



# Representation Learning

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UPDATE EXAMPLES

## Dataset

- Two types of words
  - Vehicles
  - Fruits
- Learn a representation with two dimensions
- Word2Vec skipgram negative sampling
- $\alpha = 1.0$  (bad choice in practice!)
- We'll do update for one positive and one negative sample
  - Note: much of word2vec magic is sampling negative words, you'll have to take my word for it

## Word

ambulance	-0.228	0.099
apple	0.078	0.217
backhoe	-0.086	0.138
banana	0.046	0.195
crane	-0.220	0.153
firetruck	0.039	-0.047
lemon	0.008	-0.043
strawberry	0.202	-0.081

## Context

ambulance	0.000	0.000
apple	0.000	0.000
backhoe	0.000	0.000
banana	0.000	0.000
crane	0.000	0.000
firetruck	0.000	0.000
lemon	0.000	0.000
strawberry	0.000	0.000

$$z = \mathbf{w}_{\text{focus}}^{\top} \cdot \mathbf{c}_s \quad (1)$$

$$E_s = \begin{cases} 1 - \sigma(z), & \text{if } s \text{ positive example} \\ 0 - \sigma(z), & \text{if } s \text{ negative example} \end{cases} \quad (2)$$

$$\Delta \vec{w}_{\text{focus}} = \alpha E_s \vec{c}_s \quad (3)$$

$$\Delta \vec{c}_s = \alpha E_s \vec{w}_{\text{focus}} \quad (4)$$

$$\alpha = 0.1$$

## POS (focus: banana, sample: lemon)

## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot c_{\text{lemon}}$

## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot c_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000$

## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot c_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$

## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot c_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 1.0 - \pi = 1.0 - \sigma(0.000) =$



## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot c_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 1.0 - \pi = 1.0 - \sigma(0.000) = 0.500$

## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot q_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 1.0 - \pi = 1.0 - \sigma(0.000) = 0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot q_{\text{lemon}} =$

## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot q_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 1.0 - \pi = 1.0 - \sigma(0.000) = 0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot q_{\text{lemon}} = 0.10 \cdot 0.500 \cdot (0.000, 0.000) =$

## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot q_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 1.0 - \pi = 1.0 - \sigma(0.000) = 0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot q_{\text{lemon}} = 0.10 \cdot 0.500 \cdot (0.000, 0.000) = (0.000, 0.000)$

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- $z = w_{\text{banana}}^T \cdot q_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 1.0 - \pi = 1.0 - \sigma(0.000) = 0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot q_{\text{lemon}} = 0.10 \cdot 0.500 \cdot (0.000, 0.000) = (0.000, 0.000)$
- $\Delta q_{\text{lemon}} = \alpha e \cdot w_{\text{banana}} =$

## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot q_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 1.0 - \pi = 1.0 - \sigma(0.000) = 0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot q_{\text{lemon}} = 0.10 \cdot 0.500 \cdot (0.000, 0.000) = (0.000, 0.000)$
- $\Delta q_{\text{lemon}} = \alpha e \cdot w_{\text{banana}} = 0.10 \cdot 0.500 \cdot (0.046, 0.195) =$

## POS (focus: banana, sample: lemon)

- $z = w_{\text{banana}}^T \cdot q_{\text{lemon}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 1.0 - \pi = 1.0 - \sigma(0.000) = 0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot q_{\text{lemon}} = 0.10 \cdot 0.500 \cdot (0.000, 0.000) = (0.000, 0.000)$
- $\Delta q_{\text{lemon}} = \alpha e \cdot w_{\text{banana}} = 0.10 \cdot 0.500 \cdot (0.046, 0.195) = (0.002, 0.010)$

## NEG (focus: banana, sample: firetruck)

- $z = w_{\text{banana}}^T \cdot c_{\text{firetruck}}$



## NEG (focus: banana, sample: firetruck)

- $z = w_{\text{banana}}^T \cdot c_{\text{firetruck}} = 0.046 * 0.000 + 0.195 * 0.000$

