Adversarial Search

Hal Daumé III

Computer Science
University of Maryland

me@hal3.name

CS 421: Introduction to Artificial Intelligence

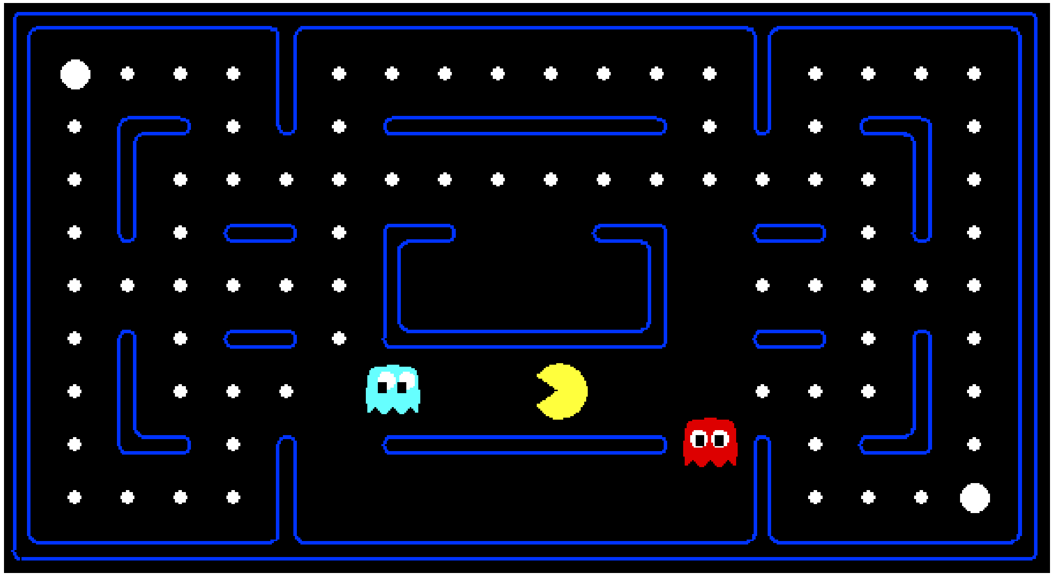
9 Feb 2012

Many slides courtesy of
Dan Klein, Stuart Russell,
or Andrew Moore

Announcements

- None

Adversarial Search



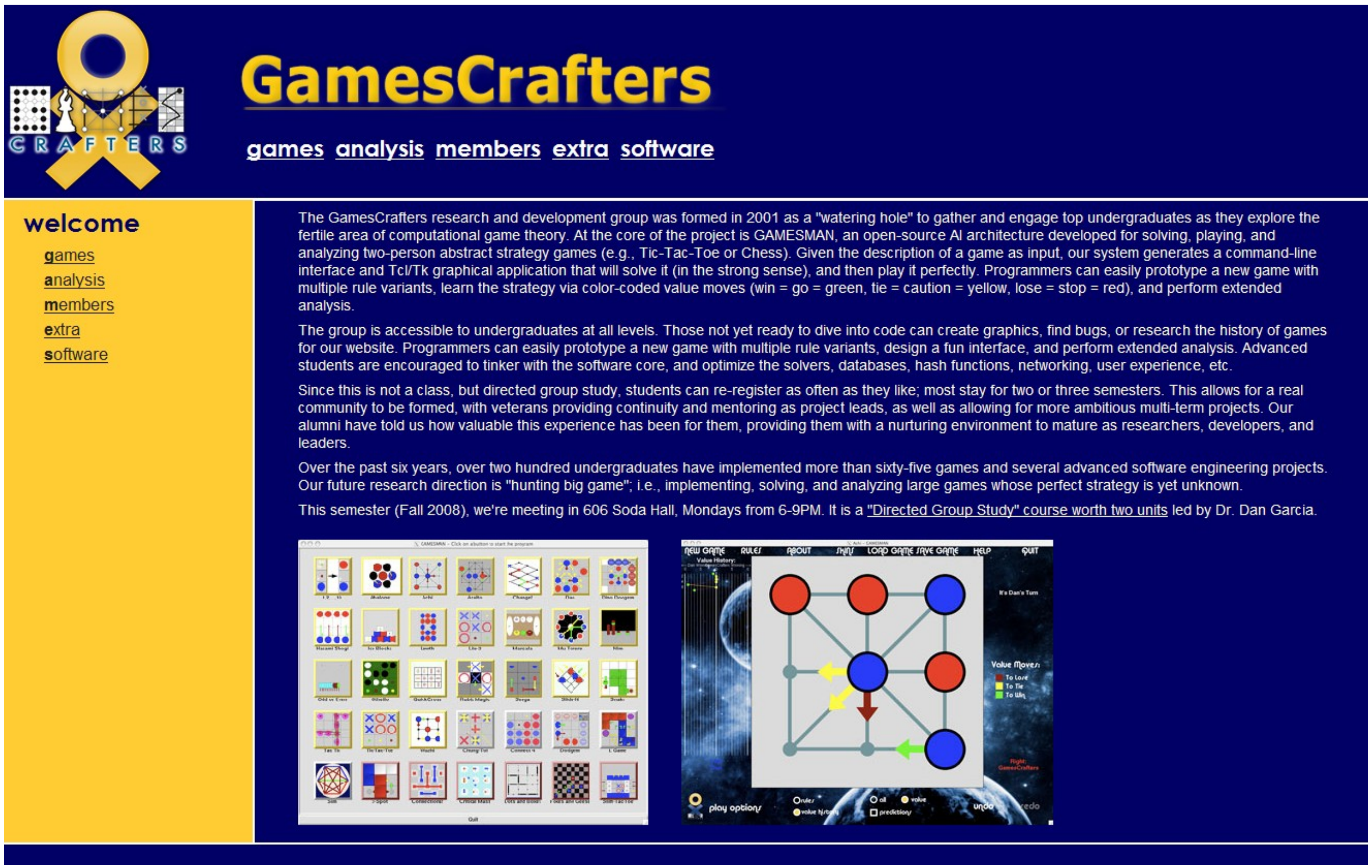
[DEMO: mystery pacman]

Game Playing State-of-the-Art

- **Checkers:** Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions. Checkers is now solved!
- **Chess:** Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue examined 200 million positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply.
- **Othello:** human champions refuse to compete against computers, which are too good.
- **Go:** human champions refuse to compete against computers, which are too bad. In go, b > 300, so most programs use pattern knowledge bases to suggest plausible moves.
- **Pacman:** unknown

GamesCrafters

http://gamescrafters.berkeley.edu/



Game Playing

- Many different kinds of games!
- Axes:
  - Deterministi
  - One, two or
  - Perfect infor
- Want algorit
  which recon

Examples?

