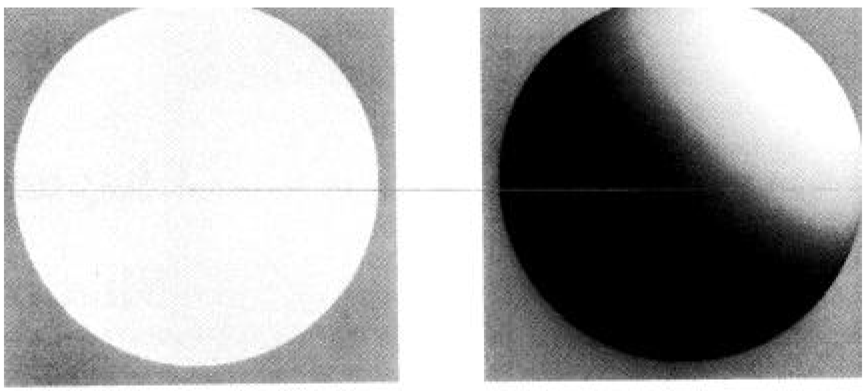
Shape from Shading

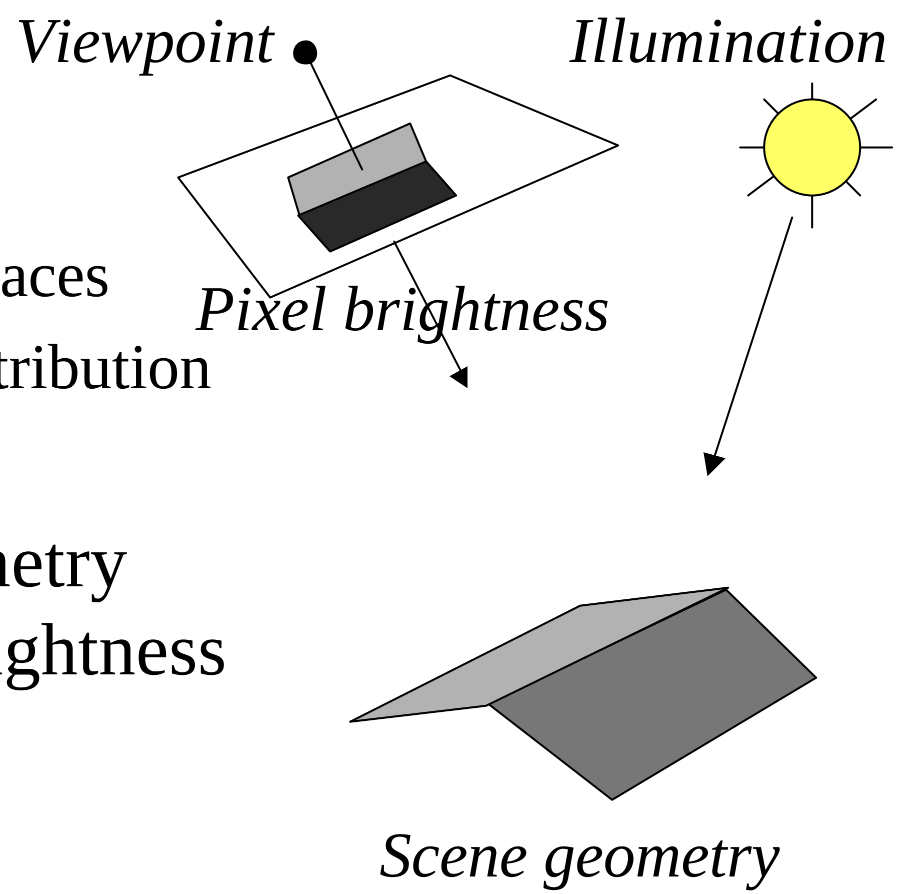
Perception of Shape from Shading

- Continuous image brightness variation due to shape variations is called *shading*
- Our perception of shape depends on shading
- Circular region on left is perceived as a flat disk
- Circular region on right has a varying brightness and is perceived as a sphere



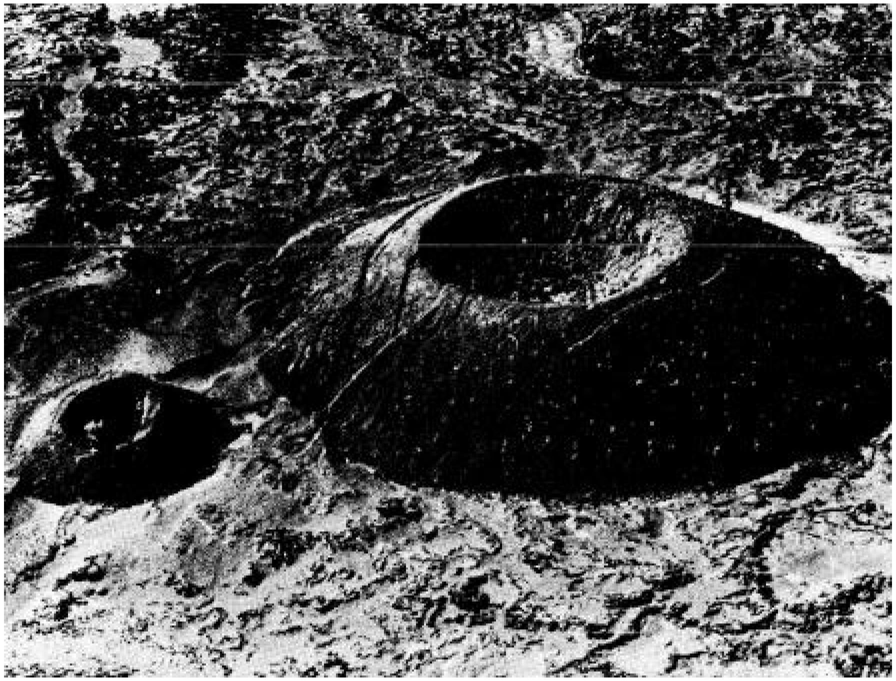
From Image to Shape

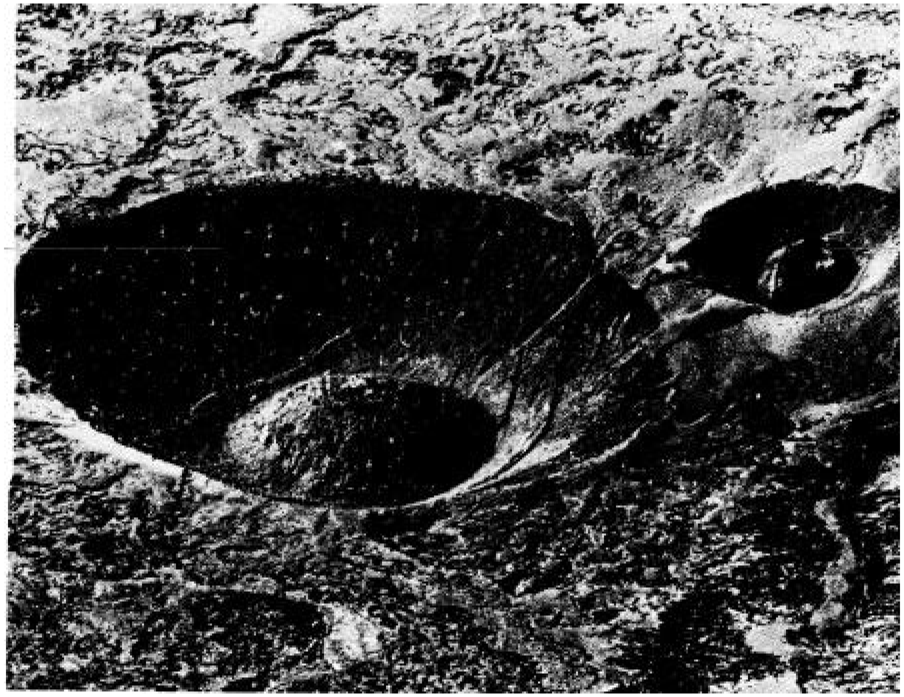
- Four main factors
  - Geometry of the scene
  - Reflectance of the visible surfaces
  - Illumination direction and distribution
  - Viewpoint
- Can we *compute* scene geometry from distribution of pixel brightness in scene image?
  - Only in very simple situations
  - Too many unknowns in general



How Do We Do It?

