Robot Locomotion and Kinematics

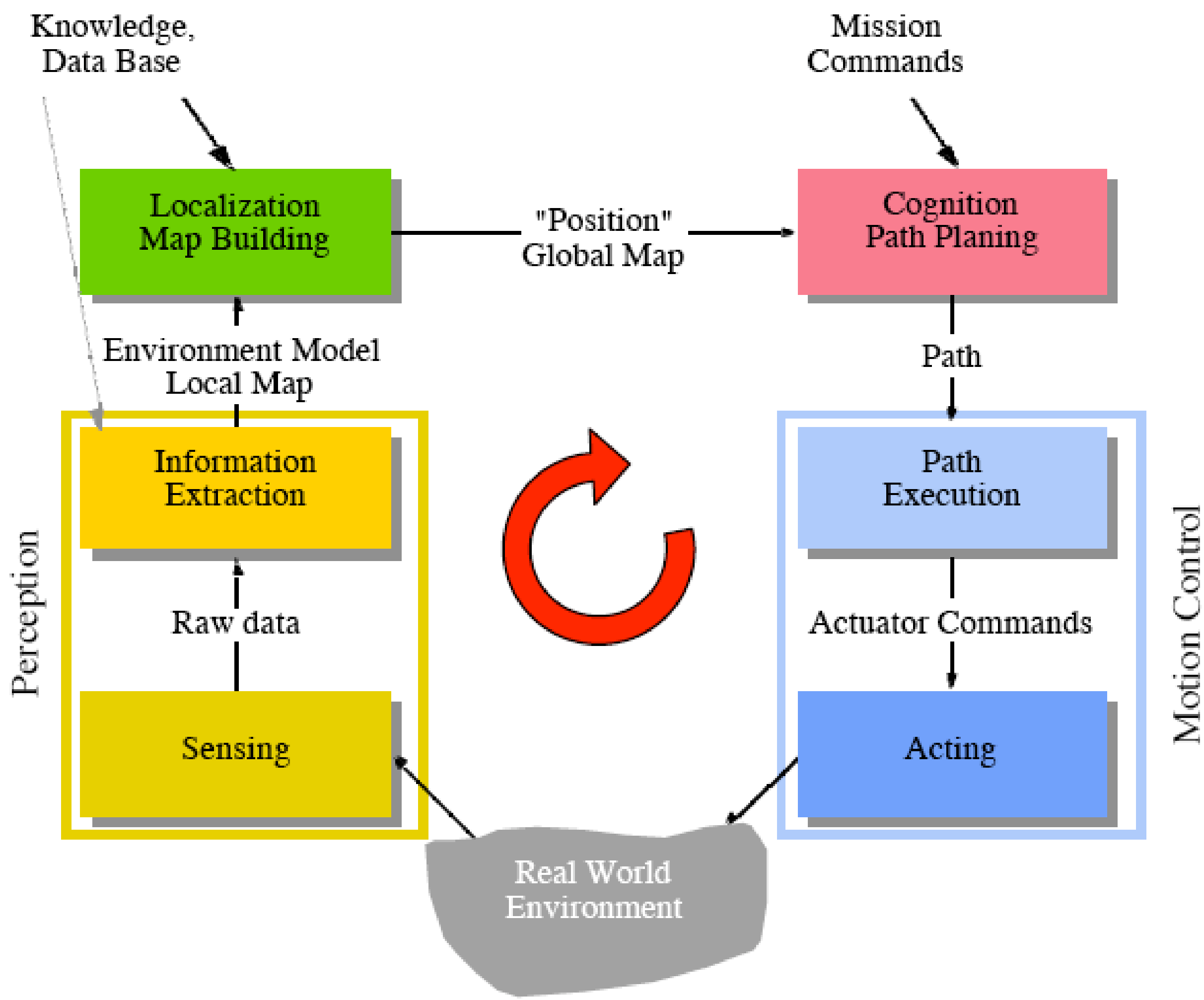
CSE 390-MEAM420/520

Fall 05

Lecture 2

Some notes taken from John Xiao, and Siegwart&Nourbakhsh

General Control Scheme for Mobile Robot Systems



Characterization of locomotion concept

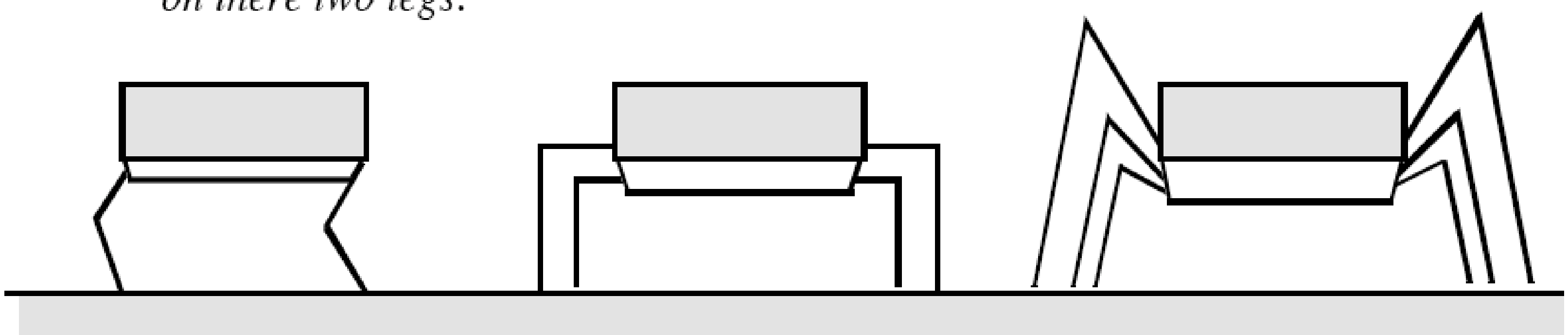
- Locomotion
  - *physical interaction between the vehicle and its environment.*
- Locomotion is concerned with *interaction forces*, and the *mechanisms* and *actuators* that generate them.
- The most important issues in locomotion are:

- stability
  - *number of contact points*
  - *center of gravity*
  - *static/dynamic stabilization*
  - *inclination of terrain*
- characteristics of contact
  - *contact point or contact area*
  - *angle of contact*
  - *friction*
- type of environment
  - *structure*
  - *medium (water, air, soft or hard ground)*

Legged Locomotion

Mobile Robots with legs (walking machines)

- The fewer legs the more complicated becomes locomotion
  - *stability, at least three legs are required for static stability*
- During walking some legs are lifted
  - *thus loosing stability?*
- For static walking at least 6 legs are required
  - *babies have to learn for quite a while until they are able to st and or even walk on there two legs.*



mammal
four legs (two)

reptiles
four legs

insects
six legs

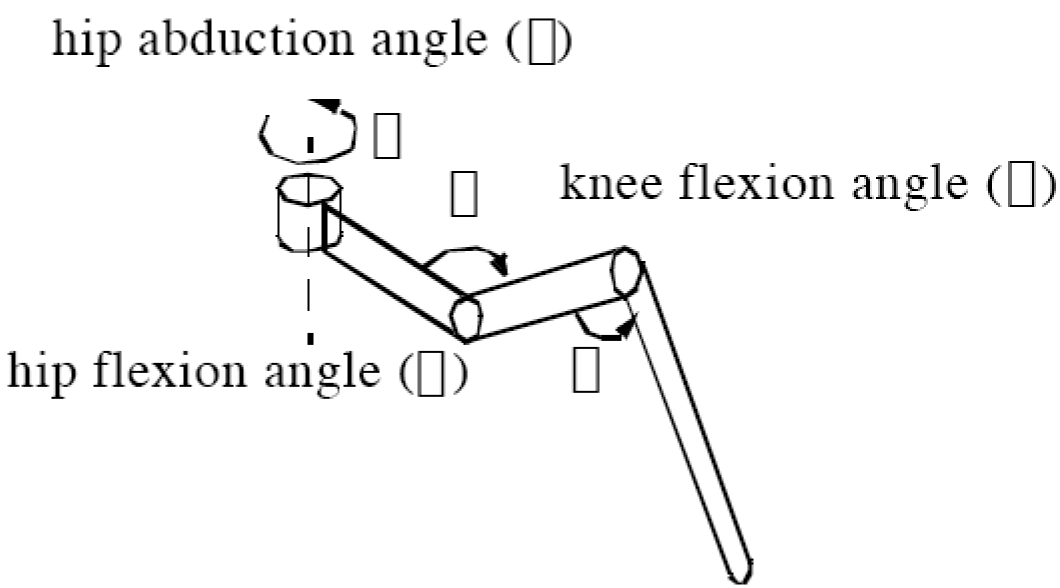
Number of Joints of Each Leg (DOF: degrees of freedom)

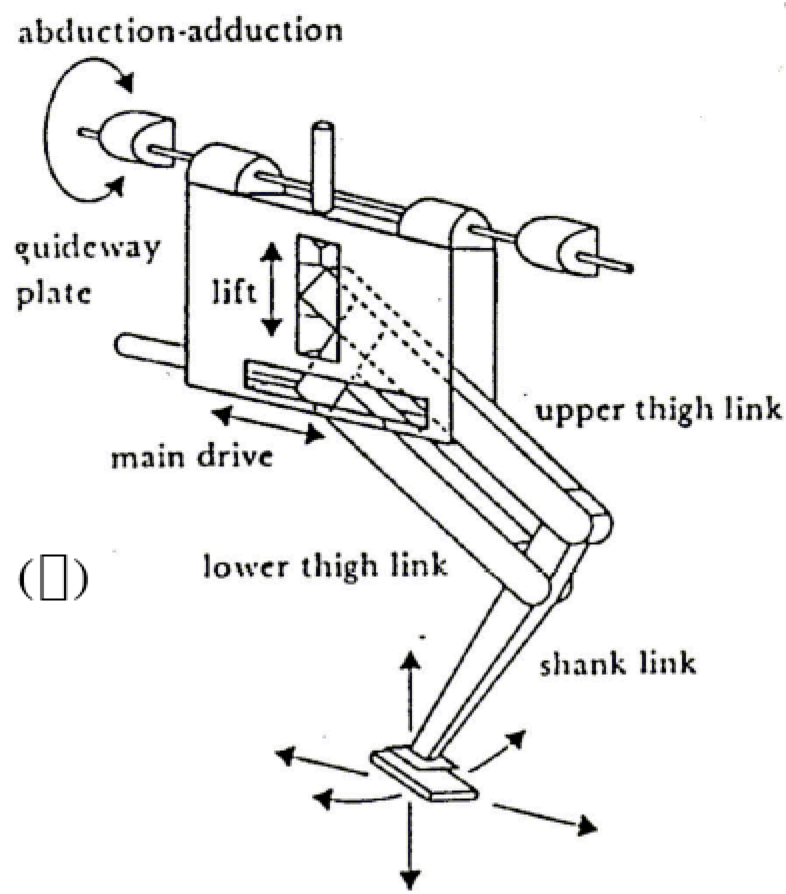
- A minimum of two DOF is required to move a leg forward
  - *a* lift *and a* swing *motion.*
  - *sliding free motion in more then only one direction not possible*

- Three DOF for each leg in most cases

- Fourth DOF for the ankle joint
  - *might improve walking*
  - *however, additional joint (DOF) increase the complexity of the design and especially of the locomotion control.*