Deterministic, 1 player, perfect information?
Deterministic, 1 player, imperfect information?

Deterministic, >1 player, perfect information?
Deterministic, >1 player, imperfect information?

Stochastic, 1 player, perfect information?
Stochastic, 1 player, imperfect information?

Stochastic, >1 player, perfect information?
Stochastic, >1 player, imperfect information?

http://u.hal3.name/ic.pl?q=game

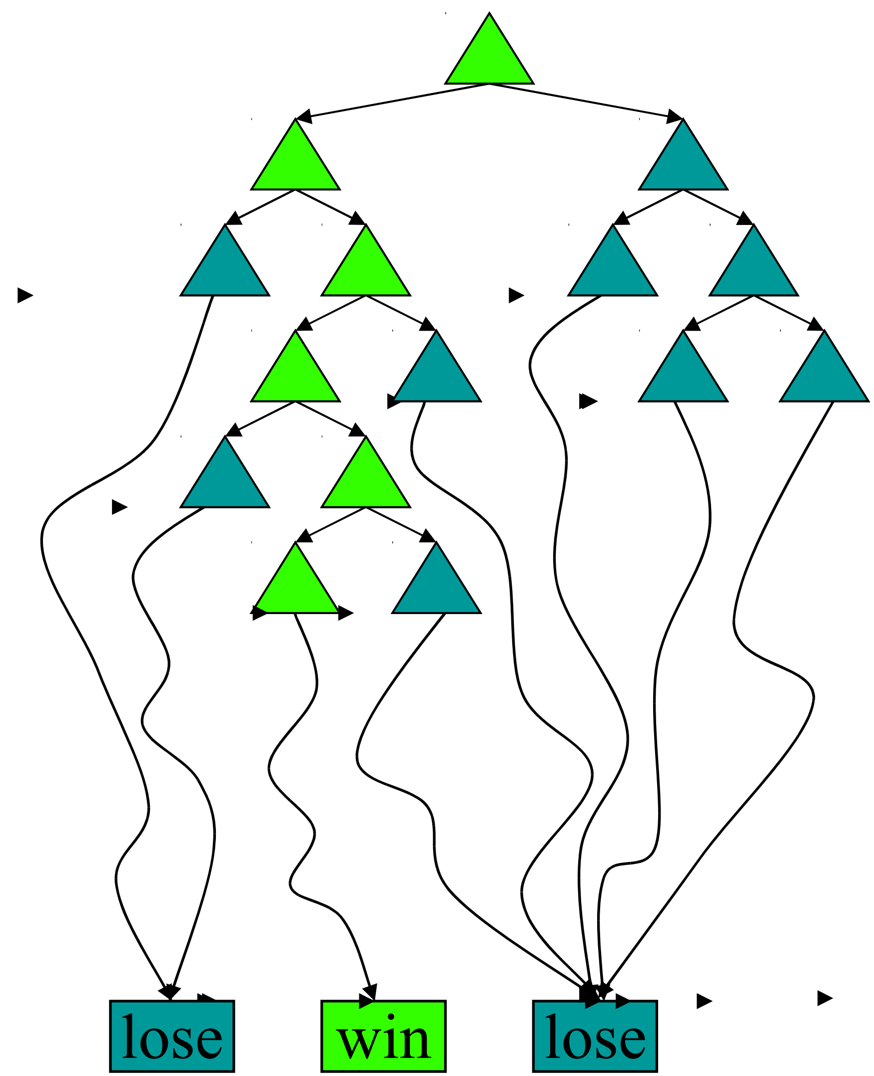
Deterministic Games

- Many possible formalizations, one is:
  - States: S (start at s_0)
  - Players: P={1...N} (usually take turns)
  - Actions: A (may depend on player / state)
  - Transition Function: SxA $\rightarrow$ S
  - Terminal Test: S $\rightarrow$ {t,f}
  - Terminal Utilities: SxP $\rightarrow$ R

- Solution for a player is a policy: S $\rightarrow$ A

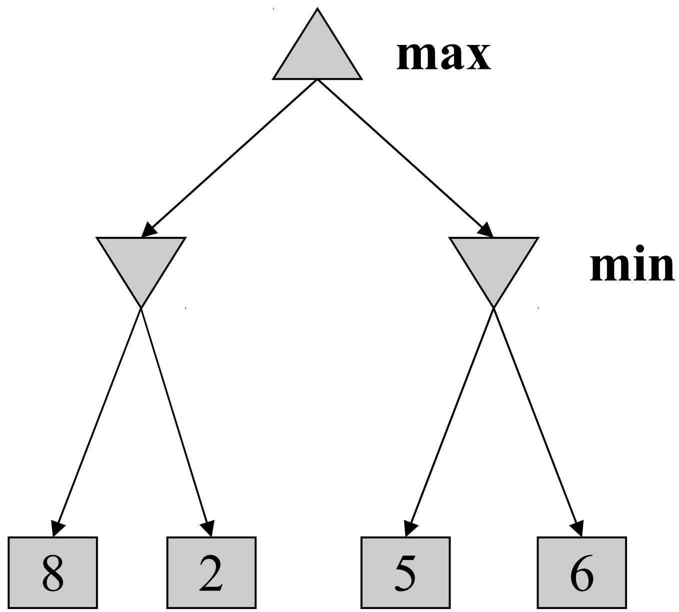
Deterministic Single-Player?