- Humans have to make assumptions about illumination: bump (left) is perceived as hole (right) when upside down

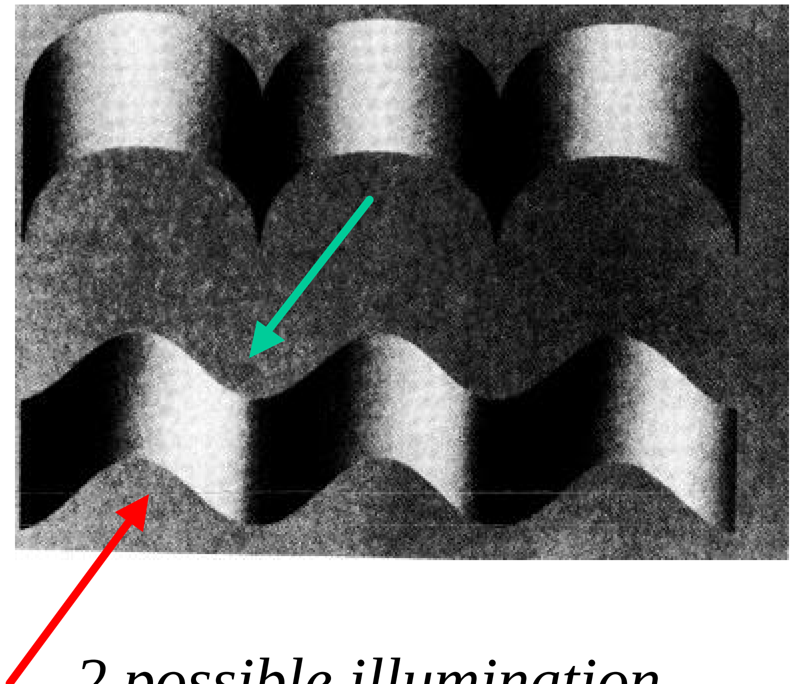




Illumination direction is unknown. It is assumed to come from above

Does Shading Play a Central Role?

- Contour plays a more important role
  - Variations in intensity are same on both shapes
  - Upper region is perceived as composed of three cylindrical pieces illuminated from above
  - Lower region is perceived as sinusoidal, illuminated from one side
    - Note the ambiguities of the surface perceptions, depending on assumed illumination direction

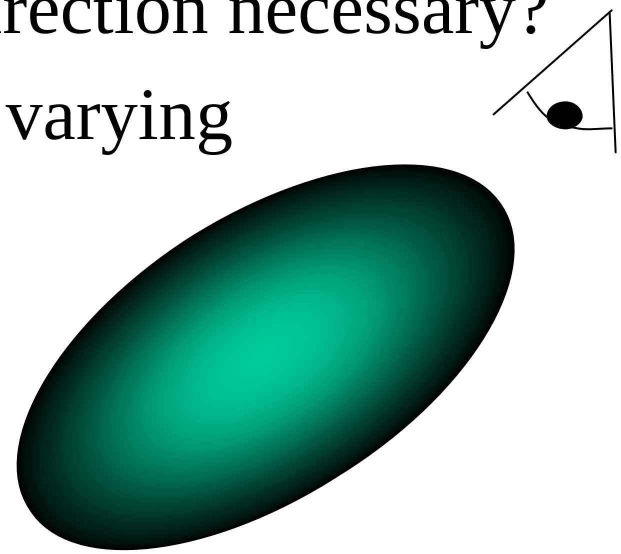


2 possible illumination hypotheses

Psychophysics

(Perception of Solid Shape from Shading, Mingolla & Todd, 1986)

- What assumptions do people make about surface reflectance?
- Is an estimate of illumination direction necessary?
- Stimuli: Shaded ellipsoids with varying
  - Elongations
  - Directions of light source
  - Reflectance
  - Cast shadows



- Test: *judge direction of light and surface orientations at discrete points*

Results

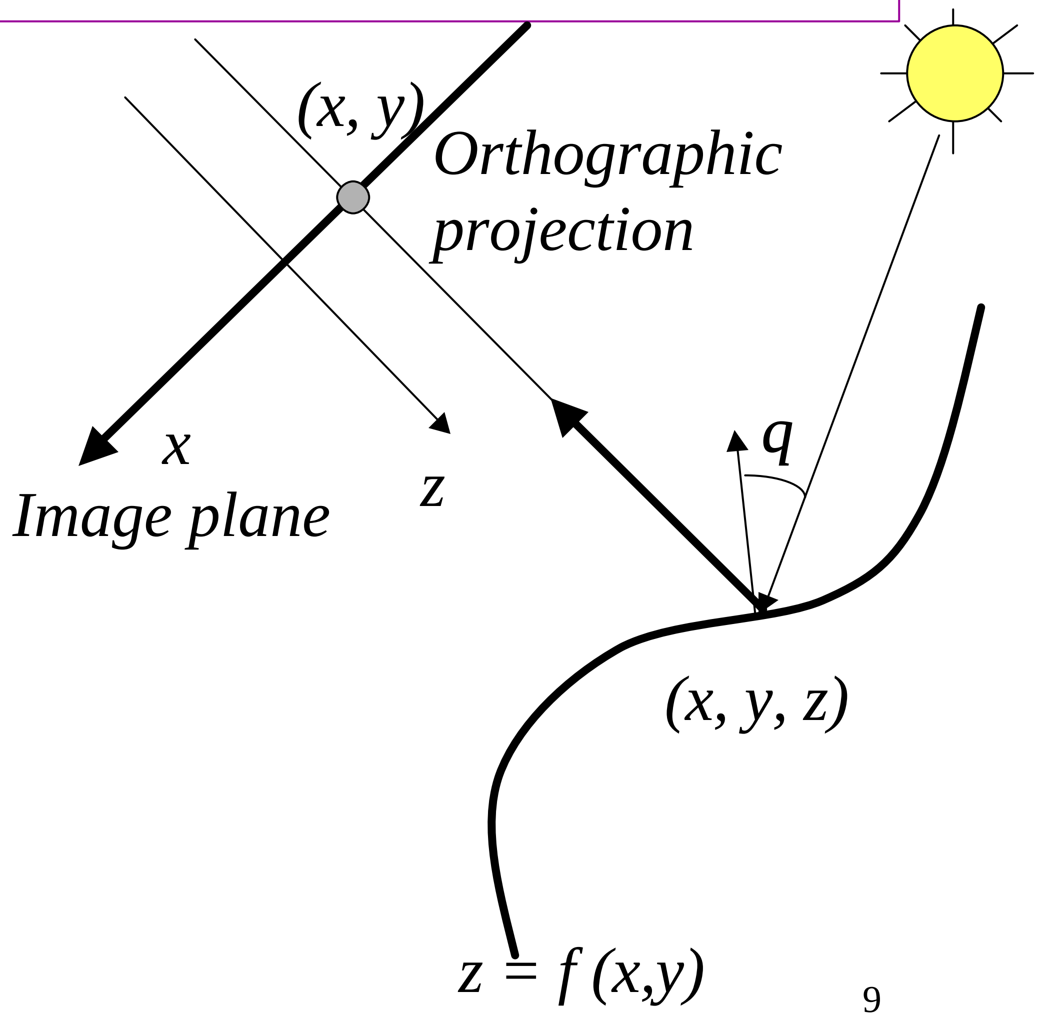
- Task is hard: errors 15 to 20 degrees
- No effect of glossiness, no Lambertian surface assumption
- No correlation between judgement of light directions and shape
  - No prior estimate of light direction
- Poor discrimination between elongated and rounded ellipsoids
  - Qualitative information