Examples of Legs with 3 DOF





The number of possible gaits

- The gait is characterized as the sequence of lift and release events of the individual legs
  - *it depends on the number of legs.*
  - *the number of possible events N for a walking machine with k legs is:*

$$(2k - 1)\,!$$

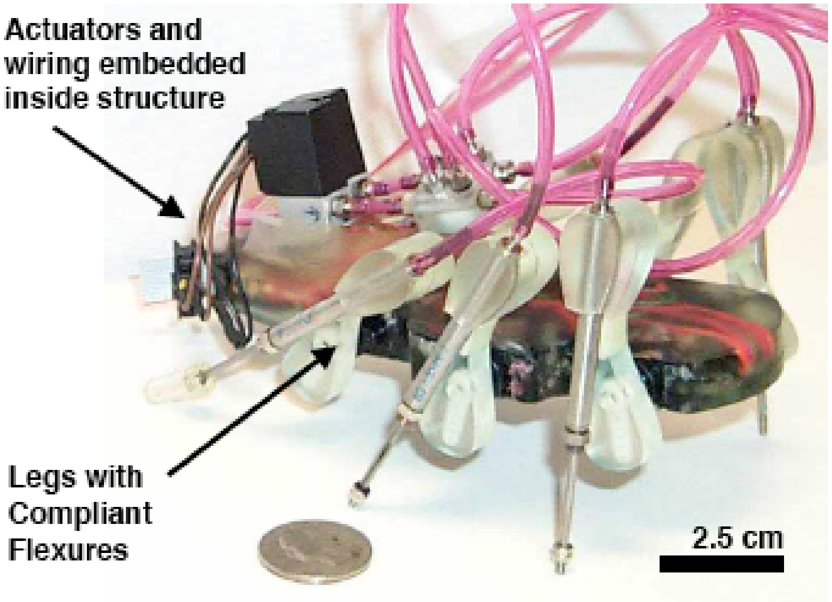
- For a biped walker (k=2) the number of possible events N is:

$$(2*2-1)\,! = 3*2*1 = 6$$

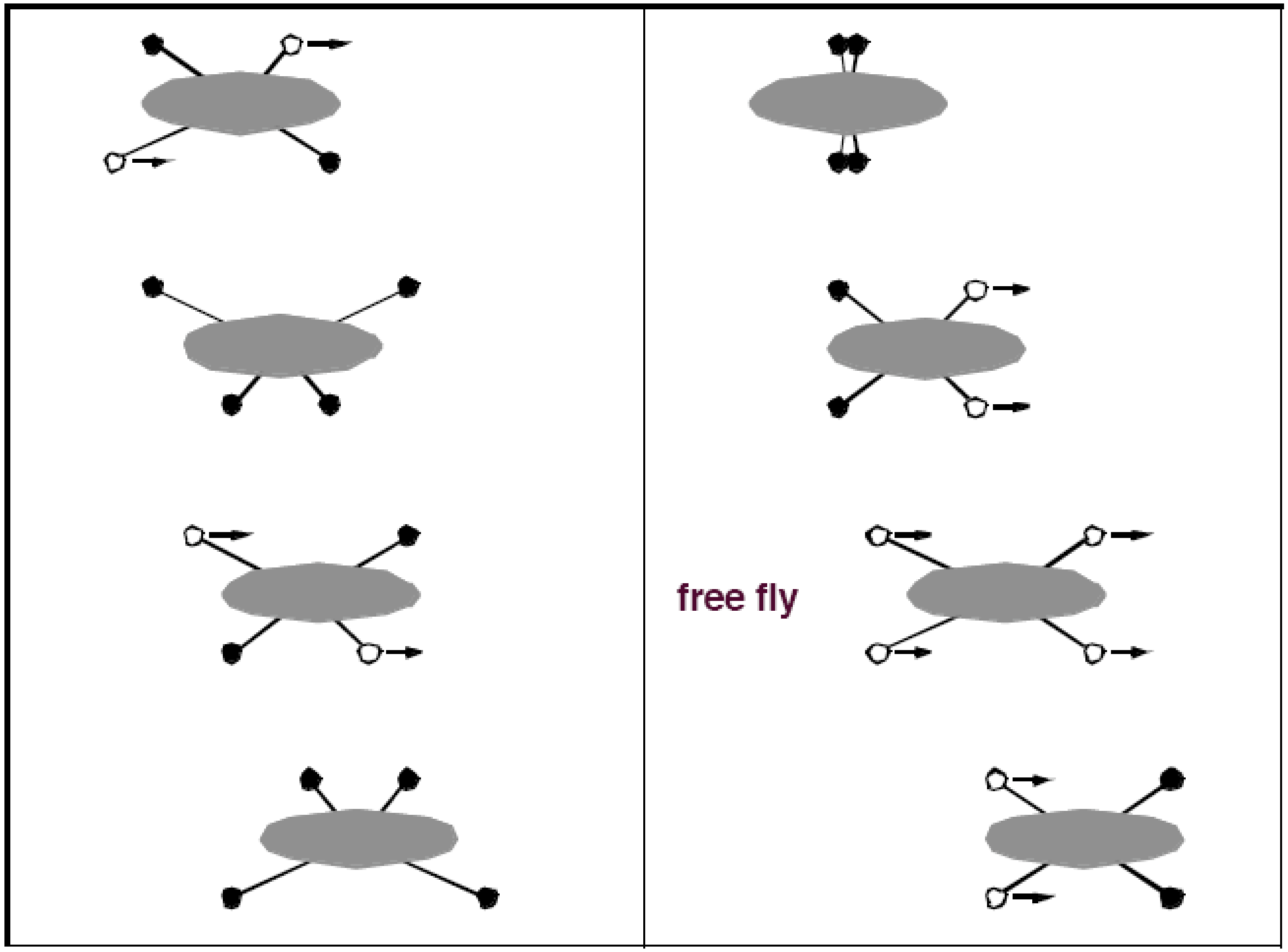
- *The 6 different events are:*
  lift right leg / lift left leg / release right leg / release left leg / lift both legs together / release both legs together

- For a robot with 6 legs (hexapod) N is already

$$N = 11! = 39'916'800$$

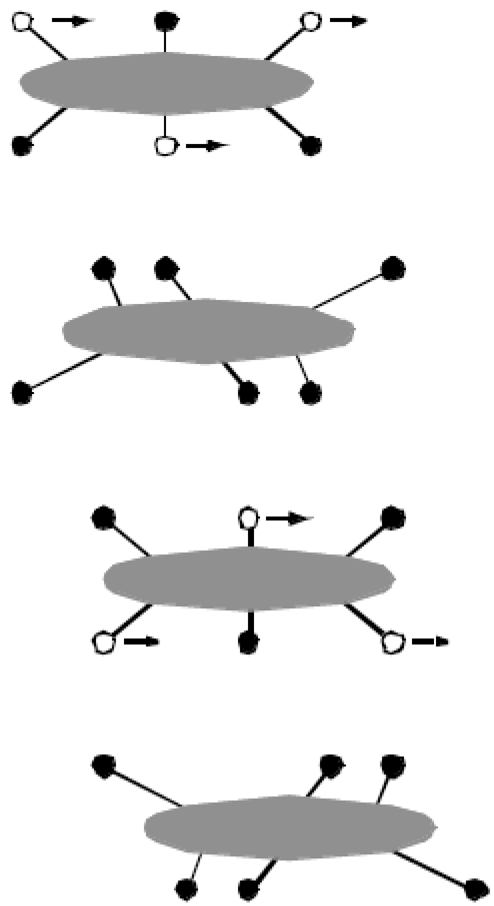


Most Obvious Gaits with 4 legs



Changeover Walking Galloping

Most Obvious Gait with 6 legs (static)

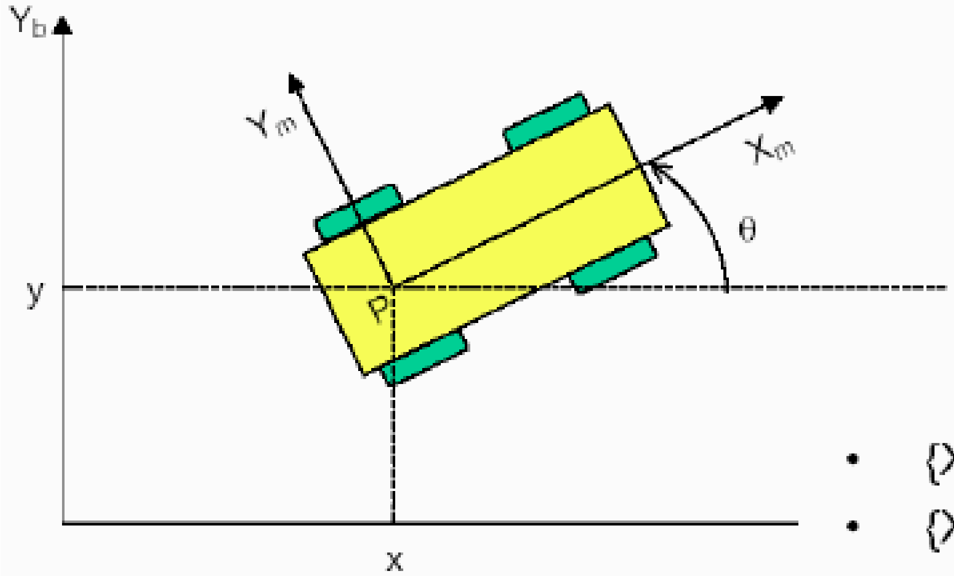


Locomotion on wheel

Mobile Robots with Wheels

- Wheels are the most appropriate solution for most applications
- Three wheels are sufficient and to guarantee stability
- With more than three wheels a flexible suspension is required
- Selection of wheels depends on the application

Notation

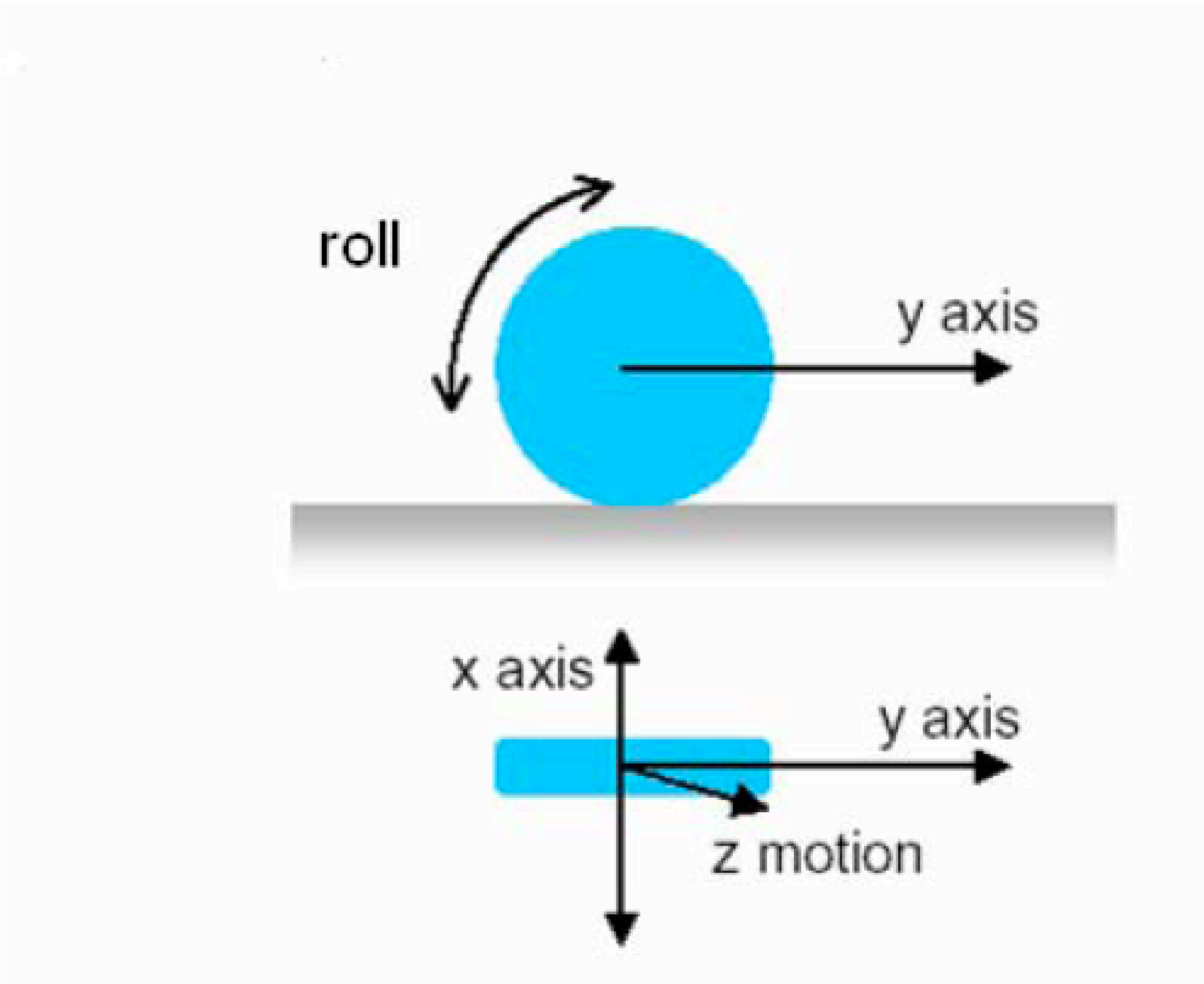


Posture: position(x, y) and orientation θ

- $\{X_m, Y_m\}$ – moving frame
- $\{X_b, Y_b\}$ – base frame

$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$ robot posture in base frame