- Deterministic, single player, perfect information:
  - Know the rules
  - Know what actions do
  - Know when you win
  - E.g. Freecell, 8-Puzzle, Rubik's cube
- … it's just search!
- Slight reinterpretation:
  - Each node stores a value: the best outcome it can reach
  - This is the maximal outcome of its children
  - Note that we don't have path sums as before (utilities at end)
- After search, can pick move that leads to best node

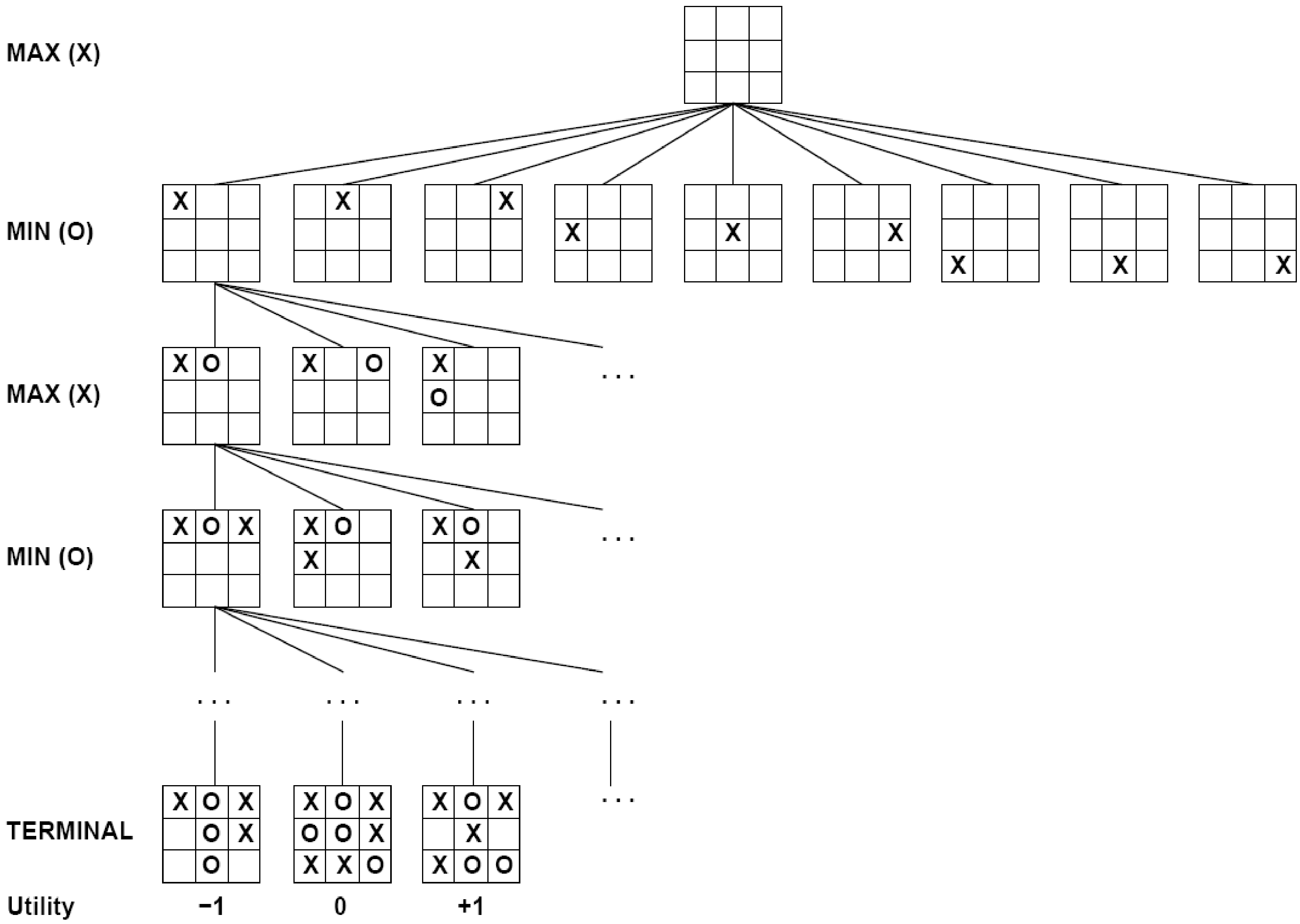


Deterministic Two-Player

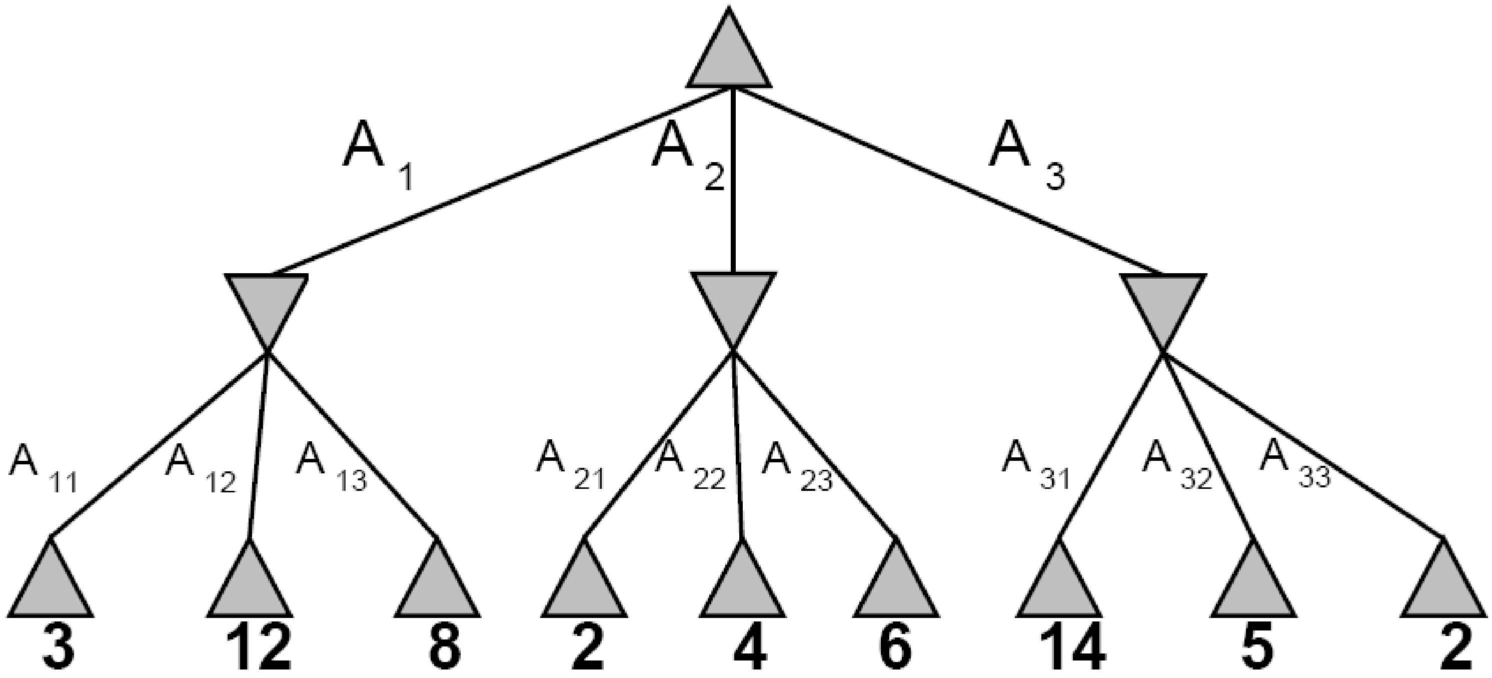
- E.g. tic-tac-toe, chess, checkers
- **Minimax search**
  - A state-space search tree
  - Players alternate
  - Each layer, or ply, consists of a round of moves
  - Choose move to position with highest minimax value = best achievable utility against best play
- Zero-sum games
  - One player maximizes result
  - The other minimizes result