Human Shading Interpretation

- Is it metric or ordinal?
  - Metric: depth
  - Ordinal: depth order
- Answer:
  - Ordinal, qualitative
  - Magnitude of shading gradient is not important

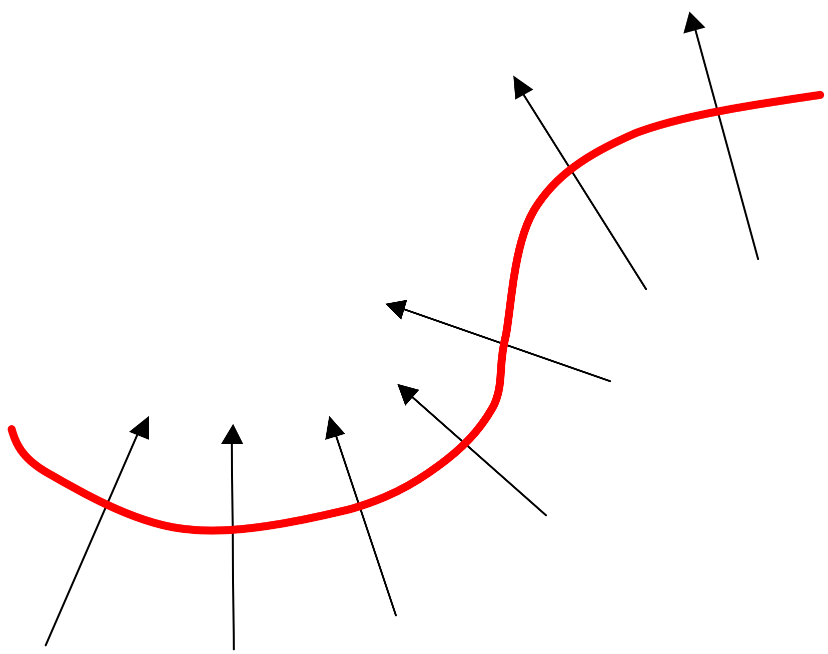
Quantitative Shape Recovery

- Orthographic projection
- We have gray levels at pixels (x, y)
- We want to recover the orientations of the normals at points (x, y, z)
- By integration, we want to obtain $z = f(x,y)$



From Normals to Surface Shape

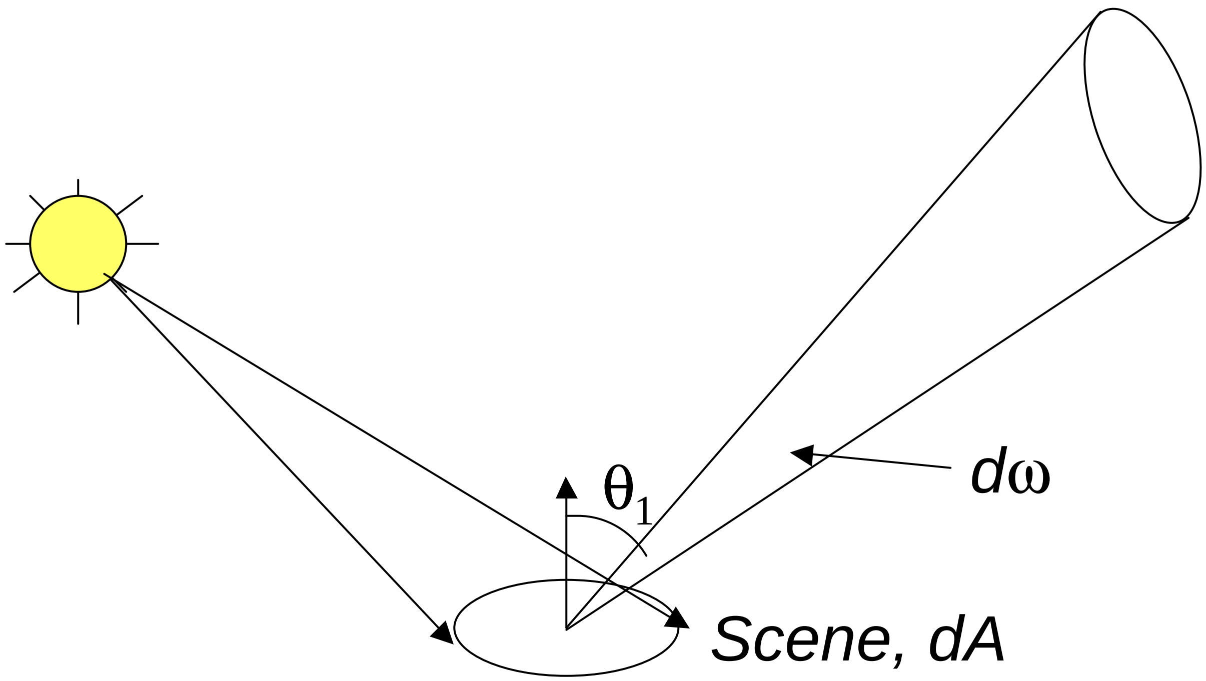
- Fit a surface that is locally perpendicular to the normals



Review: Radiance

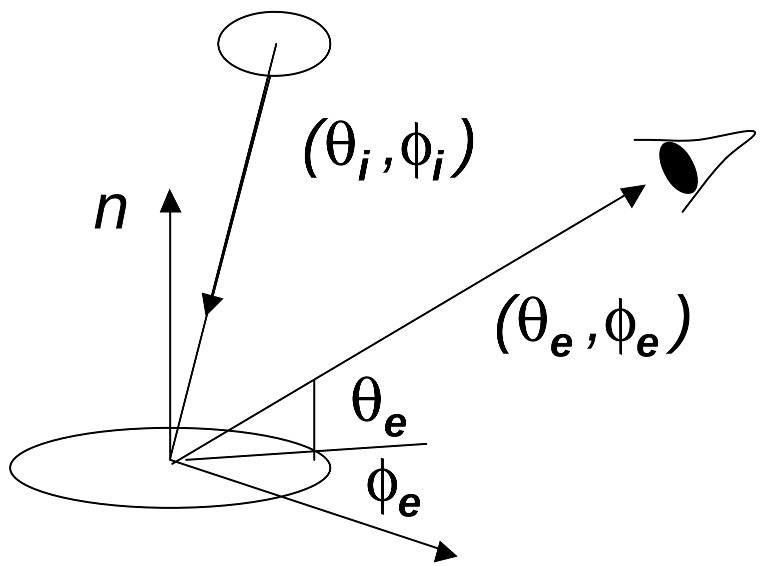
- Radiance $L\ (\theta_1)$ is power emitted per unit area (flux) into a cone having unit solid angle
  - Area used is foreshortened area in direction q_{1}