Wheels

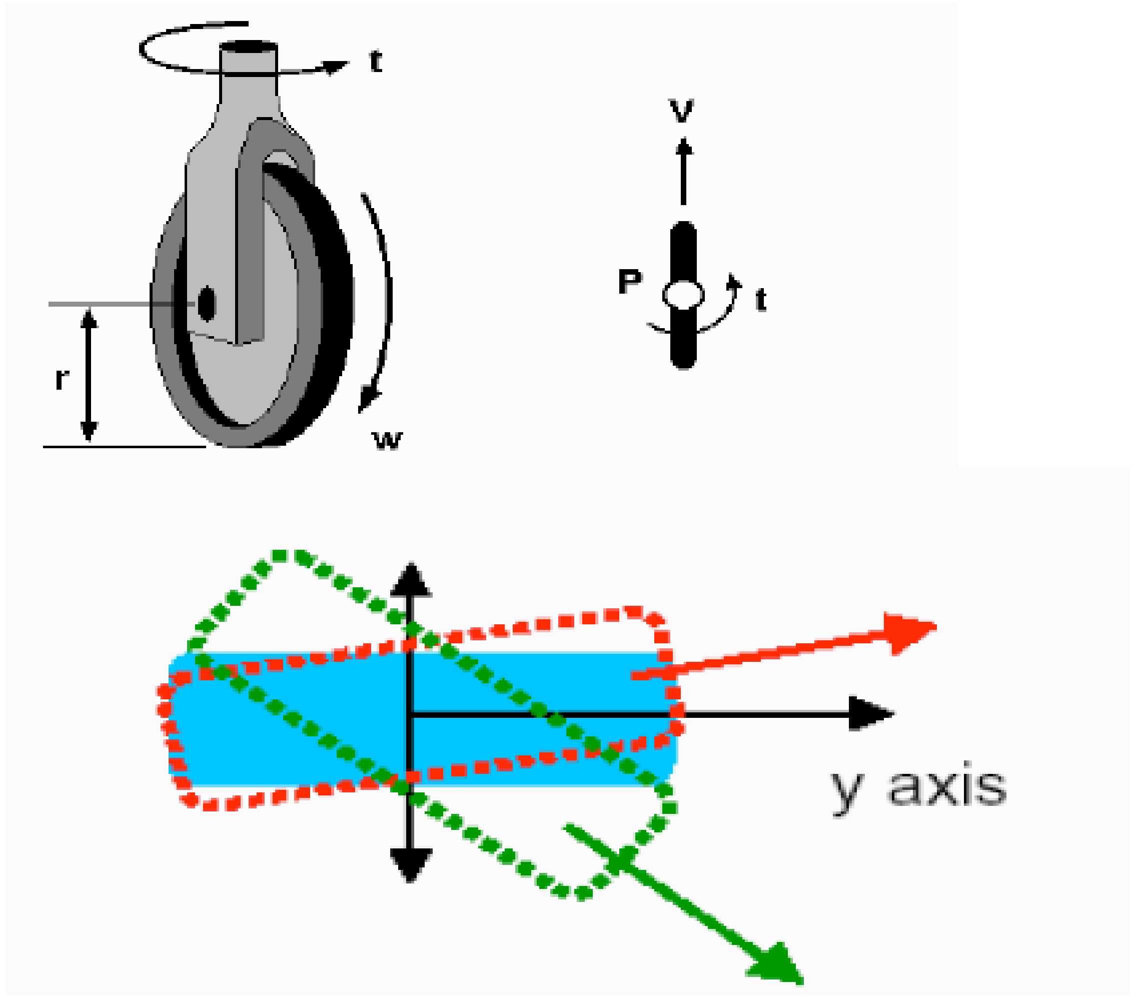


Rolling motion

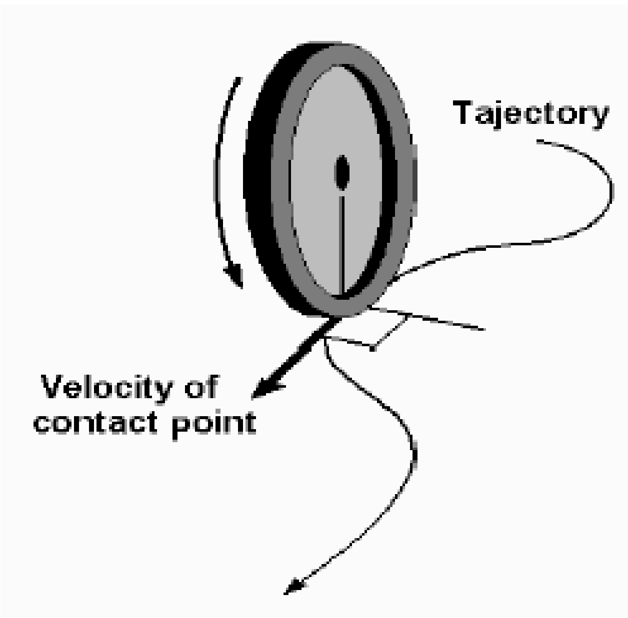
Lateral slip

Steered Wheel

- **Steered wheel**
  - The orientation of the rotation axis can be controlled



Idealized Rolling Wheel



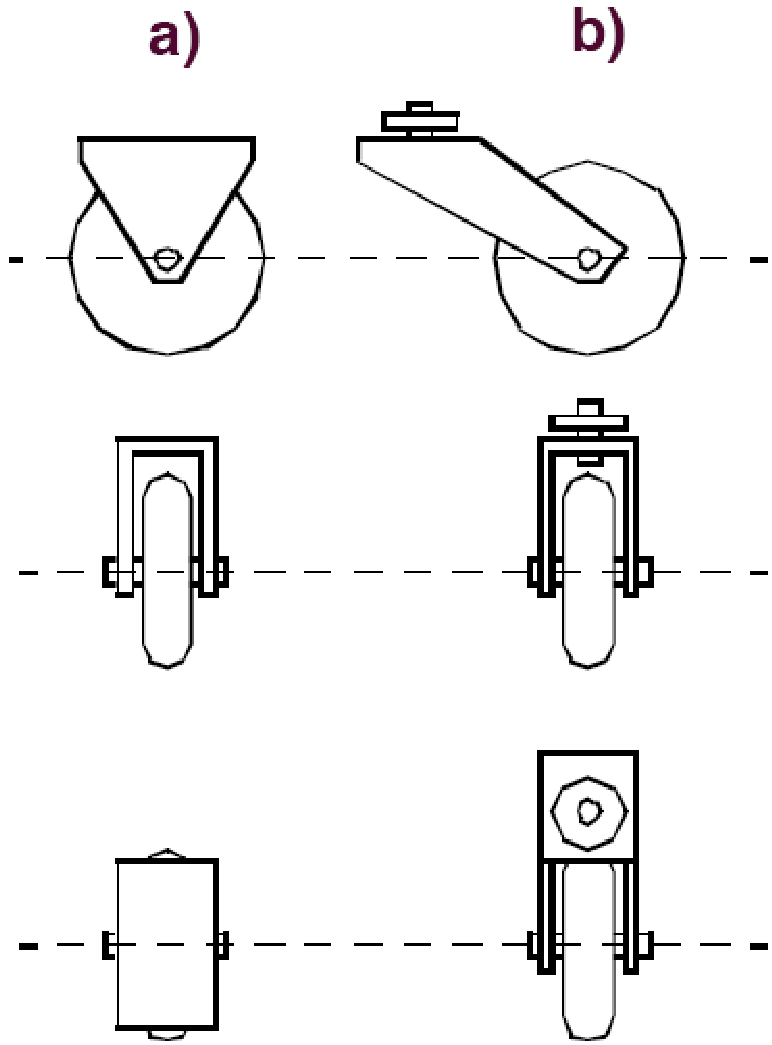
Non-slipping and pure rolling

- **Assumptions**

1. The robot is built from rigid mechanisms.
2. No slip occurs in the orthogonal direction of rolling (non-slipping).
3. No translational slip occurs between the wheel and the floor (pure rolling).
4. The robot contains at most one steering link per wheel.
5. All steering axes are perpendicular to the floor.

The Four Basic Wheels Types

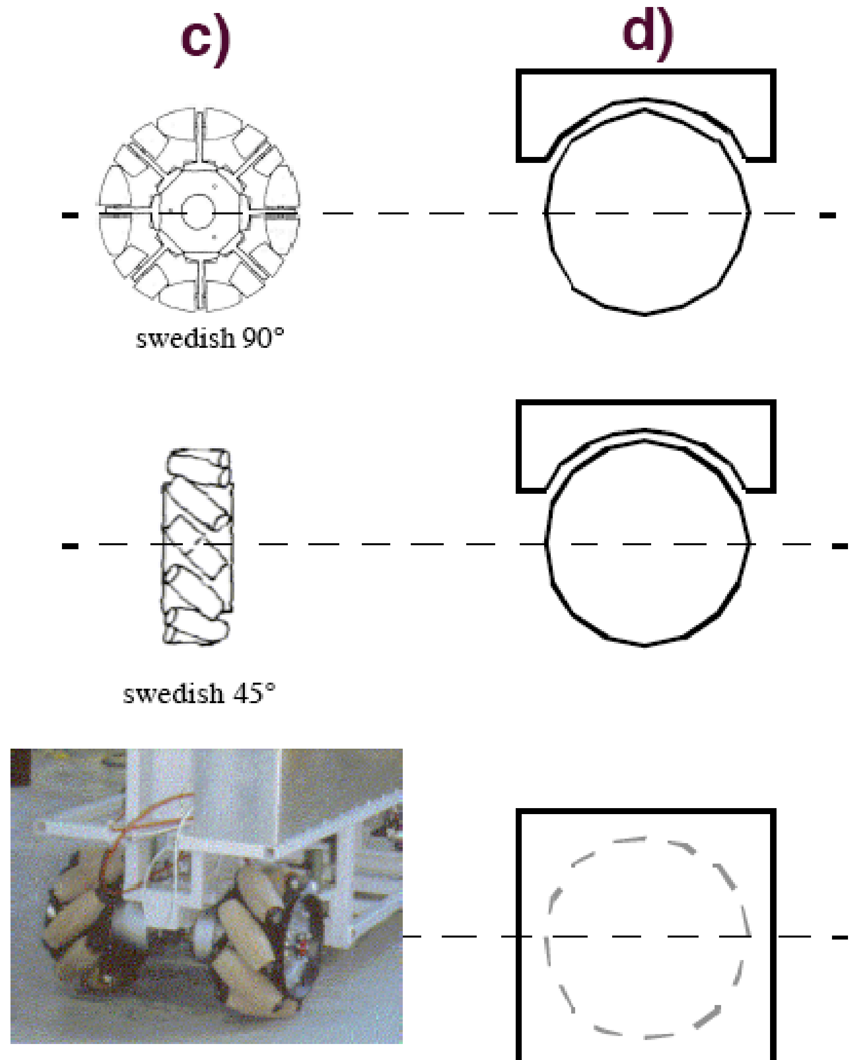
- a) Standard wheel: Two degrees of freedom; rotation around the (motorized) wheel axle and the contact point
- b) Castor wheel: Three degrees of freedom; rotation around the wheel axle, the contact point and the castor axle



The Four Basic Wheels Types

- c) Swedish wheel: Three degrees of freedom; rotation around the (motorized) wheel axle, around the rollers and around the contact point

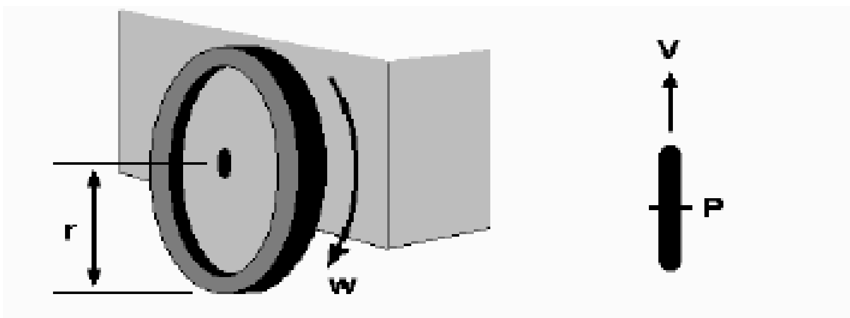
- d) Ball or spherical wheel: Suspension technically not solved



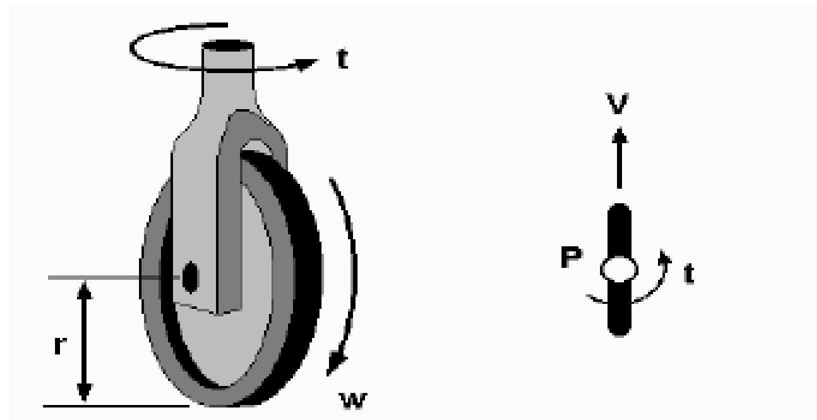
Robot wheel parameters

- For low velocities, rolling is a reasonable wheel model.
  - This is the model that will be considered in the kinematics models of WMR
- Wheel parameters:
  - r = wheel radius
  - v = wheel linear velocity
  - w = wheel angular velocity
  - t = steering velocity

