant



$$p(w = \mathbf{b} | \dots) = \frac{1}{5} + \frac{1}{5} \left( \frac{1}{6 + 1.0} + \frac{1.0}{6 + 1.0} \frac{1}{5} \right) \quad (2)$$

**Example:**  $p(w = \mathbf{b} | \vec{s}, \theta = 1.0, u = \mathbf{a})$

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### <s> Restaurant



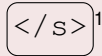
### a Restaurant



### b Restaurant



### c Restaurant



$$p(w = \mathbf{b} | \dots) = \frac{1}{5} + \frac{1}{5} \left( \frac{1}{7} + \frac{1}{7 \cdot 5} \right) = 0.24 \quad (2)$$



## Discounting

- Empirically, it helps favor the backoff if you have more tables
- Otherwise, it gets too close to maximum likelihood
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## Interpolated Kneser-Ney!

## More advanced models

- Interpolated Kneser-Ney assumes **one table with a dish (word)** per restaurant
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- Neural language models . . .