$$L\ (\theta_1) = d^2P / (dA \cos \theta_1\, d\omega)\ ,\ in\ W/m^2/sr$$



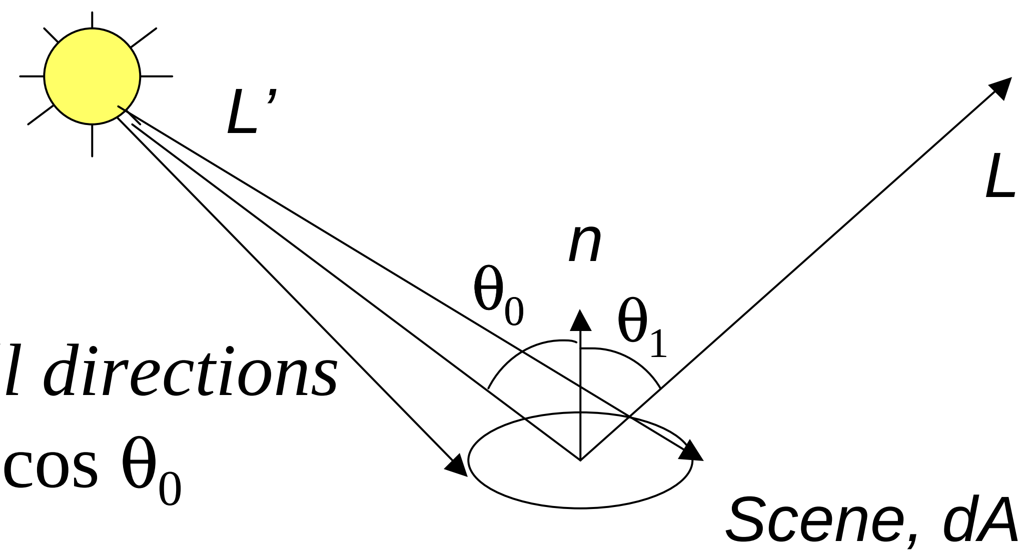
Review: Reflectance

- Reflection is characterized by reflectance
- Reflectance is ratio radiance/irradiance
- Described by a function called
  Bidirectional Reflectance Distribution Function BRDF
- BRDF = $f\,(\theta_i, \phi_i, \theta_e, \phi_e) = L\,(\theta_e, \phi_e) / dE(\theta_i, \phi_i)$



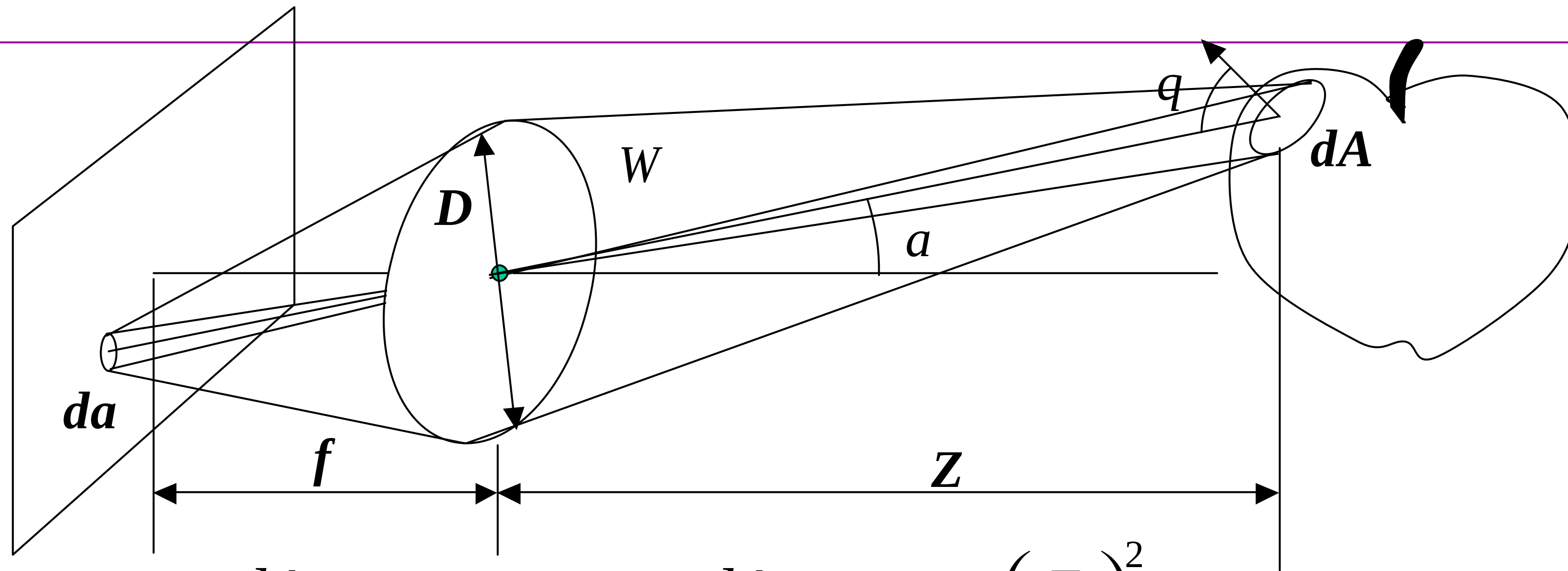
Review: *Lambertian Surfaces*

- If BRDF is a constant K, surface is called a Lambertian surface
- $dE = L' \cos\theta_0\, d\omega = k\, L' \cos\theta_0$
- $L = K\, dE = K_1\, L' \cos\theta_0$



- *Radiance is same in all directions and is proportional to* $\cos\theta_0$

Pixel Brightness and Scene Brightness



$$\frac{da\cos a}{(f/\cos a)^2} = \frac{dA\cos q}{(Z/\cos a)^2} \Longrightarrow \frac{dA}{da} = \frac{\cos a}{\cos q}\left(\frac{Z}{f}\right)^2$$

$$dP = L dA\,\Omega\cos q \Longrightarrow dP = L dA\frac{p}{4}\left(\frac{D}{Z}\right)^2\cos^3 a\cos q$$

$$E = \frac{dP}{da} = L\frac{dA}{da}\frac{p}{4}\left(\frac{D}{Z}\right)^2\cos^3 a\cos q \Longrightarrow E = \frac{p}{4}\left(\frac{D}{f}\right)^2\cos^4 a\; L$$