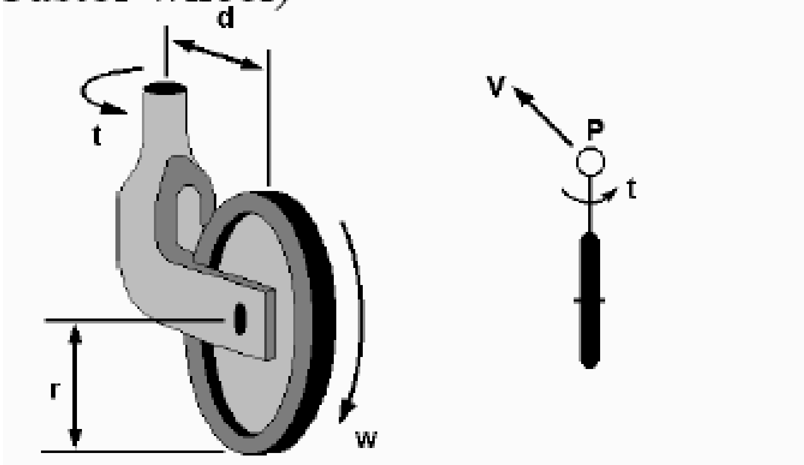
Fixed wheel



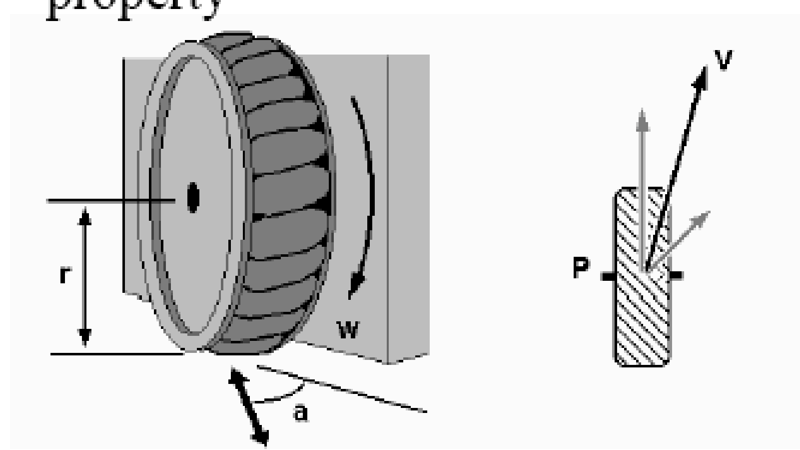
Centered orientable wheel



Off-centered orientable wheel
(Castor wheel)



Swedish wheel: omnidirectional property



Fixed wheel

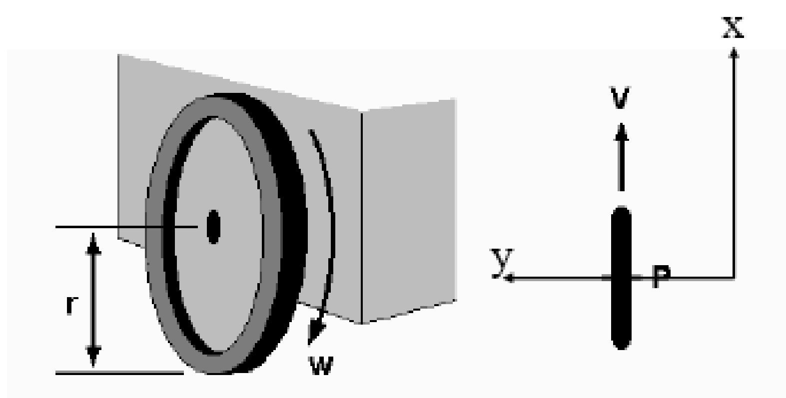
– Velocity of point **P**

$$V = (\mathbf{r} \times w)a_x$$

where, a_x : A unit vector to X axis

– Restriction to the robot mobility

Point **P** cannot move to the direction perpendicular to plane of the wheel.



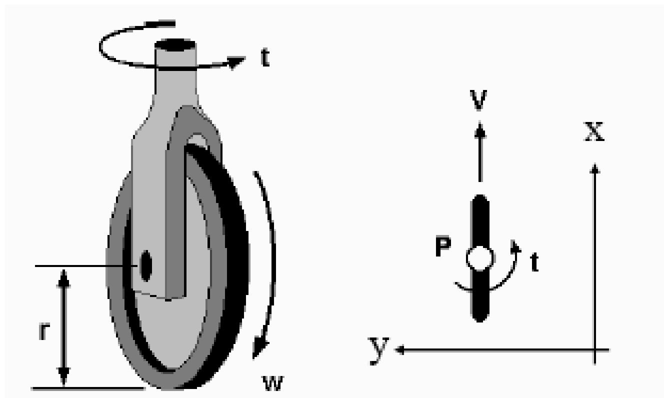
Centered orientable wheels

- Velocity of point **P**

$$V = (r \times w) a_x$$

where, a_x : A unit vector of x axis
a_y : A unit vector of y axis

- Restriction to the robot mobility



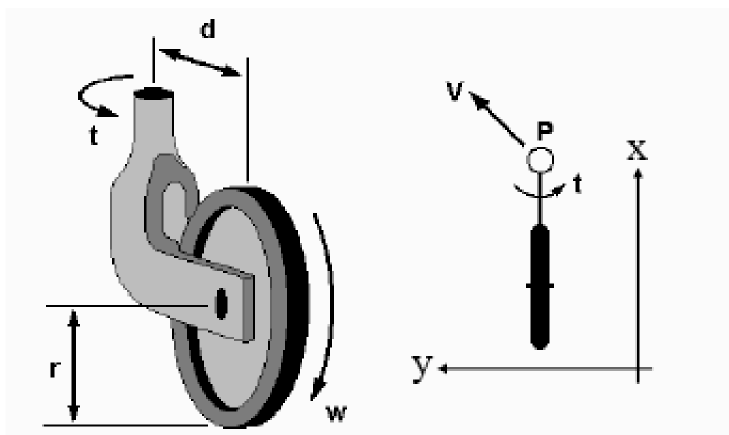
Off-Centered orientable wheels
(caster wheels)

– Velocity of point **P**

$$v = (r \times w)a_x + (d \times t)a_y$$

where, a_x : A unit vector of x axis
a_y : A unit vector of y axis

– Restriction to the robot mobility



Swedish wheel

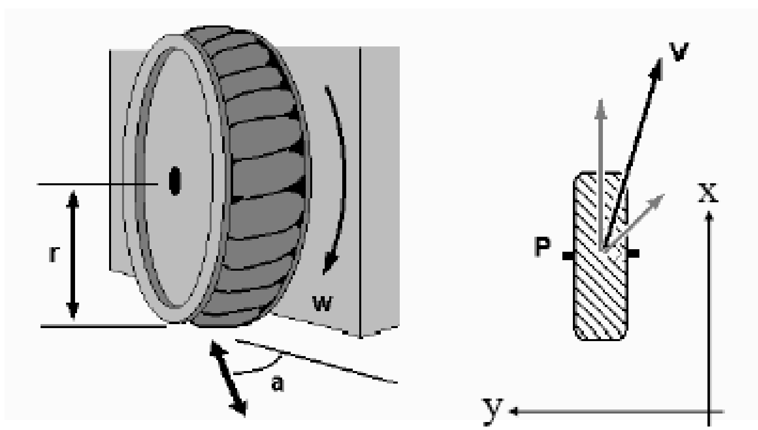
– Velocity of point **P**

$$v = (r \times w)a_x + Ua_s$$

where, a_x : A unit vector of x axis

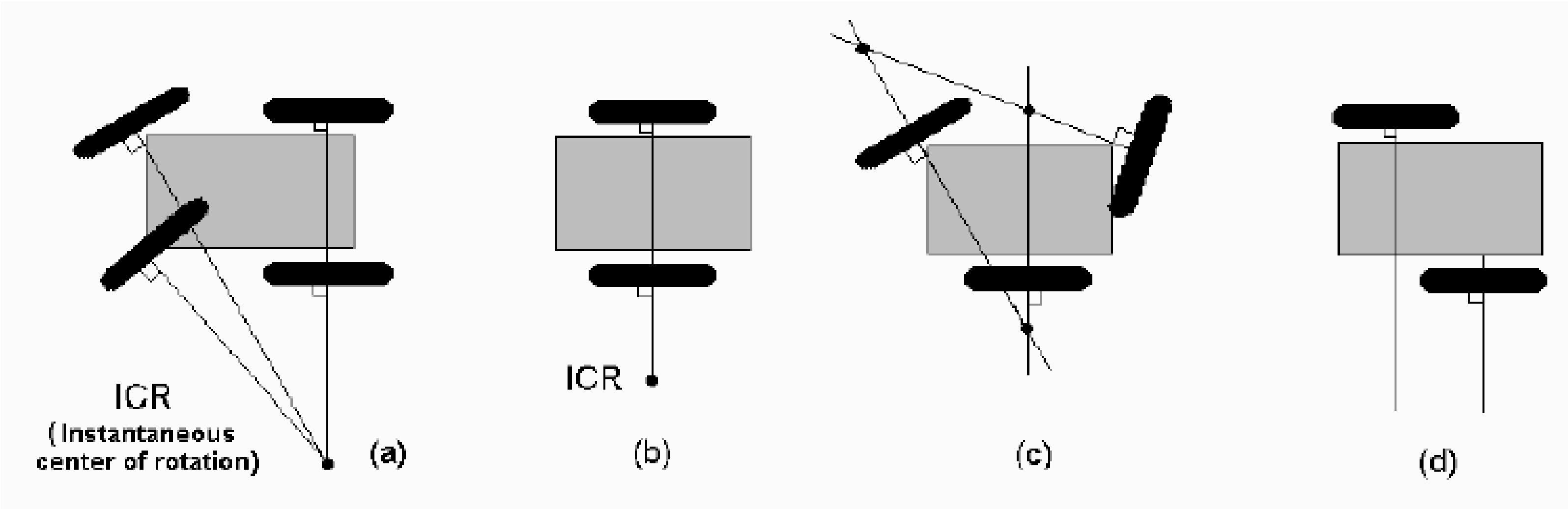
a_s : A unit vector to the motion of roller

– Omnidirectional property



Mobile Robot Locomotion

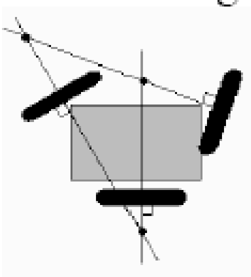
- Instantaneous center of rotation (ICR) or Instantaneous center of curvature (ICC)
  - A cross point of all axes of the wheels



Degree of Mobility

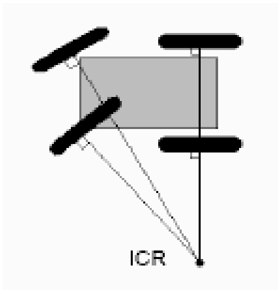
- **Degree of mobility**

  The degree of freedom of the robot motion



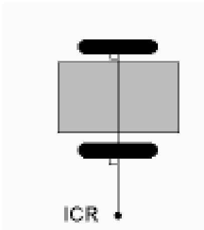
Cannot move anywhere (No ICR)

- Degree of mobility : 0



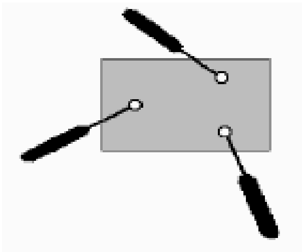
Fixed arc motion (Only one ICR)

- Degree of mobility : 1



Variable arc motion (line of ICRs)

- Degree of mobility : 2



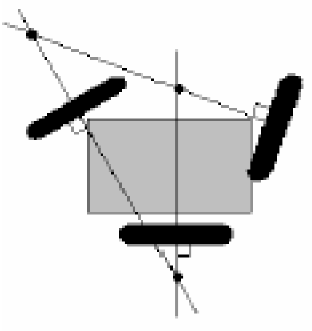
Fully free motion

(ICR can be located at any position)

- Degree of mobility : 3

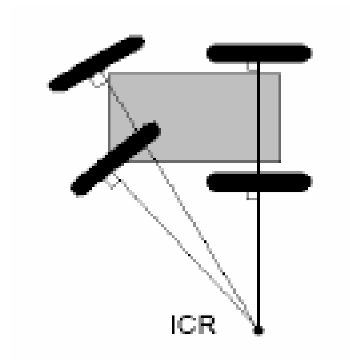
- **Degree of mobility**

  The degree of freedom of the robot motion



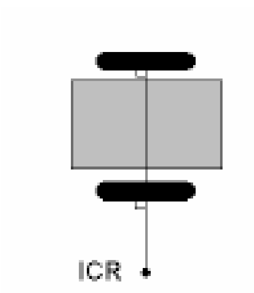
Cannot move anywhere (No ICR)

- Degree of mobility : 0

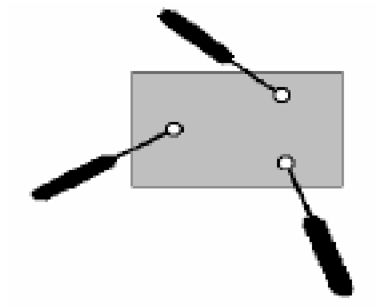


Fixed arc motion (Only one ICR)

- Degree of mobility : 1



Variable arc motion
(line of ICRs)



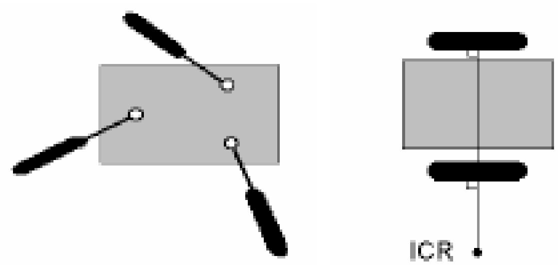
Fully free motion

(ICR can be located at any position)