Tic-tac-toe Game Tree



Minimax Example



Minimax Search

function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do** $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return v

---

function MIN-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
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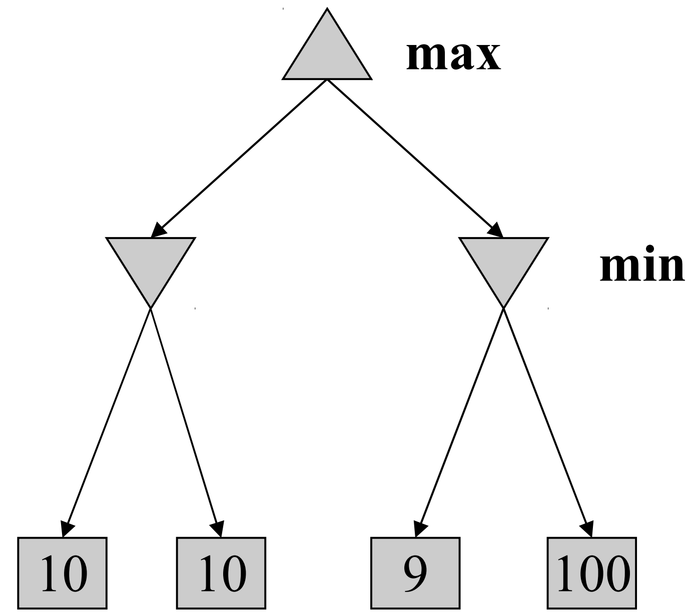
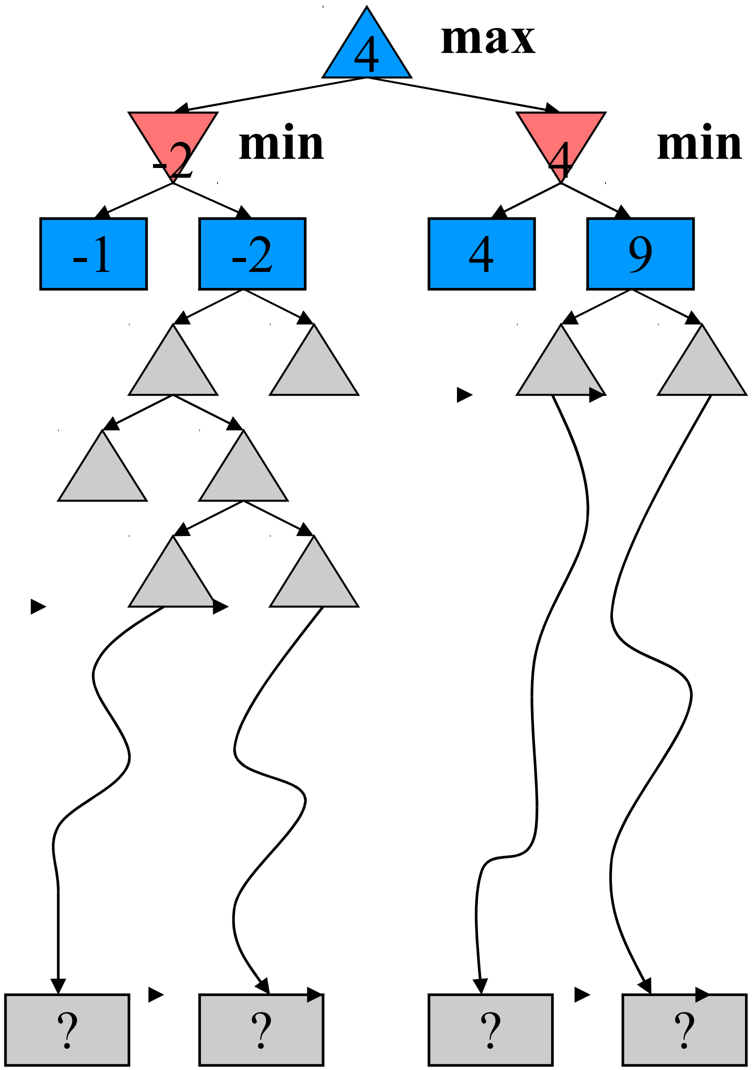
Minimax Properties

- Optimal against a perfect player. Otherwise?
- Time complexity?
  - $O(b^m)$
- Space complexity?
  - $O(bm)$
- For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