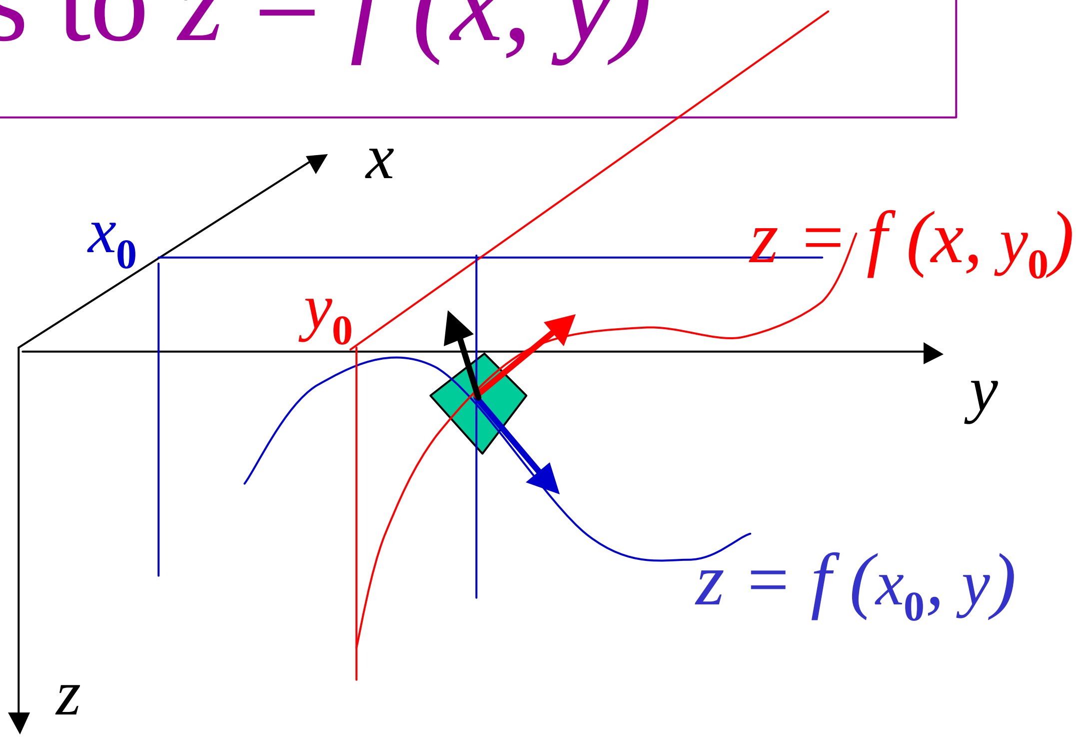
$$\boxed{E = k\,L}$$

Simple Radiometric Modeling

- Pixel Brightness is proportional to radiance of corresponding scene patch
- Radiance of scene patch is independent of viewpoint
- Radiance of scene patch is proportional to cosine of angle between normal to patch and direction of illumination source
- Therefore *pixel brightness is proportional to cosine of angle between normal to patch and direction of illumination source*

Normals to $z = f(x, y)$

- We intersect surface $z=f(x,y)$ with red plane and blue plane
- We find tangents to red curve and blue curve
- We write that normal is perpendicular to 2 tangents and is in direction of cross-product



- Red tangent $(1, 0, \partial f / \partial x)$
- Blue tangent $(0, 1, \partial f / \partial y)$
- Normal $(\partial f / \partial x, \partial f / \partial y, -1)$

Gradient Space

- Orientations of normal $(\partial f/\partial x, \partial f/\partial y, -1)$ can be represented by 2 parameters

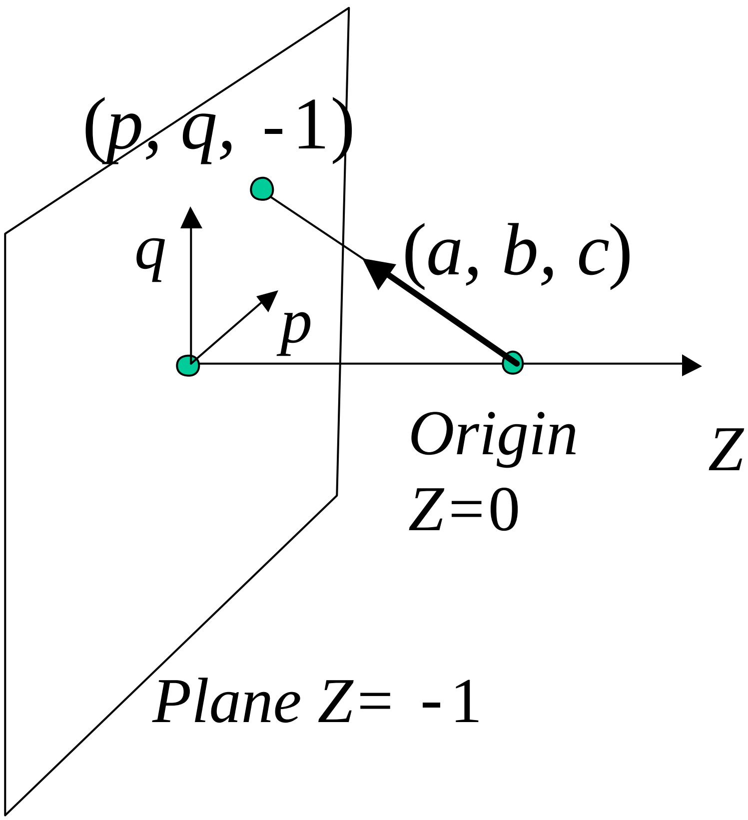
$$p = \partial f / dx$$

$$q = \partial f / dy$$

- The components p and q are called the *gradient space* coordinates of the normal
- Any direction (a, b, c) can be represented by $(-a/c, -b/c, -1)$, and by a point with 2 components ($p = -a/c$, $q = -b/c$) in the same 2D gradient space
  - Example: direction of light source can be written (p_s, q_s)

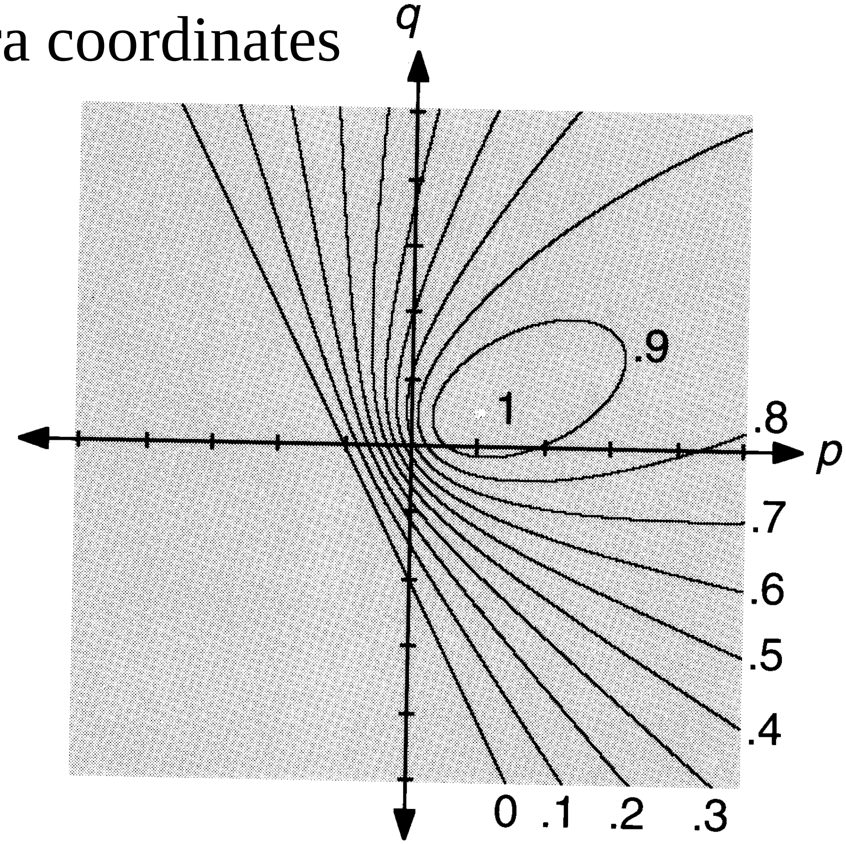
Geometric Interpretation of Gradient Space