- Degree of mobility : 2
- Degree of mobility : 3

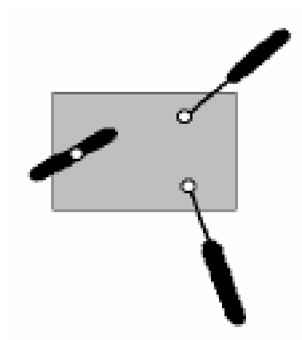
- **Degree of steerability**

The number of centered orientable wheels that can be steered independently in order to steer the robot



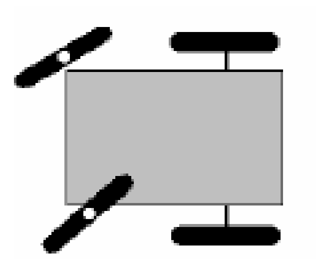
No centered orientable wheels

- Degree of steerability : 0



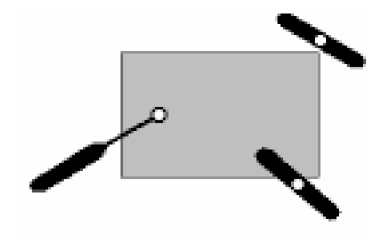
One centered orientable wheel

steerability : 1



Two mutually dependent centered orientable wheels

steerability : 1



Two mutually independent centered orientable wheels

Degree of steerability : 2

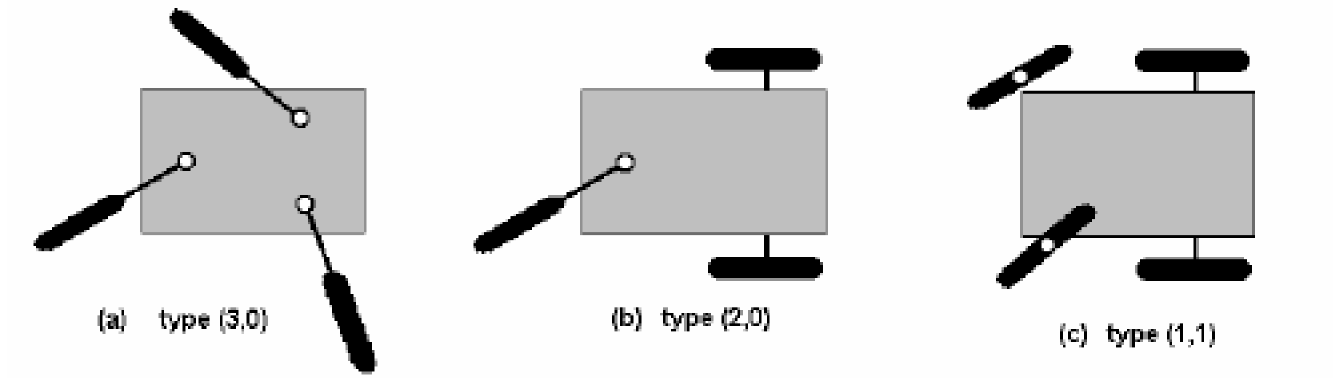
Degree of Maneuverability

- The overall degrees of freedom that a robot can manipulate:

$$\delta_M = \delta_m + \delta_s$$

Degree of Mobility	3	2	2	1	1
Degree of Steerability	0	0	1	1	2

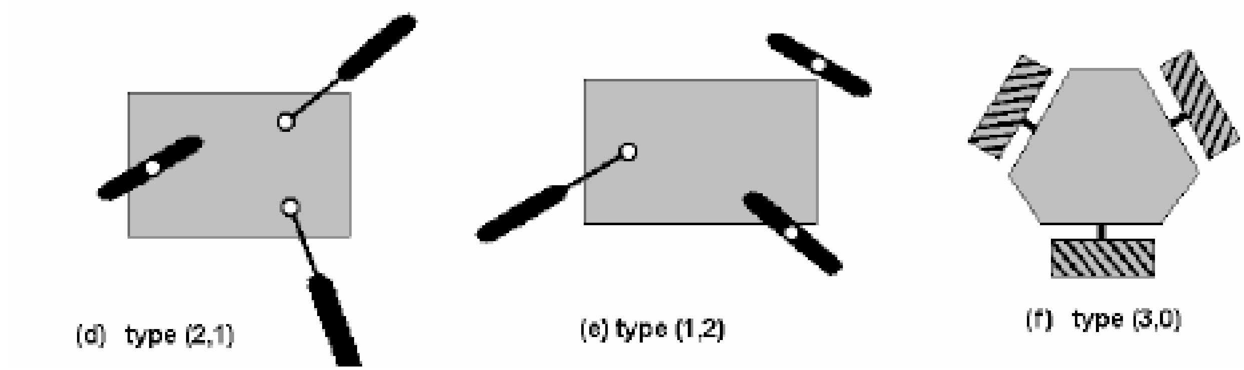
- Examples of robot types (degree of mobility, degree of steerability)



(a) type (3,0)

(b) type (2,0)

(c) type (1,1)



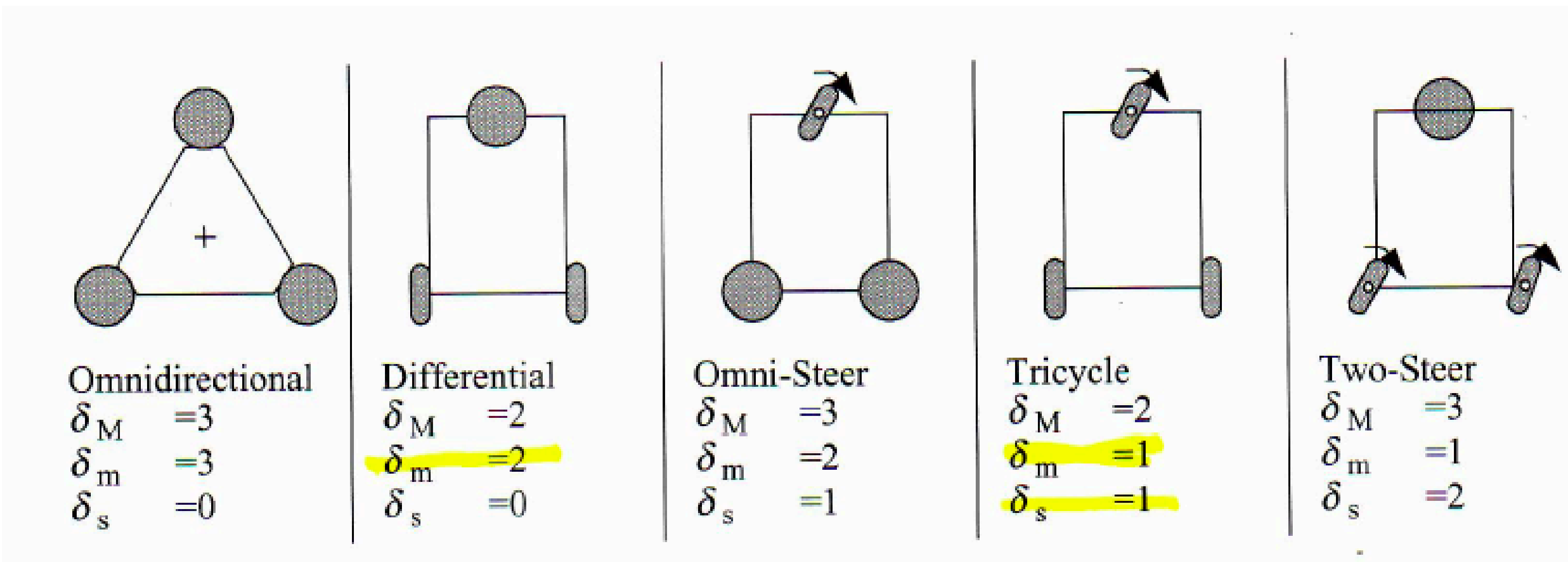
(d) type (2,1)

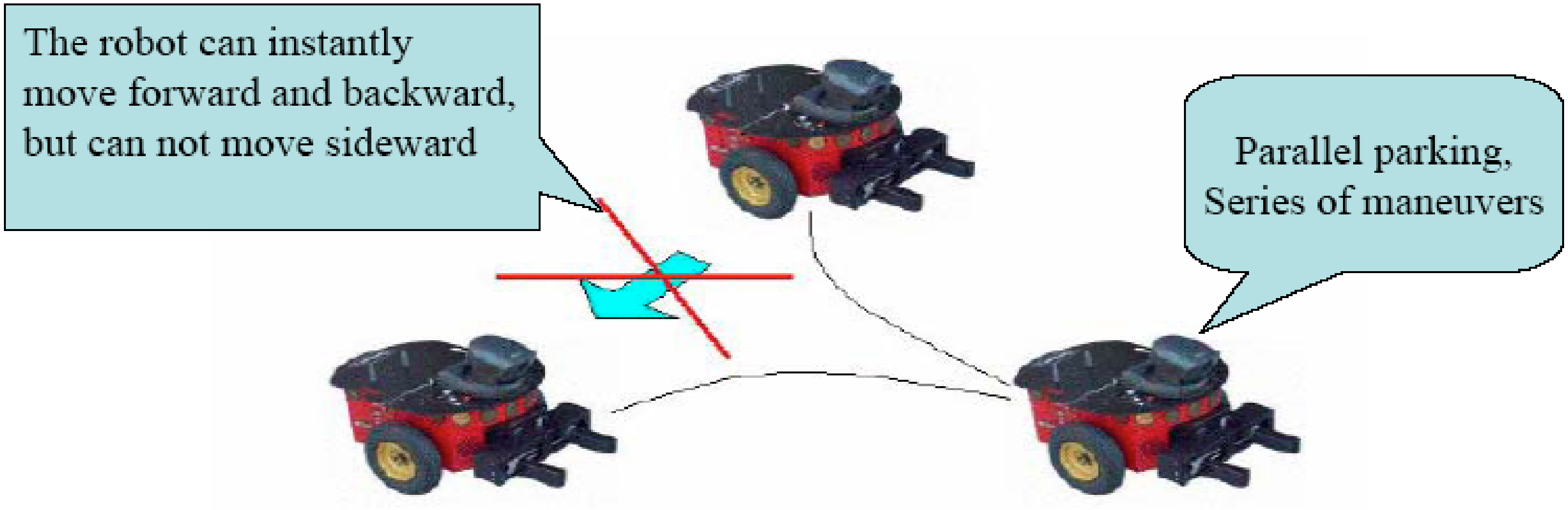
(e) type (1,2)

(f) type (3,0)

Degree of Maneuverability

$$\delta_M = \delta_m + \delta_s$$



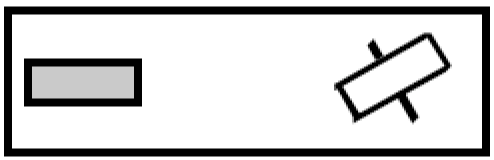


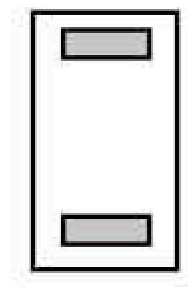
A non-holonomic constraint is a constraint on the feasible **velocities** of a body