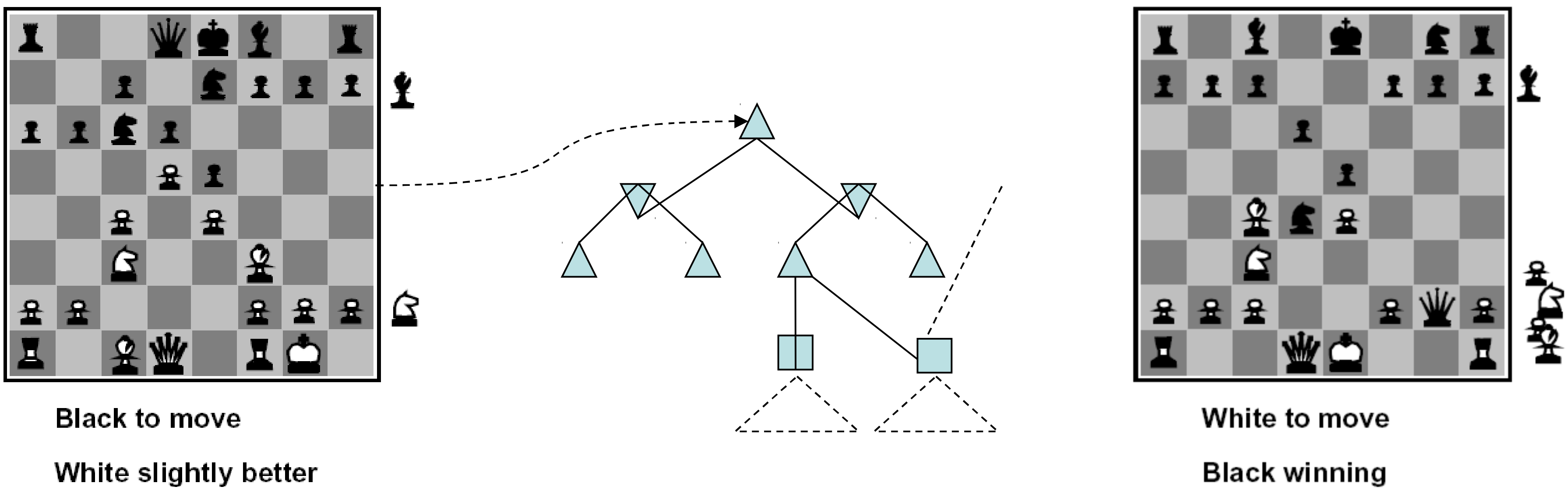
Resource Limits

- Cannot search to leaves
- Depth-limited search
  - Instead, search a limited depth of tree
  - Replace terminal utilities with an eval function for non-terminal positions
- Guarantee of optimal play is gone
- More plies makes a BIG difference
  - [DEMO: limitedDepth]
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program



Evaluation Functions

- Function which scores non-terminals

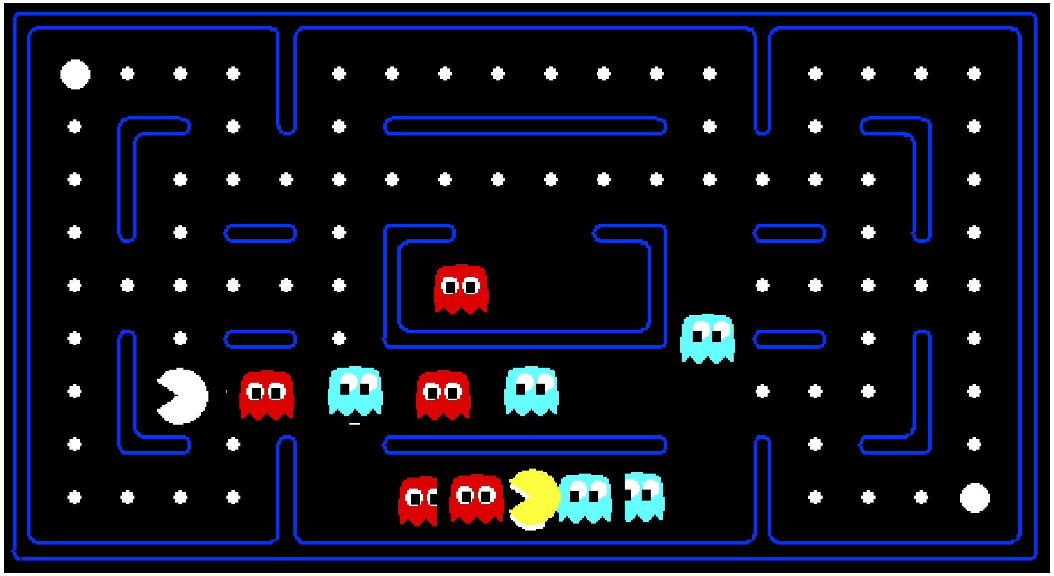


- Ideal function: returns the utility of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

Evaluation for Pacman



[DEMO: thrashing, smart ghosts]

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

Iterative Deepening

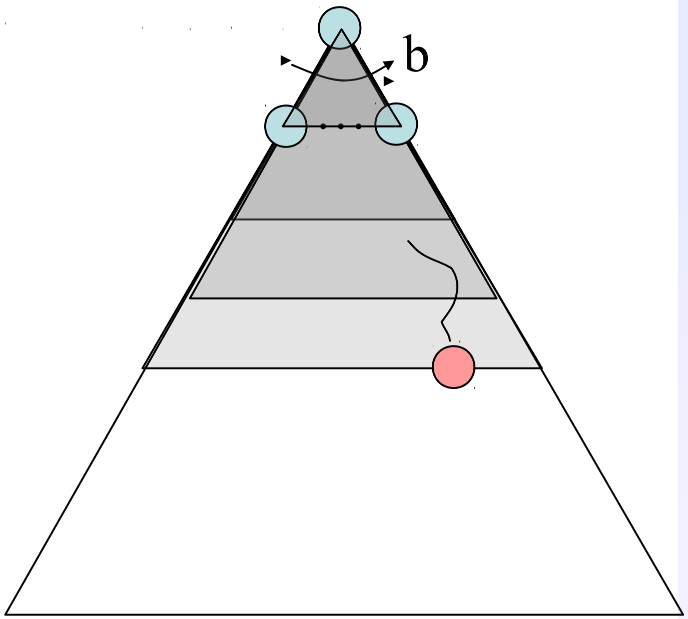
Iterative deepening uses DFS as a subroutine:

1. Do a DFS which only searches for paths of length 1 or less. (DFS gives up on any path of length 2)
2. If “1” failed, do a DFS which only searches paths of length 2 or less.
3. If “2” failed, do a DFS which only searches paths of length 3 or less.