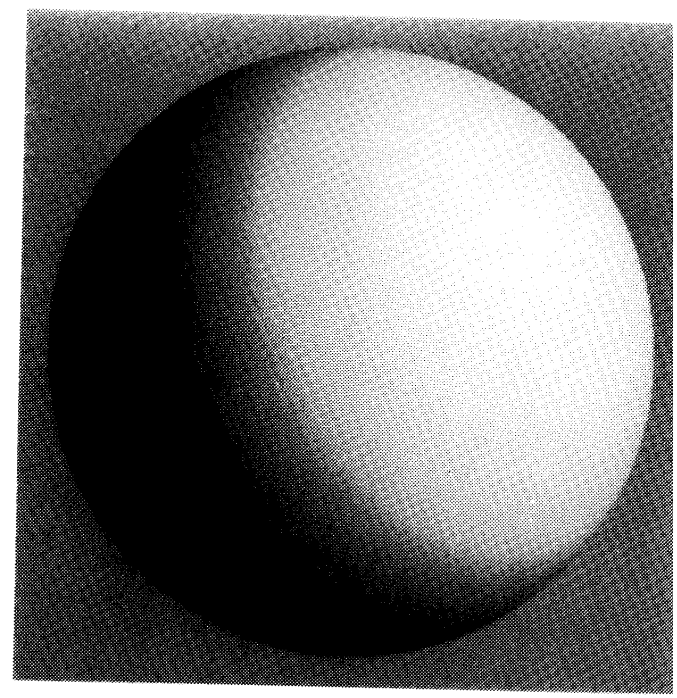
- A direction (a, b, c) can be represented by a point on the plane $Z = -1$ by constructing the intersection between the vector of same direction (drawn from the origin) and the plane



Reflectance Map

• A reflectance map is a 2D lookup table that gives the pixel brightness as a function of the orientation of the scene surface in camera coordinates





Reflectance Map for Point Light Source and Lambertian Surface

- Pixel brightness at pixel (*x*, *y*) is proportional to cosine of angle between normal to patch and direction of illumination source

$$I(x,y) = k\cos(q) = k\frac{(p_s, q_s, -1)}{\sqrt{p_s^{\,2} + q_s^{\,2} + 1}} \bullet \frac{(p\ ,q\ ,-1)}{\sqrt{p^{\,2} + q^{\,2} + 1}}$$

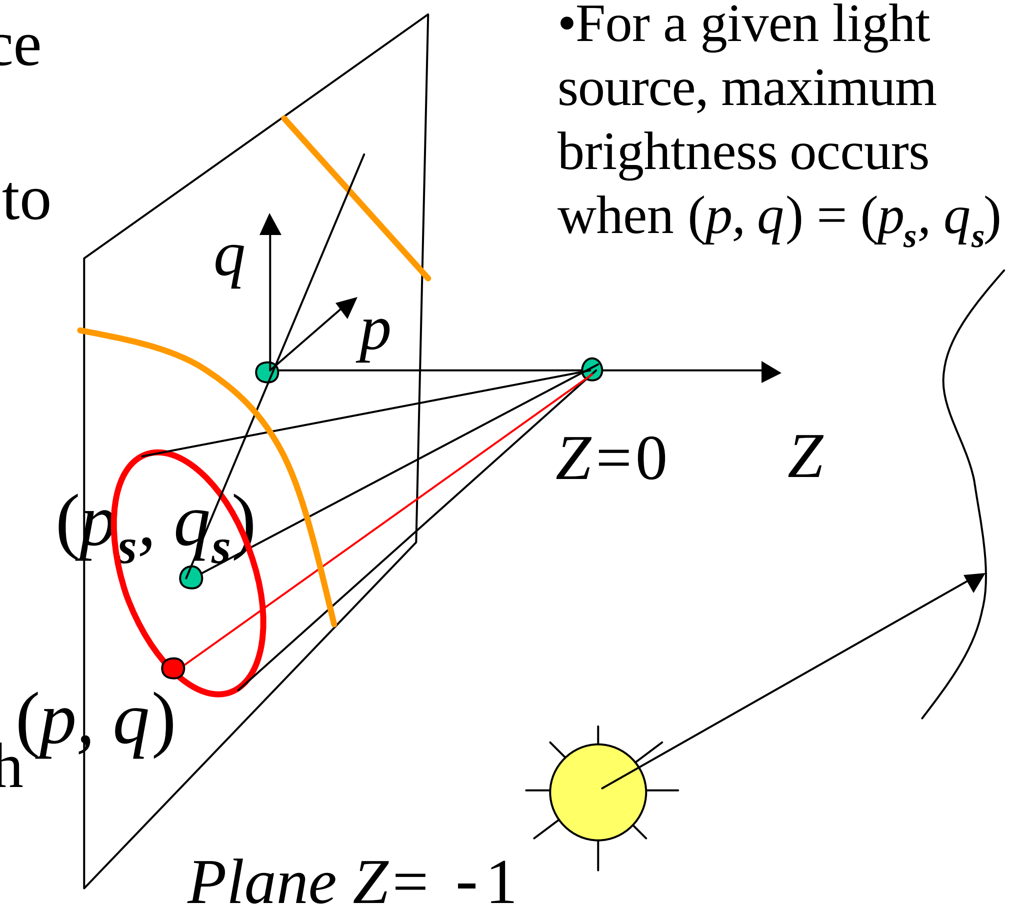
$$I(x,y)/k = k' = \frac{p_s p + q_s q + 1}{\sqrt{p_s^{\,2} + q_s^{\,2} + 1}\,\sqrt{p^{\,2} + q^{\,2} + 1}}$$

- For a given pixel brightness, the locus of possible normals (*p*,*q*) in gradient space is a conic

Locus of Iso-Brightness in Reflectance Map

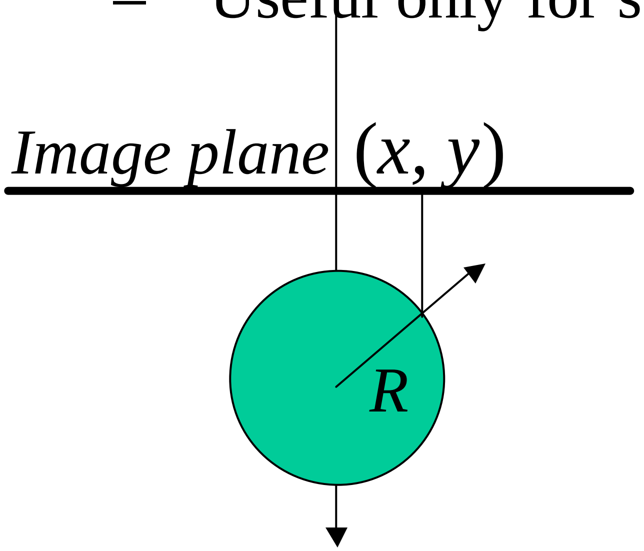
- Surface normals that produce a given brightness are at a constant angle with respect to direction of illumination
- The directions belong to a cone
- The locus corresponding to each brightness in the reflectance map is the intersection of the cone with the plane $Z = -1$

•For a given light source, maximum brightness occurs when $(p, q) = (p_s, q_s)$



Reflectance Map Obtained by Calibration Object

- A sphere can be used as a calibration object
  1. Find distance of pixel to center of sphere
  2. If distance < radius, compute direction of normal to sphere surface, and (p, q) for pixel
  3. At position (p, q) of reflectance map, store pixel value
     - Useful only for scene material similar to sphere

