# Introduction to Machine Learning 

Machine Learning: Jordan Boyd-Graber University of Maryland<br>FEATURE ENGINEERING

Content Questions

## Quiz!

## Admin Questions

- Writeup must fit in one page
- Unit tests are not comprehensive
- Don't break autograder
- HW3 due next week


## PAC Learnability: Rectangles

Is the hypothesis class of axis-aligned rectangles PAC learnable?

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A. Blumer, A. Ehrenfeucht, D. Haussler, and M.K. Warmuth. Learnability and the Vapnik-Chervonenkis dimension. Journal of the ACM (JACM), 36(4):929?965, 1989

## What's the learning algorithm

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Call this $h_{S}$, which we learned from data. $h_{s} \in c$

## Proof

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We get a bad $h_{S}$ only if we have an observation fall in this region. So let's bound this probability.

## Bounds

$$
\begin{align*}
& \operatorname{Pr}[\text { error }]=\operatorname{Pr}\left[\mathrm{u}_{i=1}^{4} x \notin R_{i}\right]  \tag{1}\\
& \text { = }  \tag{2}\\
& \left.=\sum_{=1}^{(1-P(A)}\right)^{\prime} \tag{3}
\end{align*}
$$

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\operatorname{Pr}[\text { error }] & =\operatorname{Pr}\left[\uplus_{i=1}^{4} x \notin R_{i}\right]  \tag{1}\\
& \leq \sum_{i=1}^{4} \operatorname{Pr}\left[x \notin R_{i}\right]  \tag{2}\\
& =\sum_{i=1}^{4}\left(1-P\left(R_{i}\right)\right)^{m} \tag{3}
\end{align*}
$$

If we assume that $P\left(R_{i}\right) \geq \frac{\epsilon}{4}$, then

$$
\begin{equation*}
\operatorname{Pr}[\text { error }] \leq 4\left(1-\frac{\epsilon}{4}\right)^{m} \leq 4 \cdot \exp \left\{-\frac{m \epsilon}{4}\right\} \tag{4}
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Solving for $m$ gives

$$
\begin{equation*}
m \geq \frac{4 \ln 4 / \delta}{\epsilon} \tag{5}
\end{equation*}
$$

## Concept Learning

Are Boolean conjunctions PAC learnable? Think of every feature as a Boolean variable; in a given example the variable is given the value 1 if its corresponding feature appears in the examples and 0 otherwise. In this way, if the number of measured features is $n$ the concept is represented as a Boolean function $c:\{0,1\} \mapsto\{0,1\}$. For example we could define a chair as something that has four legs and you can sit on and is made of wood.
Can you learn such a conjunction concept over $n$ variables?

## Algorithm

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## Start with

$$
\begin{equation*}
h=\bar{x}_{1} x_{1} \bar{x}_{2} x_{2} \ldots \bar{x}_{n} x_{n} \tag{6}
\end{equation*}
$$

(say no to everything)

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- After first example, $x_{1} \bar{x}_{2} \bar{x}_{3} \bar{x}_{4} x_{5}$
- After last example, $x_{1} \overline{x_{3}} \bar{x}_{4}$


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- Having seen no data, $h$ says no to everything
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- Having seen no data, $h$ says no to everything
- Our algorithm can be too specific. It might not say yes when it should.
- We make an error on a literal if we've never seen it before (there are $2 n$ literals: $x_{1}, \bar{x}_{1}$ )


## Solving for number of examples

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General learning bounds for consistent hypotheses

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\begin{equation*}
m \geq \frac{1}{\epsilon}\left(\ln |H|+\ln \frac{1}{\delta}\right) \tag{7}
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& m \geq \frac{1}{\epsilon}\left(\ln |H|+\ln \frac{1}{\delta}\right)  \tag{7}\\
& m \geq \frac{1}{\epsilon}\left(n \cdot \ln 3+\ln \frac{1}{\delta}\right) \tag{8}
\end{align*}
$$

## 3-DNF

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Not efficiently learnable unless $P=N P$.