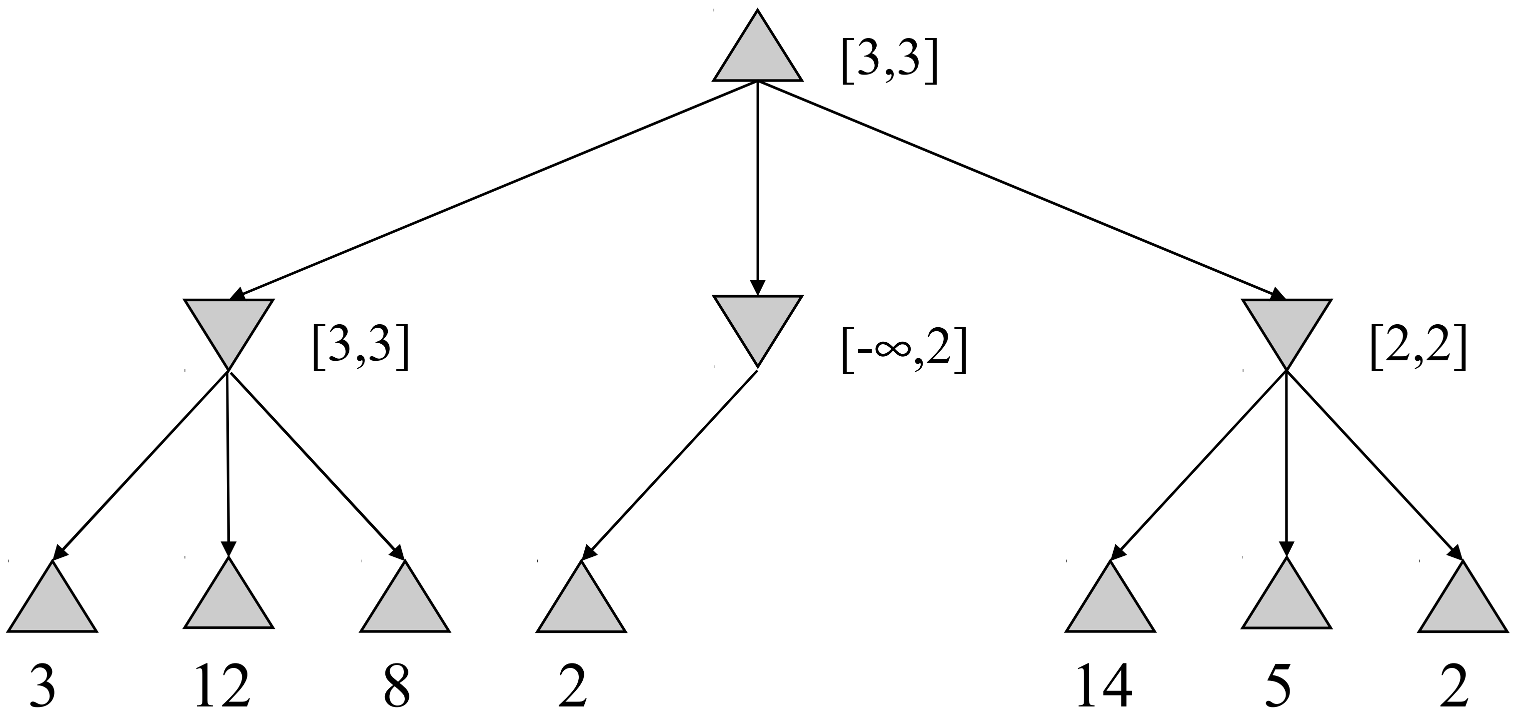
….and so on.

This works for single-agent search as well!

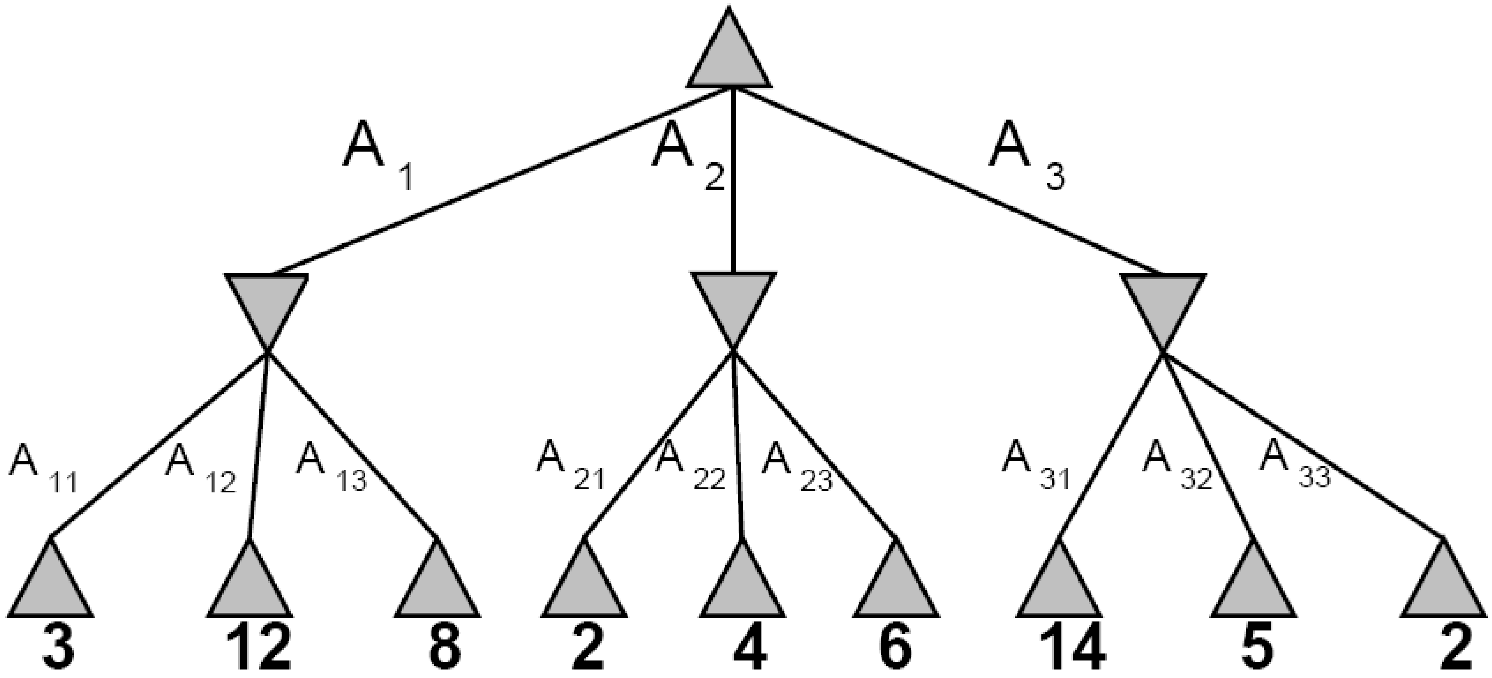
Why do we want to do this for multiplayer games?