Wheel configurations

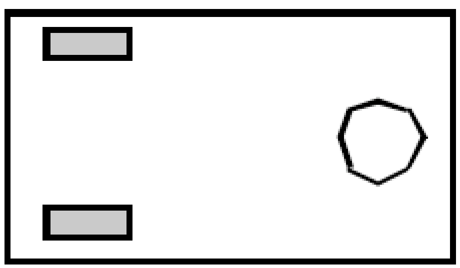
Different Arrangements of Wheels I

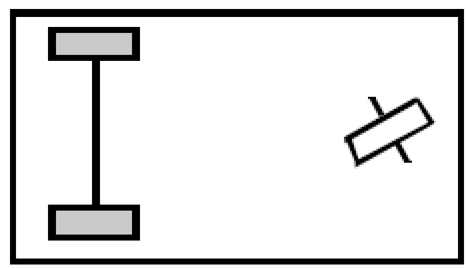
- Two wheels

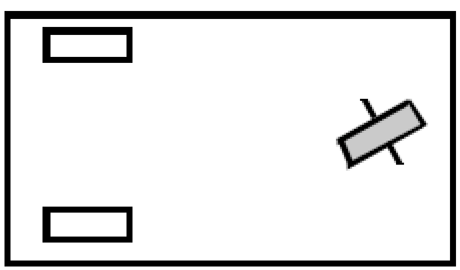


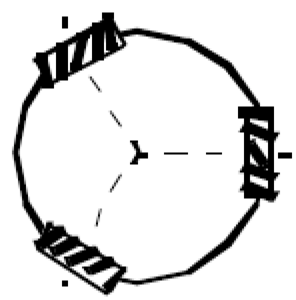


- Three wheels

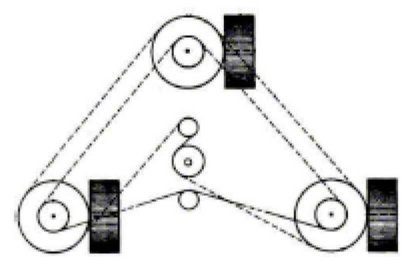






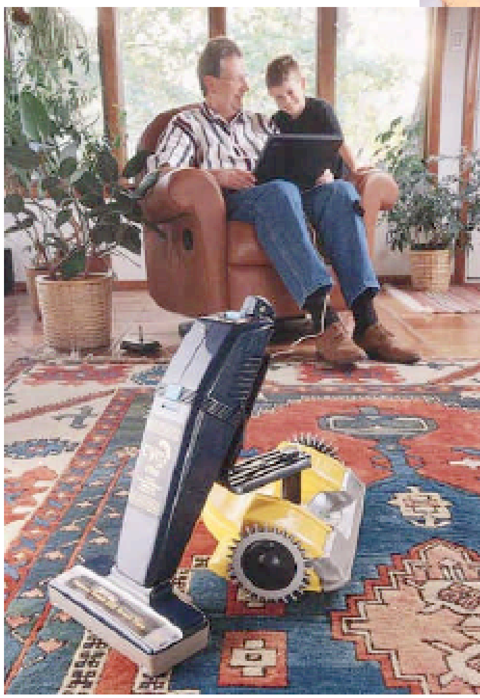


Omnidirectional Drive



Synchro Drive

Cye, a Two Wheel Differential Drive Robot

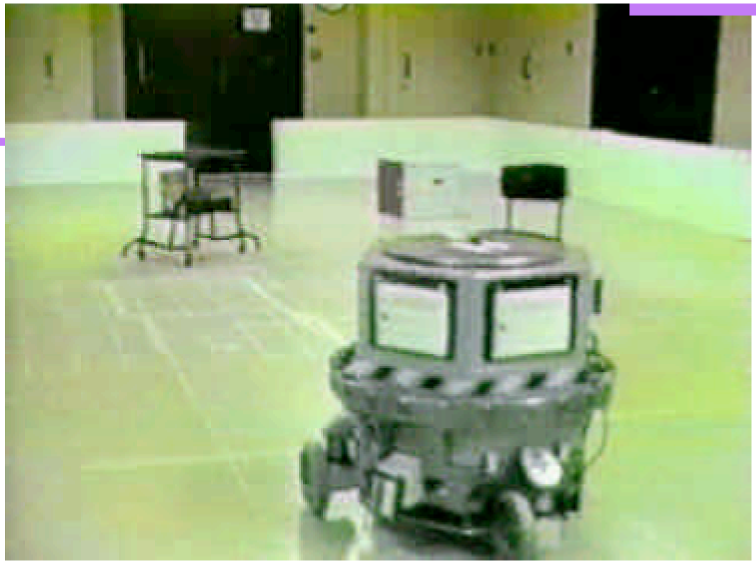


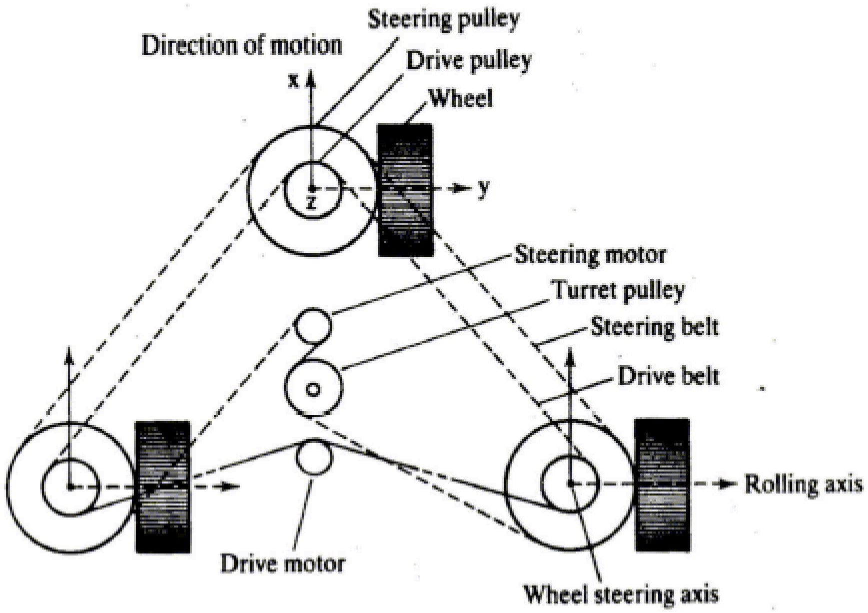
- Cye, a commercially available domestic robot that can vacuum and make deliveries in the home, is built by Probotics, Inc.

Synchro Drive

- All wheels are actuated synchronously by one motor
  - *defines the speed of the vehicle*
- All wheels steered synchronously by a second motor
  - *sets the heading of the vehicle*

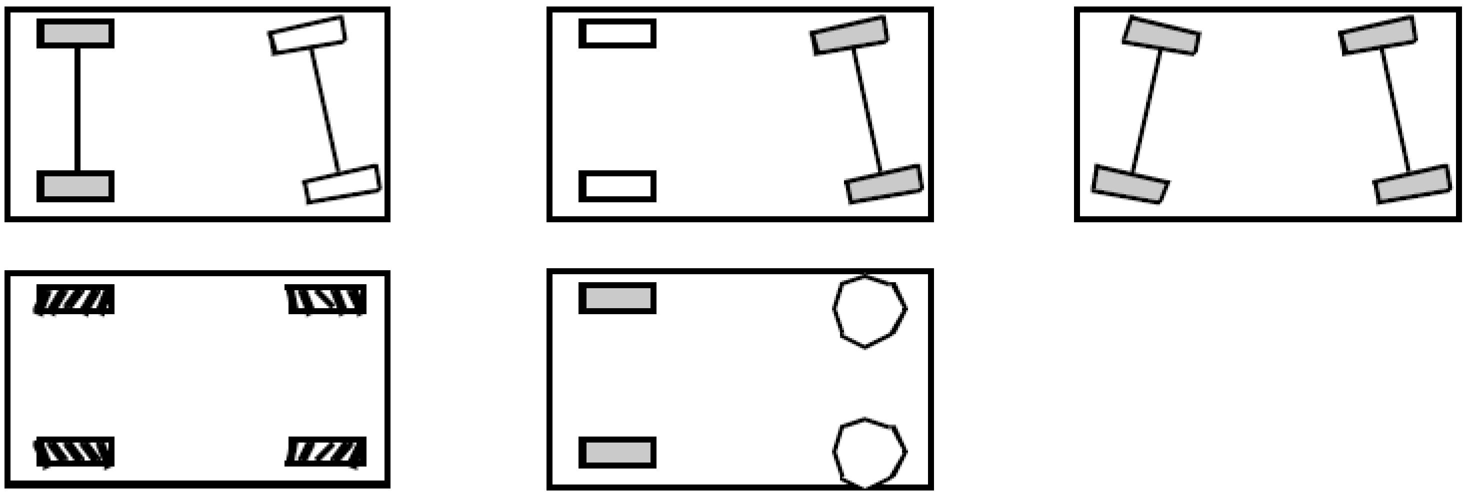
- The orientation in space of the robot frame will always remain the same
  - *It is therefore not possible to control the orientation of the robot frame.*



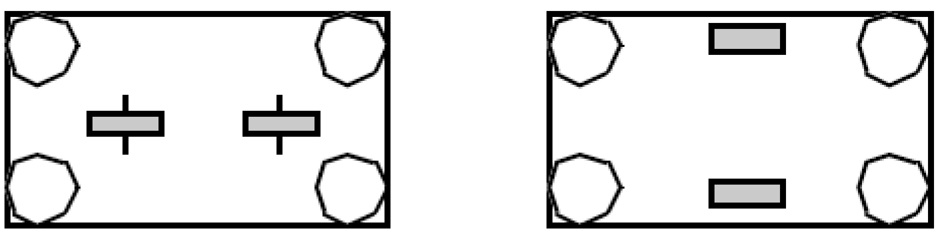


Different Arrangements of Wheels II

- Four wheels

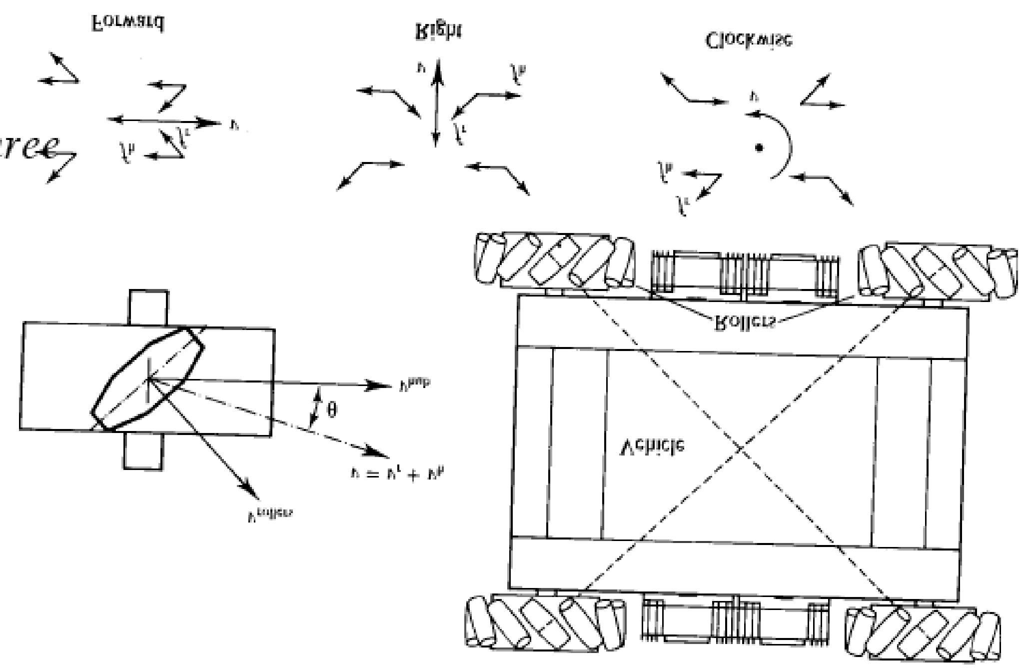


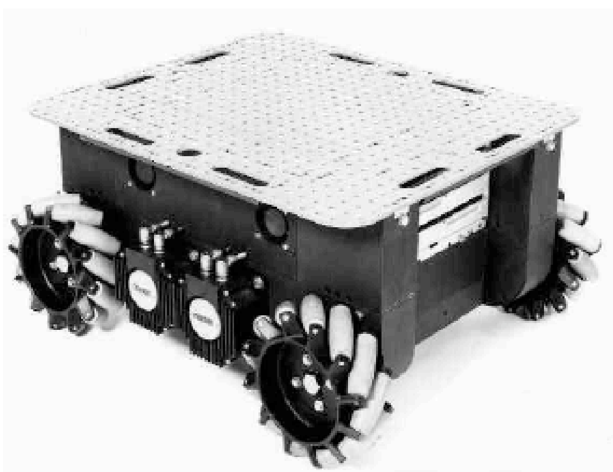
- Six wheels



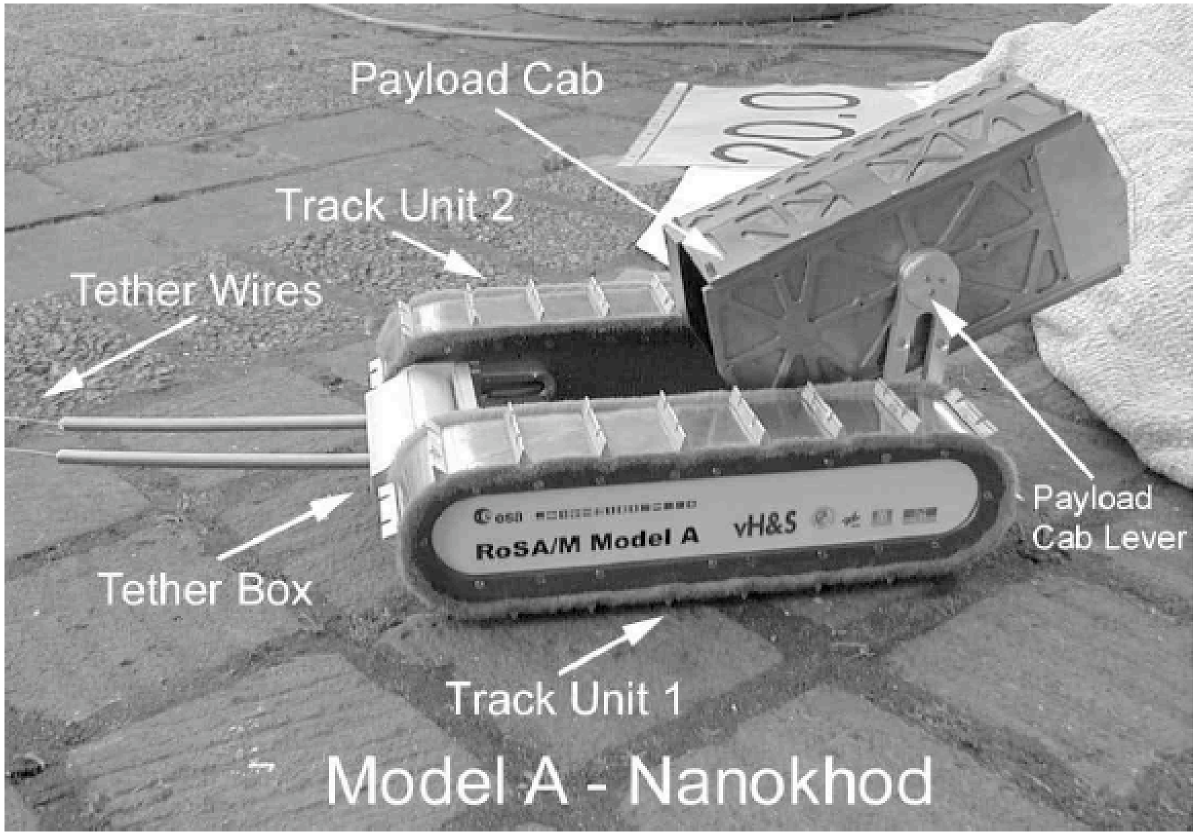
Uranus, CMU: Omnidirectional Drive with 4 Wheels