## NEG (focus: banana, sample: firetruck)

- $z = w_{\text{banana}}^T \cdot c_{\text{firetruck}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$

## NEG (focus: banana, sample: firetruck)

- $z = w_{\text{banana}}^{\top} \cdot c_{\text{firetruck}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 0.0 - \pi = 0.0 - \sigma(0.000) =$

## NEG (focus: banana, sample: firetruck)

- $z = w_{\text{banana}}^T \cdot c_{\text{firetruck}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 0.0 - \pi = 0.0 - \sigma(0.000) = -0.500$

## NEG (focus: banana, sample: firetruck)

- $z = w_{\text{banana}}^T \cdot c_{\text{firetruck}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 0.0 - \pi = 0.0 - \sigma(0.000) = -0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot c_{\text{firetruck}} =$

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- $\Delta w_{\text{banana}} = \alpha e \cdot c_{\text{firetruck}} = 0.10 \cdot -0.500 \cdot (0.000, 0.000) =$

## NEG (focus: banana, sample: firetruck)

- $z = \mathbf{w}_{\text{banana}}^T \cdot \mathbf{c}_{\text{firetruck}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 0.0 - \pi = 0.0 - \sigma(0.000) = -0.500$
- $\Delta \mathbf{w}_{\text{banana}} = \alpha e \cdot \mathbf{c}_{\text{firetruck}} = 0.10 \cdot -0.500 \cdot (0.000, 0.000) = (-0.000, -0.000)$

## NEG (focus: banana, sample: firetruck)

- $z = w_{\text{banana}}^T \cdot c_{\text{firetruck}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 0.0 - \pi = 0.0 - \sigma(0.000) = -0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot c_{\text{firetruck}} = 0.10 \cdot -0.500 \cdot (0.000, 0.000) = (-0.000, -0.000)$
- $\Delta c_{\text{firetruck}} = \alpha e \cdot w_{\text{banana}} =$



## NEG (focus: banana, sample: firetruck)

- $z = w_{\text{banana}}^T \cdot c_{\text{firetruck}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 0.0 - \pi = 0.0 - \sigma(0.000) = -0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot c_{\text{firetruck}} = 0.10 \cdot -0.500 \cdot (0.000, 0.000) = (-0.000, -0.000)$
- $\Delta c_{\text{firetruck}} = \alpha e \cdot w_{\text{banana}} = 0.10 \cdot -0.500 \cdot (0.046, 0.195) =$

## NEG (focus: banana, sample: firetruck)

- $z = w_{\text{banana}}^T \cdot c_{\text{firetruck}} = 0.046 * 0.000 + 0.195 * 0.000 = 0.000$
- $e = 0.0 - \pi = 0.0 - \sigma(0.000) = -0.500$
- $\Delta w_{\text{banana}} = \alpha e \cdot c_{\text{firetruck}} = 0.10 \cdot -0.500 \cdot (0.000, 0.000) = (-0.000, -0.000)$
- $\Delta c_{\text{firetruck}} = \alpha e \cdot w_{\text{banana}} = 0.10 \cdot -0.500 \cdot (0.046, 0.195) = (-0.002, -0.010)$

## Word

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strawberry	0.202	-0.081

## Context

ambulance	0.000	0.000
apple	0.000	0.000
backhoe	-0.002	-0.010
banana	0.000	0.000
crane	0.000	0.000
firetruck	-0.002	-0.010
lemon	0.005	0.019
strawberry	0.000	0.000

$$\alpha = 0.1$$

**Much later ...**

Vectors are starting to take shape

## Word

ambulance	-0.906	0.107
apple	0.992	0.780
backhoe	-0.902	0.459
banana	1.286	0.573
crane	-1.119	0.399
firetruck	-0.830	0.094
lemon	0.750	-0.289
strawberry	1.174	-0.379