Pruning in Minimax Search

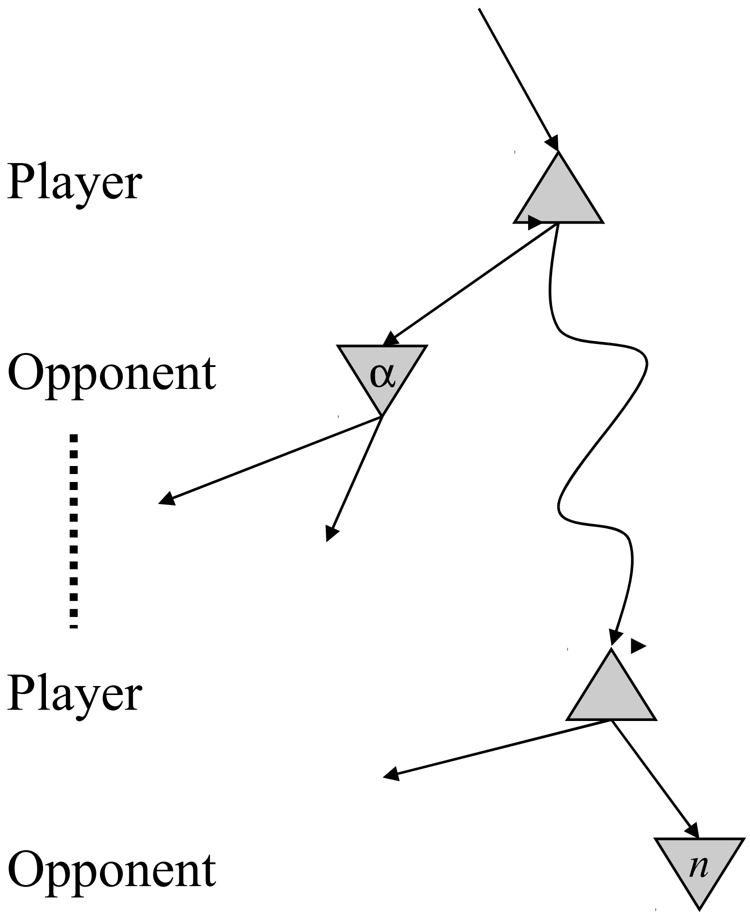


α-β Pruning Example



α-β Pruning

- General configuration
  - α is the best value that MAX can get at any choice point along the current path
  - If *n* becomes worse than α, MAX will avoid it, so can stop considering *n*'s other children
  - Define β similarly for MIN



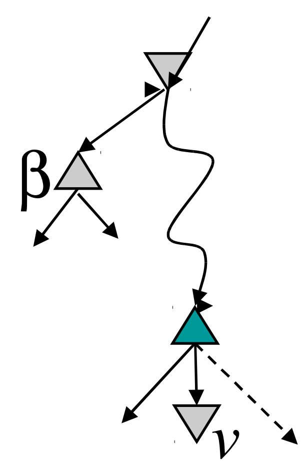
α-β Pruning Pseudocode

function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do** $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return v

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

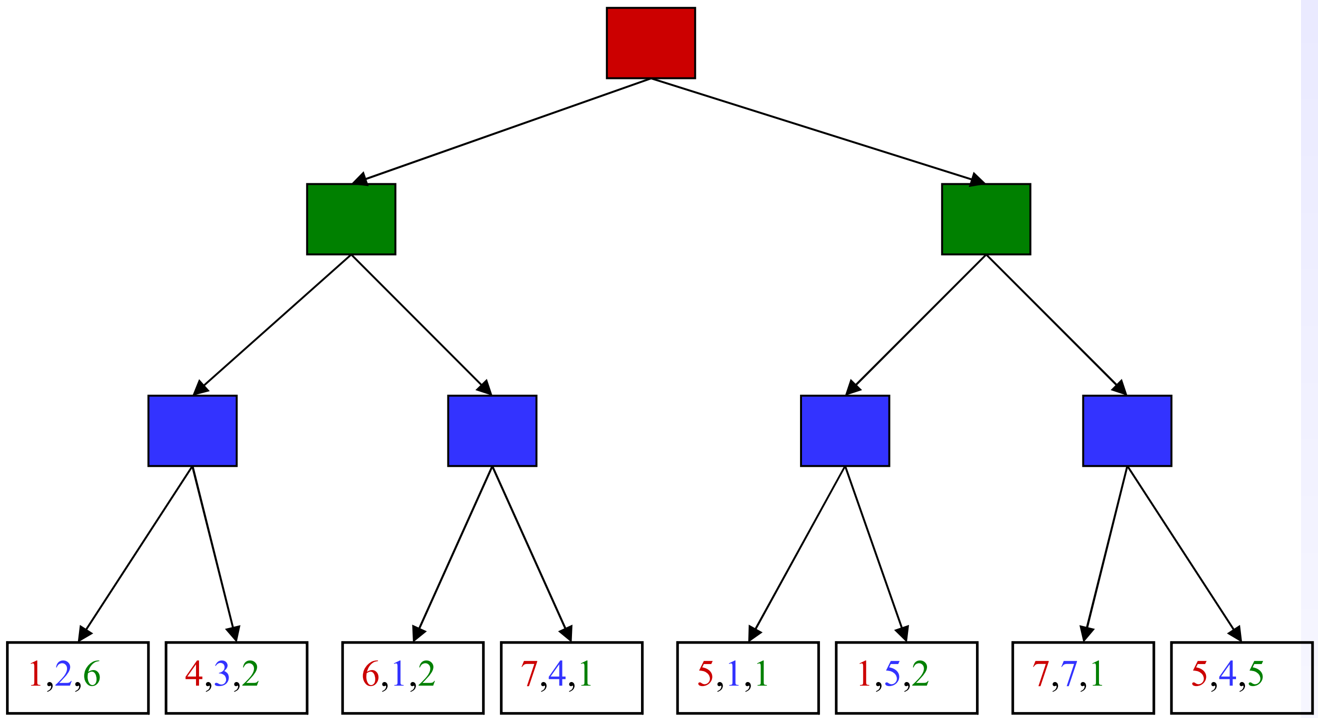


α-β Pruning Properties

- Pruning has no effect on final result
- Good move ordering improves effectiveness of pruning
- With “perfect ordering”:
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth
  - Full search of, e.g. chess, is still hopeless!
- A simple example of metareasoning, here reasoning about which computations are relevant