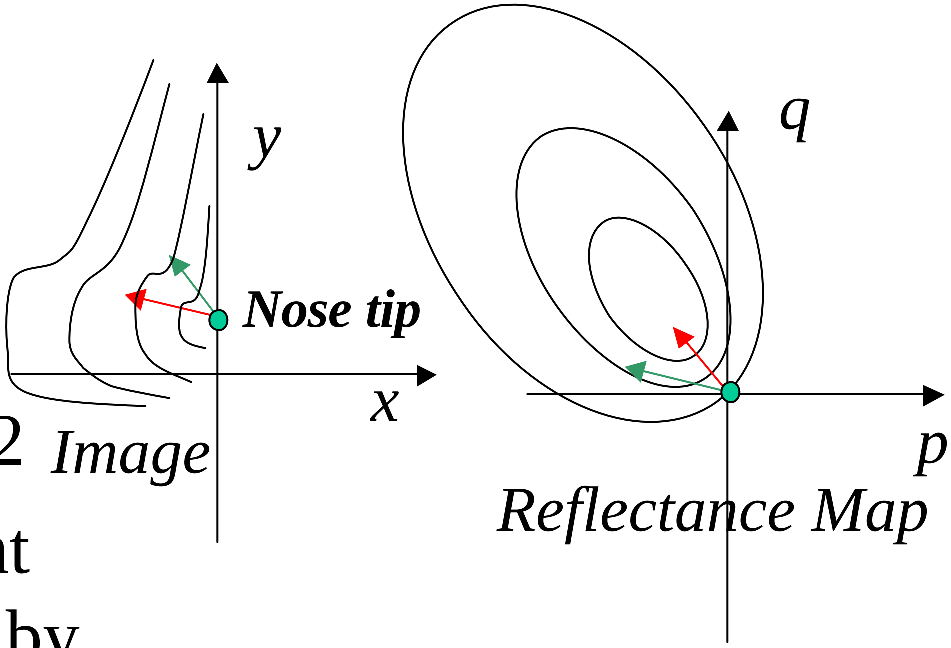


$$p = \frac{x}{\sqrt{R^2 - x^2 - y^2}},$$

$$q = \frac{y}{\sqrt{R^2 - x^2 - y^2}}$$

Using Reflectance Map to Find Normals

- We are on the image at a pixel where we know the direction of the normal, a point in the reflectance map
- Find Gradient 1 at pixel
- Find Gradient 2 at reflectance map point
- Move in image by Gradient 2
- Then the corresponding point in reflectance map is moved by Gradient 1



Proof

- Gradient 1 in image = $\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right)$
- Gradient 2 in reflectance map = $\left(\frac{\partial R}{\partial p}, \frac{\partial R}{\partial q}\right)$
- If (dx, dy) = Gradient 2, $dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy = \frac{\partial p}{\partial x} dx + \frac{\partial q}{\partial x} dy$

$$dp = \frac{\partial R}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial R}{\partial q}\frac{\partial q}{\partial x} = \frac{\partial I}{\partial x}$$

$$dq = \frac{\partial R}{\partial p}\frac{\partial p}{\partial y} + \frac{\partial R}{\partial q}\frac{\partial q}{\partial y} = \frac{\partial I}{\partial y}$$

- Then (dp, dq) = Gradient 1

From Normals to Surface Shape

- Step by step

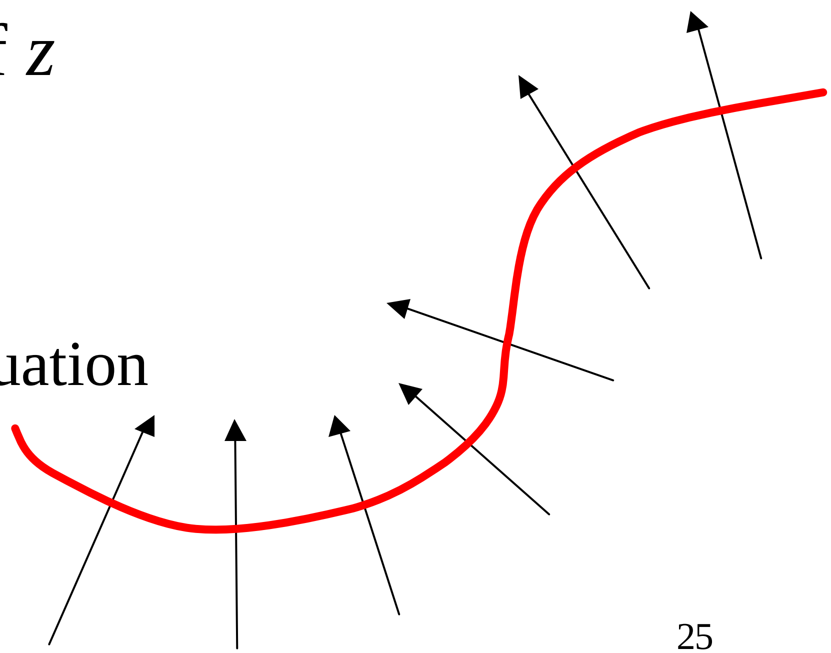
$$z(x+dx, y+dy) = z(x,y) + dz$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = p\,dx + q\,dy$$

- Global least square formulation leads to expression for Laplacian of *z*

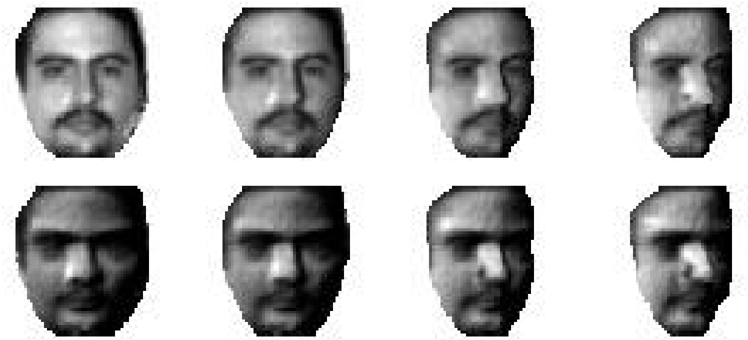
$$\Delta z = \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y}$$

  – Second order differential equation



Application to Face Recognition (Zhao and Chellappa)

