

Introduction to Machine Learning

Machine Learning: Jordan Boyd-Graber University of Maryland FEATURE ENGINEERING

Content Questions

Quiz!

Admin Questions

- Writeup must fit in one page
- Unit tests are not comprehensive
- Don't break autograder
- HW3 due next week

PAC Learnability: Rectangles

Is the hypothesis class of axis-aligned rectangles PAC learnable?

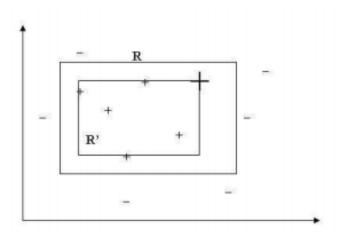
PAC Learnability: Rectangles

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A. Blumer, A. Ehrenfeucht, D. Haussler, and M.K. Warmuth. Learnability and the Vapnik-Chervonenkis dimension. Journal of the ACM (JACM), 36(4):929?965, 1989

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Call this h_S , which we learned from data. $h_s \in c$

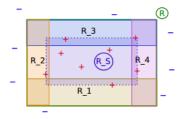
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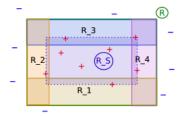
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We get a bad h_S only if we have an observation fall in this region. So let's bound this probability.

Bounds

$$Pr[error] = Pr[\bigcup_{i=1}^{4} x \notin R_i]$$

$$\leq \sum_{i=1}^{4} Pr[x \notin R_i]$$

$$= \sum_{i=1}^{4} (1 - P(R_i))^m$$
(3)

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(2)

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If we assume that $P(R_i) \ge \frac{\epsilon}{4}$, then

$$\Pr[error] \le 4 \left(1 - \frac{\epsilon}{4}\right)^m \le 4 \cdot \exp\left\{-\frac{m\epsilon}{4}\right\}$$
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Solving for *m* gives

$$m \ge \frac{4\ln 4/\delta}{\epsilon} \tag{5}$$

Concept Learning

Are Boolean conjunctions PAC learnable? Think of every feature as a Boolean variable; in a given example the variable is given the value 1 if its corresponding feature appears in the examples and 0 otherwise. In this way, if the number of measured features is *n* the concept is represented as a Boolean function $c : \{0, 1\} \mapsto \{0, 1\}$. For example we could define a chair as something that has four legs **and** you can sit on **and** is made of wood. Can you learn such a conjunction concept over *n* variables?

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$$h = \bar{x_1} x_1 \bar{x_2} x_2 \dots \bar{x_n} x_n \tag{6}$$

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- After last example, $x_1 \bar{x_3} \bar{x_4}$

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- We make an error on a literal if we've never seen it before (there are 2n literals: $x_1, \bar{x_1}$)

Solving for number of examples

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General learning bounds for consistent hypotheses

$$m \ge \frac{1}{\epsilon} \left(\ln |\mathcal{H}| + \ln \frac{1}{\delta} \right) \tag{7}$$

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$$m \ge \frac{1}{\epsilon} \left(\ln |H| + \ln \frac{1}{\delta} \right)$$

$$m \ge \frac{1}{\epsilon} \left(n \cdot \ln 3 + \ln \frac{1}{\delta} \right)$$
(8)

3-DNF

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Not efficiently learnable unless P = NP.