## Context

ambulance	-0.927	-0.090
apple	0.973	-0.923
backhoe	-0.984	-0.379
banana	0.634	-0.486
crane	-1.258	-0.188
firetruck	-1.224	-0.060
lemon	1.087	-0.081
strawberry	1.054	0.410

$$z = \mathbf{w}_{\text{focus}}^T \cdot \mathbf{c}_s \quad (5)$$

$$E_s = \begin{cases} 1 - \sigma(z), & \text{if } s \text{ positive example} \\ 0 - \sigma(z), & \text{if } s \text{ negative example} \end{cases} \quad (6)$$

$$\Delta \vec{w}_{\text{focus}} = \alpha E_s \vec{c}_s \quad (7)$$

$$\Delta \vec{c}_s = \alpha E_s \vec{w}_{\text{focus}} \quad (8)$$

$$\alpha = 0.1$$

## POS (focus: firetruck, sample: backhoe)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{backhoe}}$

## POS (focus: firetruck, sample: backhoe)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379$

## POS (focus: firetruck, sample: backhoe)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379 = 0.780$



## POS (focus: firetruck, sample: backhoe)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379 = 0.780$
- $e = 1.0 - \pi = 1.0 - \sigma(0.780) =$

## POS (focus: firetruck, sample: backhoe)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379 = 0.780$
- $e = 1.0 - \pi = 1.0 - \sigma(0.780) = 0.314$

## POS (focus: firetruck, sample: backhoe)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379 = 0.780$
- $e = 1.0 - \pi = 1.0 - \sigma(0.780) = 0.314$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{backhoe}} =$

## POS (focus: firetruck, sample: backhoe)

- $z = \mathbf{w}_{\text{firetruck}}^T \cdot \mathbf{c}_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379 = 0.780$
- $e = 1.0 - \pi = 1.0 - \sigma(0.780) = 0.314$
- $\Delta \mathbf{w}_{\text{firetruck}} = \alpha e \cdot \mathbf{c}_{\text{backhoe}} = 0.10 \cdot 0.314 \cdot (-0.984, -0.379) =$

## POS (focus: firetruck, sample: backhoe)

- $z = \mathbf{w}_{\text{firetruck}}^T \cdot \mathbf{c}_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379 = 0.780$
- $e = 1.0 - \pi = 1.0 - \sigma(0.780) = 0.314$
- $\Delta \mathbf{w}_{\text{firetruck}} = \alpha e \cdot \mathbf{c}_{\text{backhoe}} = 0.10 \cdot 0.314 \cdot (-0.984, -0.379) = (-0.031, -0.012)$

## POS (focus: firetruck, sample: backhoe)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379 = 0.780$
- $e = 1.0 - \pi = 1.0 - \sigma(0.780) = 0.314$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{backhoe}} = 0.10 \cdot 0.314 \cdot (-0.984, -0.379) = (-0.031, -0.012)$
- $\Delta c_{\text{backhoe}} = \alpha e \cdot w_{\text{firetruck}} =$

## POS (focus: firetruck, sample: backhoe)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379 = 0.780$
- $e = 1.0 - \pi = 1.0 - \sigma(0.780) = 0.314$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{backhoe}} = 0.10 \cdot 0.314 \cdot (-0.984, -0.379) = (-0.031, -0.012)$
- $\Delta c_{\text{backhoe}} = \alpha e \cdot w_{\text{firetruck}} = 0.10 \cdot 0.314 \cdot (-0.830, 0.094) =$

## POS (focus: firetruck, sample: backhoe)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{backhoe}} = -0.830 * -0.984 + 0.094 * -0.379 = 0.780$
- $e = 1.0 - \pi = 1.0 - \sigma(0.780) = 0.314$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{backhoe}} = 0.10 \cdot 0.314 \cdot (-0.984, -0.379) = (-0.031, -0.012)$
- $\Delta c_{\text{backhoe}} = \alpha e \cdot w_{\text{firetruck}} = 0.10 \cdot 0.314 \cdot (-0.830, 0.094) = (-0.026, 0.003)$



## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}}$

## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188$

## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188 = 1.025$

## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188 = 1.025$
- $e = 0.0 - \pi = 0.0 - \sigma(1.025) =$

## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^{\top} \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188 = 1.025$
- $e = 0.0 - \pi = 0.0 - \sigma(1.025) = -0.736$

## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188 = 1.025$
- $e = 0.0 - \pi = 0.0 - \sigma(1.025) = -0.736$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{crane}} =$

## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188 = 1.025$
- $e = 0.0 - \pi = 0.0 - \sigma(1.025) = -0.736$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{crane}} = 0.10 \cdot -0.736 \cdot (-1.258, -0.188) =$

## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188 = 1.025$
- $e = 0.0 - \pi = 0.0 - \sigma(1.025) = -0.736$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{crane}} = 0.10 \cdot -0.736 \cdot (-1.258, -0.188) = (0.093, 0.014)$



## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188 = 1.025$
- $e = 0.0 - \pi = 0.0 - \sigma(1.025) = -0.736$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{crane}} = 0.10 \cdot -0.736 \cdot (-1.258, -0.188) = (0.093, 0.014)$
- $\Delta c_{\text{crane}} = \alpha e \cdot w_{\text{firetruck}} =$

## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188 = 1.025$
- $e = 0.0 - \pi = 0.0 - \sigma(1.025) = -0.736$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{crane}} = 0.10 \cdot -0.736 \cdot (-1.258, -0.188) = (0.093, 0.014)$
- $\Delta c_{\text{crane}} = \alpha e \cdot w_{\text{firetruck}} = 0.10 \cdot -0.736 \cdot (-0.830, 0.094) =$

## NEG (focus: firetruck, sample: crane)

- $z = w_{\text{firetruck}}^T \cdot c_{\text{crane}} = -0.830 * -1.258 + 0.094 * -0.188 = 1.025$
- $e = 0.0 - \pi = 0.0 - \sigma(1.025) = -0.736$
- $\Delta w_{\text{firetruck}} = \alpha e \cdot c_{\text{crane}} = 0.10 \cdot -0.736 \cdot (-1.258, -0.188) = (0.093, 0.014)$
- $\Delta c_{\text{crane}} = \alpha e \cdot w_{\text{firetruck}} = 0.10 \cdot -0.736 \cdot (-0.830, 0.094) = (0.061, -0.007)$

## Word

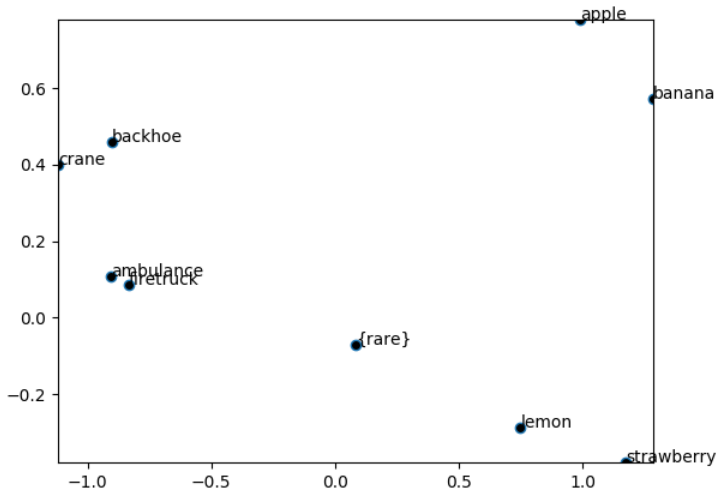
ambulance	-0.906	0.107
apple	0.992	0.780
backhoe	-0.902	0.459
banana	1.286	0.573
crane	-1.119	0.399
firetruck	-0.833	0.086
lemon	0.750	-0.289
strawberry	1.174	-0.379

## Context

ambulance	-0.927	-0.090
apple	0.973	-0.923
backhoe	-1.035	-0.373
banana	0.634	-0.486
crane	-1.196	-0.195
firetruck	-1.224	-0.060
lemon	1.110	-0.083
strawberry	1.054	0.410

$$\alpha = 0.1$$

## Word Vectors



## Context Vectors