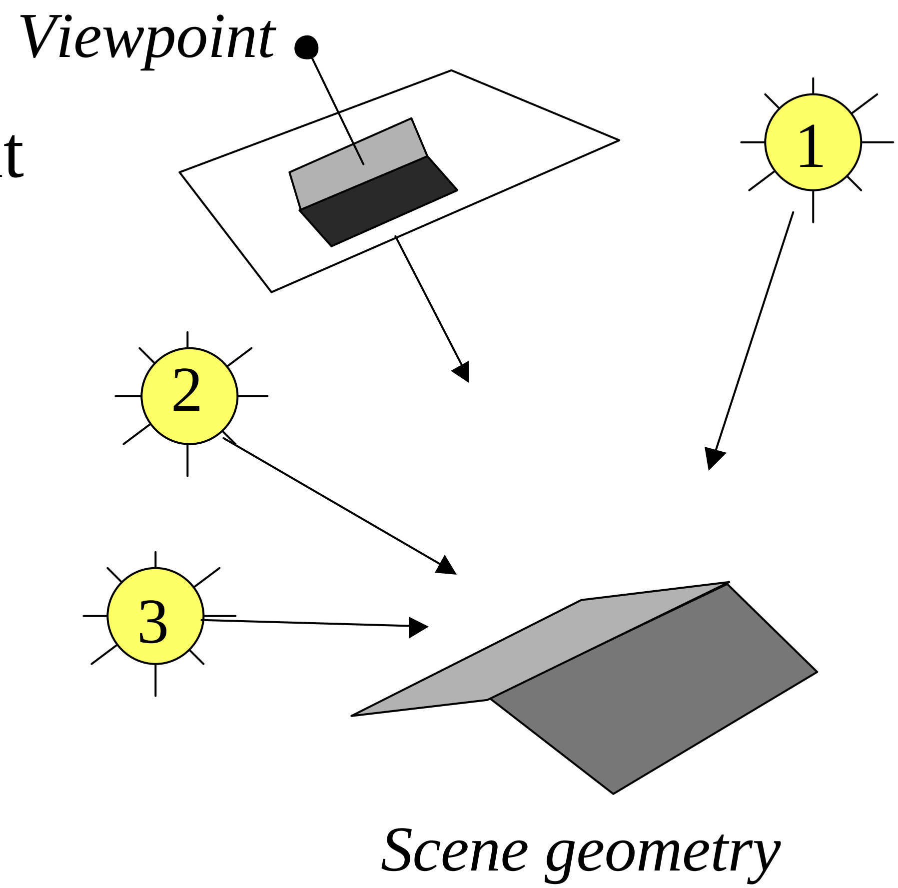
- Appearance of faces changes when viewing and lighting directions change
- Face databases use front views and frontal lighting
- If we can reconstruct 3D face shape, we can convert any face image into a front-view with frontal lighting and compare to the database faces
- Use shape from shading and symmetry of face
- Or assume generic shape, but varying albedo, and remove unknown albedo by using symmetry of face



Synthetic faces for 4 angles and 2 illuminations

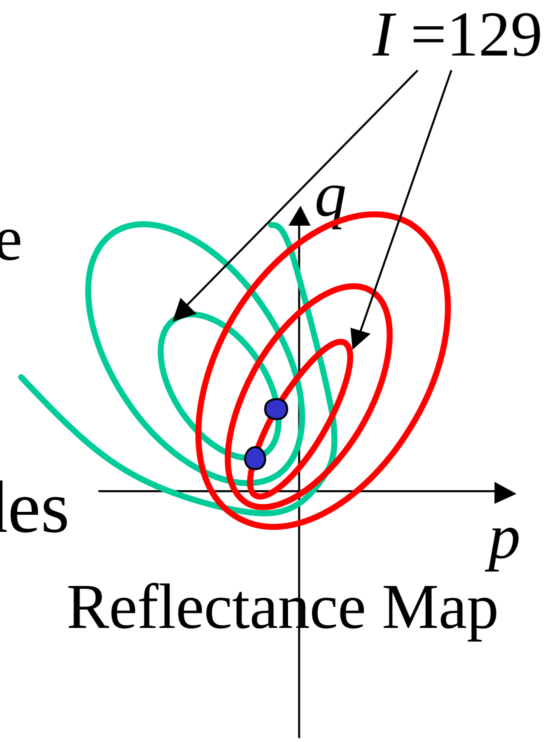
Photometric Stereo

- Move light source at different known positions to obtain different shadings of object with unknown geometry
- Find geometry from shading information



Photometric Stereo

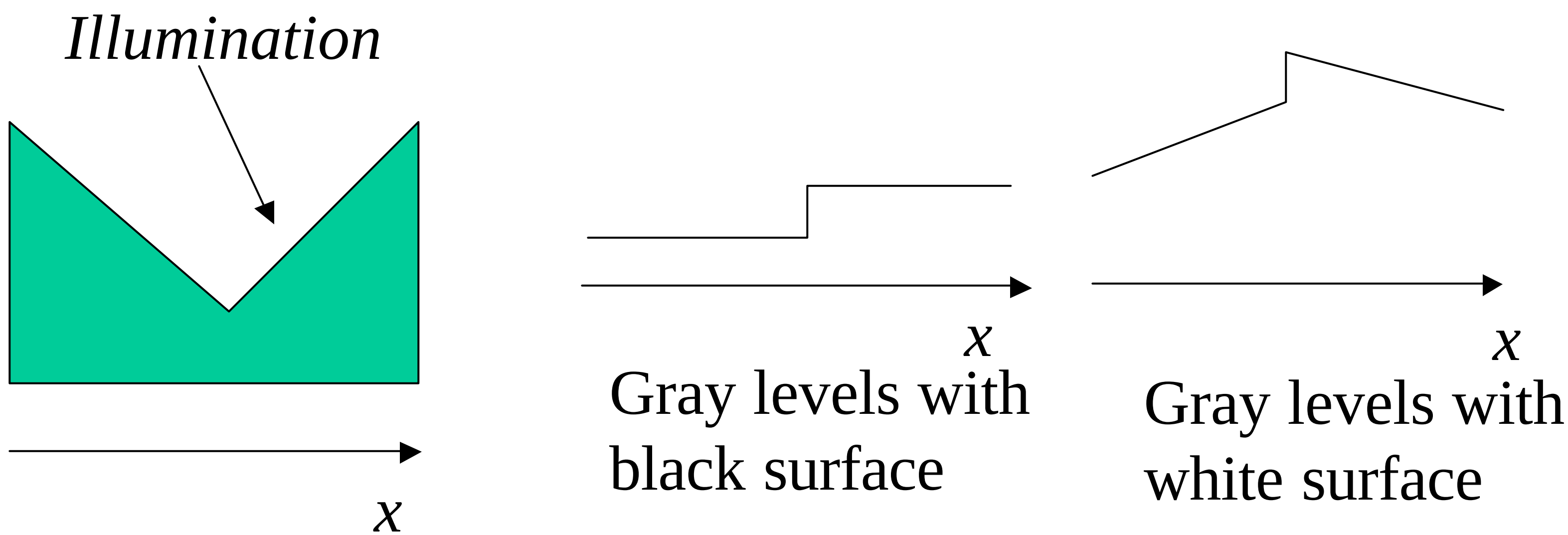
- Different illumination conditions lead to different reflectance maps
  - Each reflectance map can be computed if we know position of point light source
- Intersection of 2 iso-brightness contours corresponding to same brightness provides 2 possible normal directions for pixels having that brightness value
- Three maps give unambiguous normals for each pixel



Assumptions of Shape from Shading

- Surfaces with constant albedo
- Orthographic projection
- Distant point sources
- Absence of cast shadows
- **Insignificance of secondary illumination**
  - This one is a real problem: inter-reflections are everywhere

Inter-Reflections



- Accurate computation of shape from shading is unlikely to succeed in real world
- Shape from shading may be used as a complementary process
- Edges are more reliable indicators of shape

The Real World

- Diffuse light sources (overcast sky)
- Interreflections between surfaces generate secondary light sources
- Surfaces have varying light absorption (albedo)
- Surface reflections range from Lambertian to specular
- Surfaces cast shadows on each other

Conclusions

- Accurate computation of shape from shading is unlikely to succeed in the real world
- Edges are more reliable indicators of shape
- Shape from shading may be used as a complementary process in combination with shape inference from edges
- There is still a lot of research activity in this area, so it is useful to have an idea of the terminology and the techniques (reflectance map, etc.)

References

- A Guided Tour of Computer Vision, Vishvjit S. Nalwa, AT&T Press, 1993
- Robot Vision, B.K.P. Horn, MIT Press
- “Perceiving Shape from Shading”, V.S. Ramachandran, Scientific American, 1988, pp. 76-83
- “SFS Based View Synthesis for Robust Face Recognition”, W.Y. Zhao and R. Chellappa, www.cfar.umd.edu/~wyzhao