- Movement in the plane has 3 DOF
  - *thus only three wheels can be independently controlled*
  - *It might be better to arrange three swedish wheels in a triangle*





Caterpillar

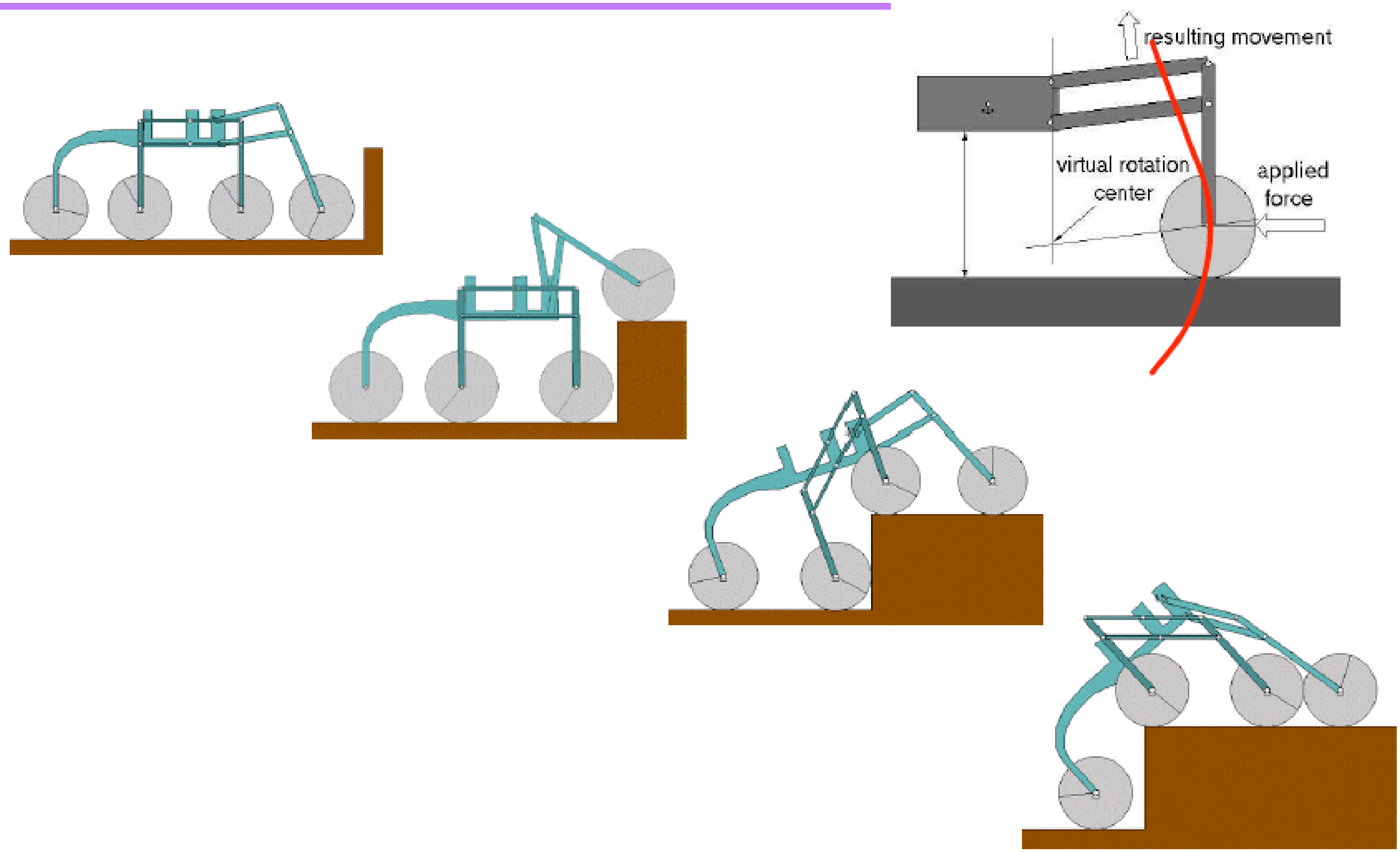


- The NANOKHOD II, developed by von Hoerner & Sulger GmbH and Max Planck Institute, Mainz for European Space Agency (ESA) will probably go to Mars

SHRIMP, a Mobile Robot with Excellent Climbing Abilities

- Objective
  - *Passive locomotion concept for rough terrain*
- Results: The Shrimp
  - *6 wheels*
    - *one fixed wheel in the rear*
    - *two boogies on each side*
    - *one front wheel with spring suspension*
  - *robot sizing around 60 cm in length and 20 cm in height*
  - *highly stable in rough terrain*
  - *overcomes obstacles up to 2 times its wheel diameter*

The SHRIMP Adapts Optimally to Rough Terrain



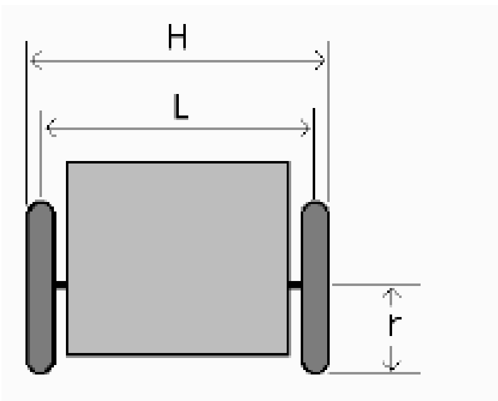
Introduction: Mobile Robot Kinematics

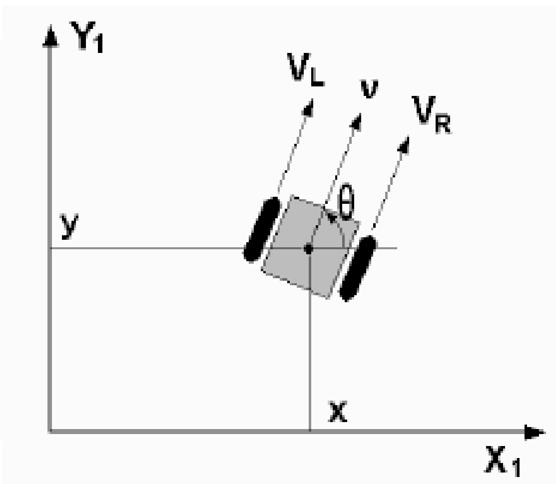
- Aim
  - *Description of mechanical behavior of the robot for* design *and* control
  - *Similar to robot manipulator kinematics*
  - *However, mobile robots can move unbound with respect to its environment*
    - *there is no direct way to measure the robot's position*
    - *Position must be integrated over time*
    - *Leads to inaccuracies of the position (motion) estimate*
      -> the number 1 challenge in mobile robotics
  - *Understanding mobile robot motion starts with understanding whee l* constraints *placed on the robots mobility*

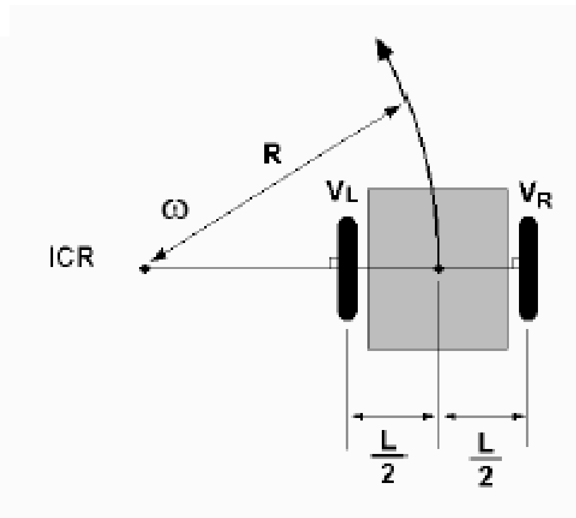
Mobile Robot Locomotion

- Differential Drive
  - two driving wheels (plus roller-ball for balance)
  - simplest drive mechanism
  - sensitive to the relative velocity of the two wheels (small error result in different trajectories, not just speed)
- Steered wheels (tricycle, bicycles, wagon)
  - Steering wheel + rear wheels
  - cannot turn $\pm 90^o$
  - limited radius of curvature
- Synchronous Drive
- Omni-directional
- Car Drive (Ackerman Steering)

Differential Drive







- Posture of the robot

$$P = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

(x,y) : Position of the robot

θ : Orientation of the robot

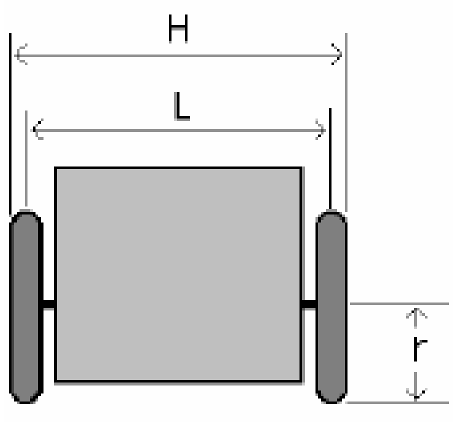
- Control input

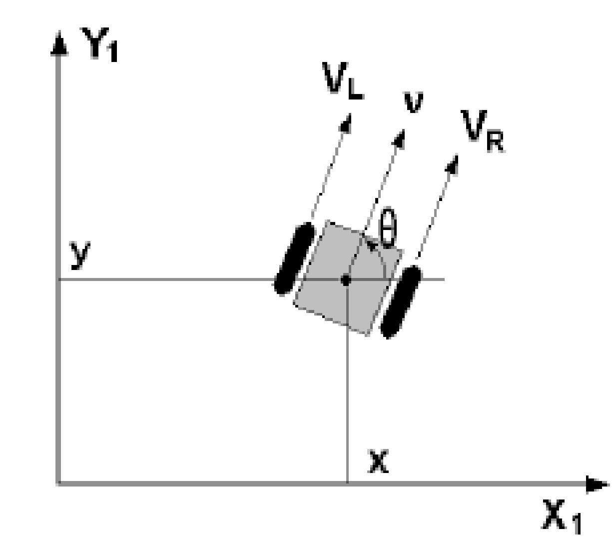
$$U = \begin{pmatrix} v \\ w \end{pmatrix}$$

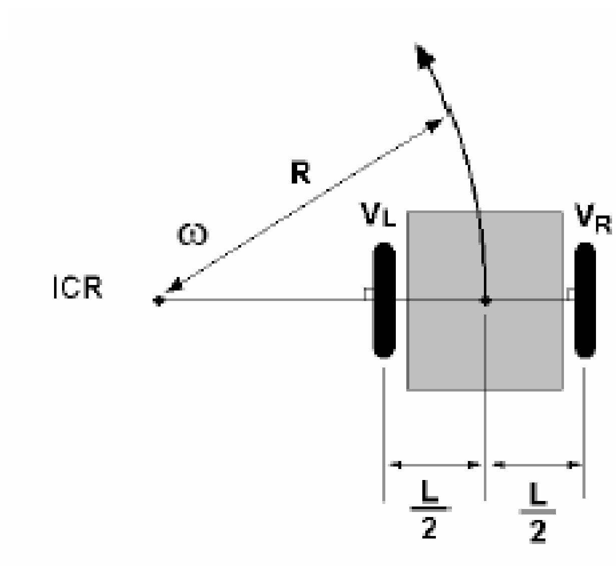
v : Linear velocity of the **robot**

w : Angular velocity of the **robot**

(notice: not for each wheel)