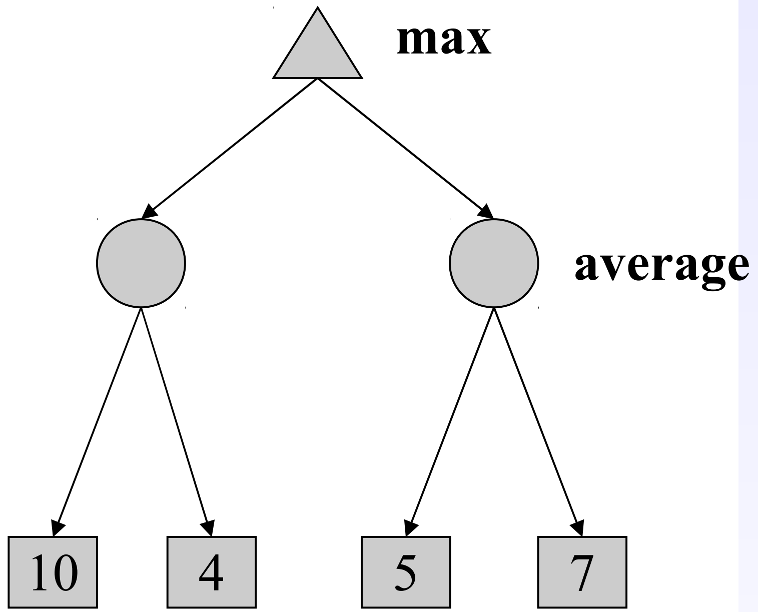
Non-Zero-Sum Games

- Similar to minimax:
  - Utilities are now tuples
  - Each player maximizes their own entry at each node
  - Propagate (or back up) nodes from children



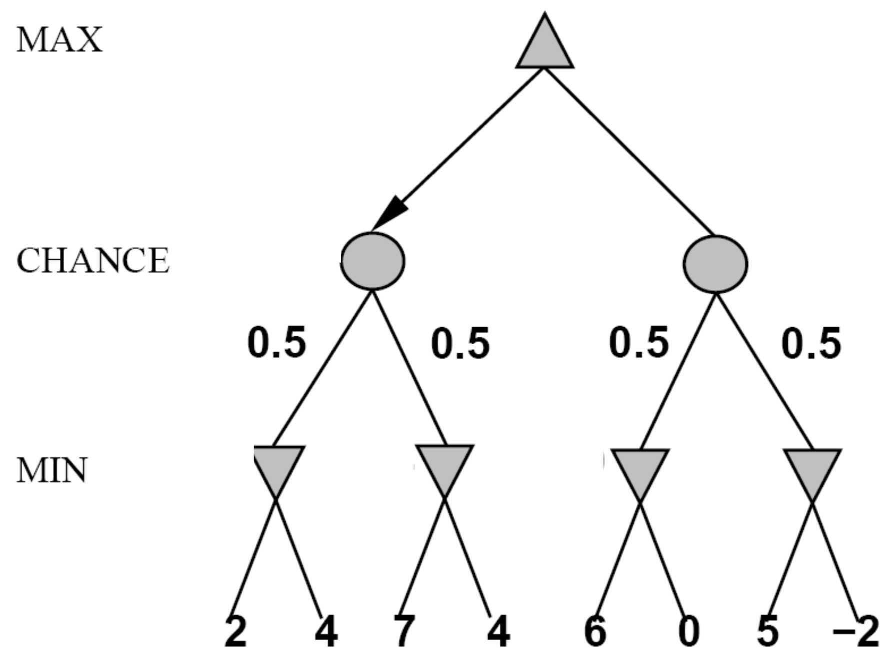
Stochastic Single-Player

- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, shuffle is unknown
  - In minesweeper, mine locations
  - In pacman, ghosts!
- Can do **expectimax search**
  - Chance nodes, like actions except the environment controls the action chosen
  - Calculate utility for each node
  - Max nodes as in search
  - Chance nodes take average (expectation) of value of children
- Later, we'll learn how to formalize this as a **Markov Decision Process**



Stochastic Two-Player

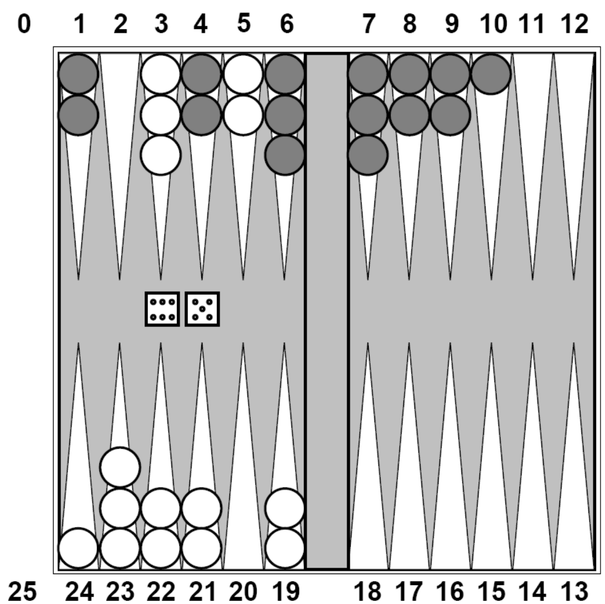
- E.g. backgammon
- Expectiminimax (!)
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax



if *state* is a MAX node **then**
    return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a MIN node **then**
    return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a chance node **then**
    return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

Stochastic Two-Player

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 4 = 20 x (21 x 20)^3 1.2 x 10^9
- As depth increases, probability of reaching a given node shrinks
  - So value of lookahead is diminished
  - So limiting depth is less damaging
  - But pruning is less possible…
- TDGammon uses depth-2 search + very good eval function + reinforcement learning: world-champion level play



What's Next?

- Make sure you know what:
  - Probabilities are
  - Expectations are
  - You should be able to do any exercise from:
    - http://www.cs.umd.edu/class/fall2011/cmsc250-0x0x/hw/HW11.pdf
    - Username and password are both "250"
  - If you can't, review your probability discrete math!
    - http://www.cs.umd.edu/class/fall2011/cmsc250-0x0x/notes/CRASH.pdf

- Next topics:
  - Dealing with uncertainty
  - How to learn evaluation functions
  - Markov Decision Processes