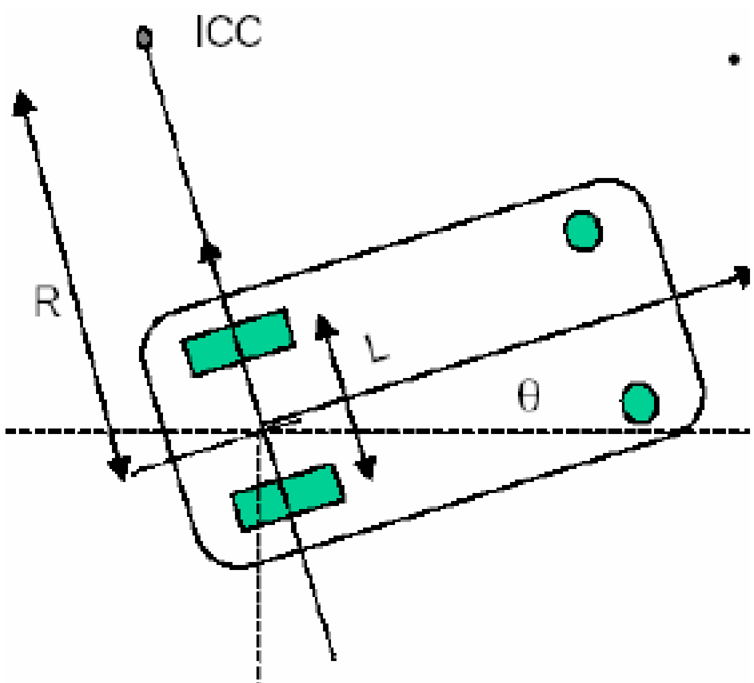


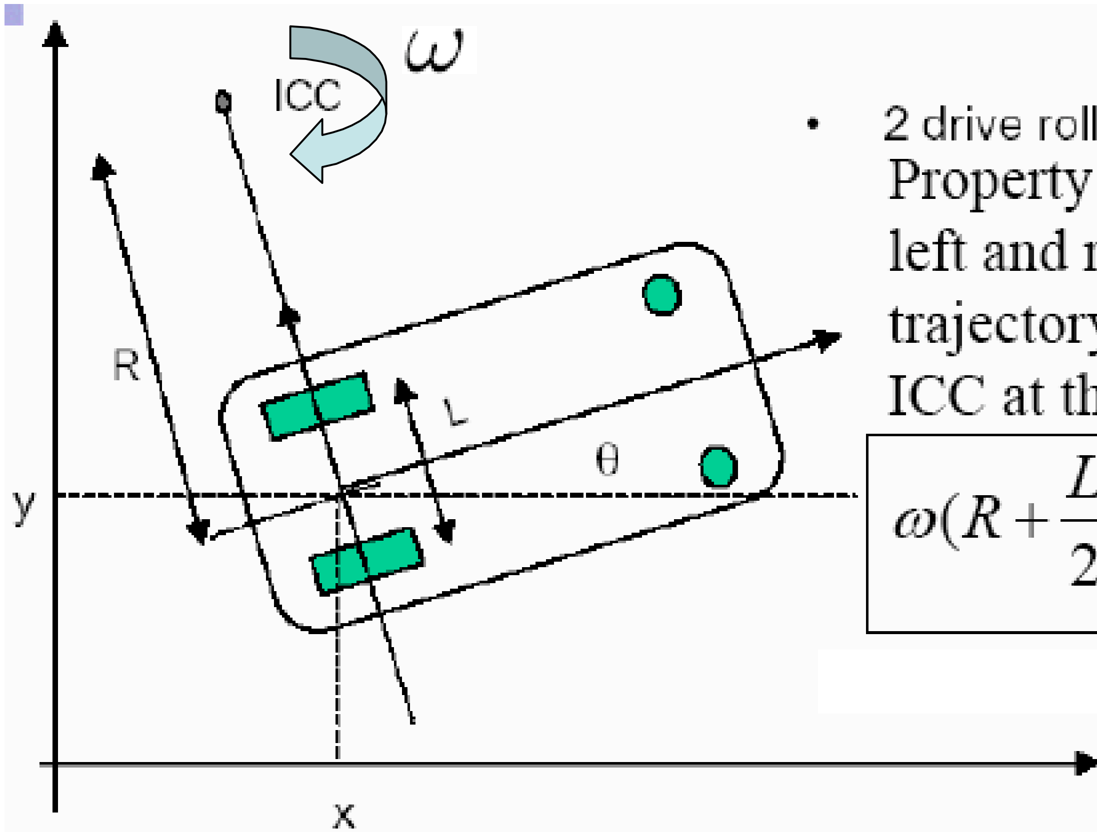




Differential Drive

$V_R(t)$ – linear velocity of right wheel
$V_L(t)$ – linear velocity of left wheel
r – nominal radius of each wheel
R – instantaneous curvature radius of the robot trajectory (distance from ICC to the midpoint between the two wheels).

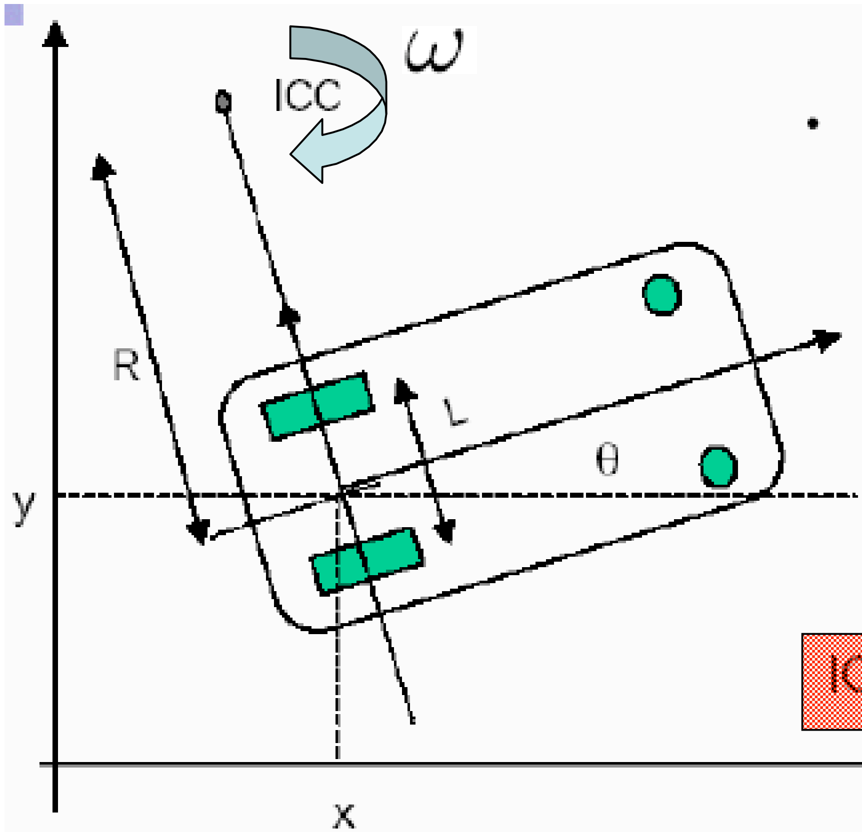




- 2 drive rolling wheels

Property: At each time instant, the left and right wheels must follow a trajectory that moves around the ICC at the same angular rate ω, i.e.,

$$\omega(R+\frac{L}{2})=V_R \qquad \omega(R-\frac{L}{2})=V_L$$



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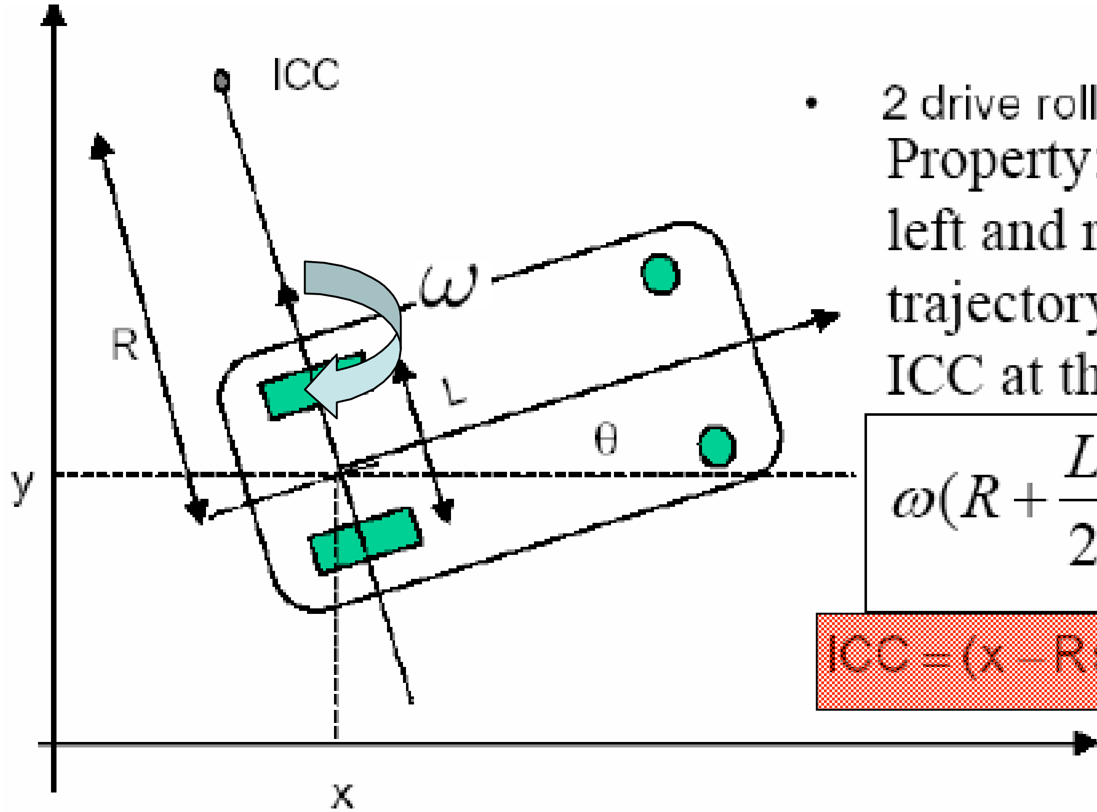
$ICC = (x - R\sin\theta, y + R\cos\theta)$



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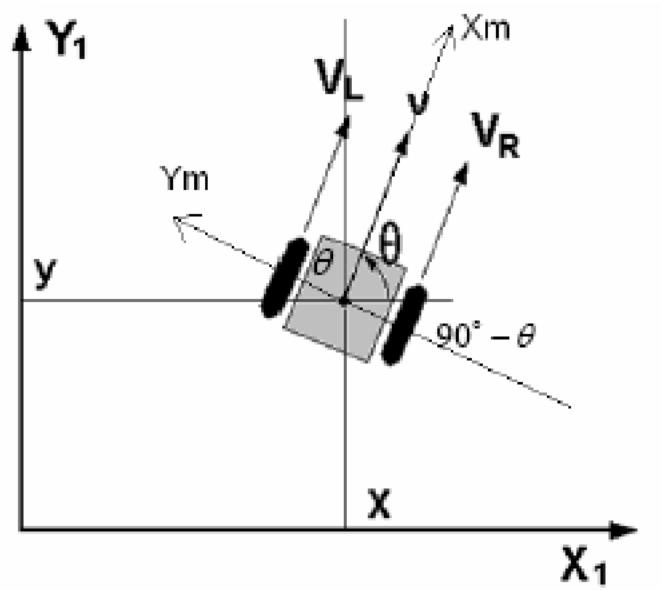
ICC = (x – R sinθ, y + R cosθ)

- Relation between the control input and speed of wheels

$$V_L = r\,\omega_L \qquad V_R = r\,\omega_R$$

$$\omega = \frac{V_R - V_L}{L}$$



Kinematic equation

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

Nonholonomic Constraint

$$\begin{bmatrix} \sin\theta & -\cos\theta \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \quad \dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

Differential Drive

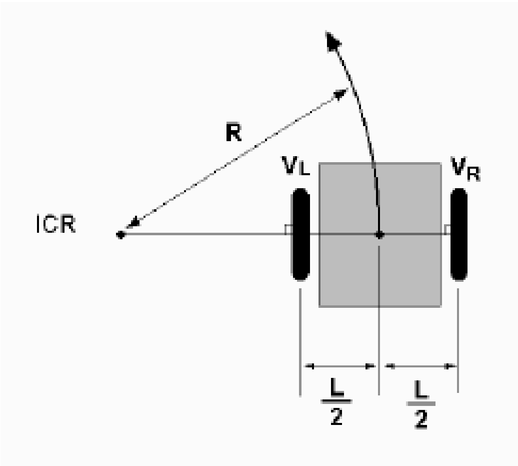
Kinematics model in robot frame
---configuration kinematics model

$$\begin{bmatrix} v_x(t) \\ v_y(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} w_l(t) \\ w_r(t) \end{bmatrix}$$

- $w_r(t)$ – angular velocity of right wheel
- $w_l(t)$ – angular velocity of left wheel

Basic Motion Control

- Instantaneous center of rotation



$$(V_R - V_L)/\ L \ = \ V_R\ /(R + \frac{L}{2})$$

$$R = \frac{L}{2}\ \frac{V_R + V_L}{V_R - V_L}$$

R : Radius of rotation

- Straight motion

  R = Infinity → $V_R = V_L$

- Rotational motion

  R = 0 → $V_R = -V_L$

Legged robot:

1) More legs the better(stable)
2) Exponential number of possible gaits

Wheeled robot:

1) Wheels can be moved in any directions
2) Maneuverability = mobility + steerability
3) Differential drive allows variable ICR